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Subject: Report on the scientific work of L. Hajdu

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The present dissertation consists of nine separate scientific papers, all of them devoted to diophantine equations. In particular they deal with representations for the product of (successive) elements of arithmetic progressions. A detailed introduction outlines the common feature of the 9 papers. Most of the papers are published in highly respected international journals, such as Acta Arith., Compositio Math., Proc. London Math. Soc., etc. The list of references in the introduction shows that in this area many leading number theorists from all over the world are working.

In a classical paper Erdös and Selfridge (1975) proved that the product of two or more consecutive positive integers is never a perfect power, i.e. the diophantine equation

$$X(X + 1) \ldots (x + k — 1) = y'$$

has no solution with $k, 1 > 2$ and $x > 1$. Following investigations of Saradha the present author considered a somewhat modified problem: the left hand side is replaced by the product of k successive elements of an arithmetic progression with difference d and the right hand side by by^e. In this case, solutions do exist and Saradha established an algorithm (depending on the greatest prime factor of b) to find all solutions. Based on a joint paper with Brindza and Ruzsa this algorithm is improved (joint work of L. Hajdu with L. Filakovszky). The method is based on establishing an equivalent system of elliptic equations and using the grogram package SIMATH for the resolution of such systems.

In the second paper of the dissertation (with K. Győry and k Pintér it is shown that for any positive integers x, d, k with $ged(x, d) = 1$ and $3 < k < 35$ the product of the k—term arithmetic progression with initial element x and difference d cannot be a perfect power. This is an improvement of the third paper (jointly with M.A. Bennett, N. Bruin and K. Győry), which only covered the case $k < 11$. In both papers the authors developed a new powerful approach. They use the Frey curve and Galois representations as well as algorithmic ideas and a computer search. This is definitely an important contribution.

The fourth paper of the dissertation (with Sz. Tengley and R. Tijdeman) is devoted to a similar problem. Euler proved that the product of four positive integers in an arithmetic progression is not a sequare. Győry showed that the product of three coprime positive integers in an arithmetic progression cannot be an 1-th power for $l > 3$. In the joint paper with Sz. Tengely and R. Tijdeman, L. Hajdu extended the range of Vs such that the product of k coprime integers in an arithmetic progression cannot be a cube when $2 < k < 39$. The proof is based on a classical paper of Selmer (1951) and new elementary ideas discussing divisibility by small prime factors.

In a further paper L. Hajdu obtains a result concerning the inhomogeneous case for perfect powers in arithmetic progressions. However, this result depends on the abc-conjecture. Applying very strong theorems of Faltings as well as of Darmon and Granville in a further paper N. Bruin, K. Győry, L. Hajdu and Sz. Tengely could obtain some new results about unlike powers in arithmetic progressions. Here geometric ideas and symbolic computation play an important role. Of special interest is a quite recent paper of L. Hajdu where he characterizes arithmetic progressions whose k—th term is a k—th power (for all k). By a result of Robertson (2000) such progressions have to be of length < 6 and L. Hajdu gives a complete solution of the problem. The paper heavily depends on elliptic curve algorithms.

M. Pohst asked the question if every prime can be written in the form 2' ± 3' with some non-negative integers u, v. L. Hajdu puts this problem into the framework of "diophantine properties of arithmetic progressions". Applying a deep theorem of Green and Tao (2006) he can give a negative answer to the question of Pohst. The final paper of the dissertation is a joint work of L. Hajdu with A. Bérczes and A. Pethő. It is devoted to arithmetic progressions in the solution of norm form equations and extends earlier results of Pethő and Ziegler. A crucial tool for the proof is the quantitative subspace theorem of Evertse, Schlickewei and Schmidt (Ann. Math. 2002).

Conclusion: This dissertation shows that L. Hajdu is able to obtain important new results in number theory. He applies a variety of modern techniques ranging from diophantine approximation and algebraic number theory to algorithmic tools and scientific software. Furthermore, all papers contain new original ideas. L. Hajdu is well known in the international scientific community. Thus I strongly recommend the acceptance of this dissertation.

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