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BOUNDED CIRCULAR DOMAINS AND THEIR JORDAN STRUCTURES
by *LÁSZLÓ L. STACHÓ*
Report by Professor Seán Dineen, University College Dublin

Introduction

Before going into details I would like to apologize for my delay in bringing this report to you, this was partially due to family circumstances and also to the fact that portions of this thesis are extremely technical and challenging. I was familiar with the topics in Chapters 2, 3, 4, 6, and 7 but even still the presentation was original and some results I knew, for instance the result in the short section 4.4, were given in a new way. I was unfamiliar with the material in Chapters 5 and 8 and after much effort found it interesting. The author's excellent overview of the material in Chapter 1 was an essential guide in leading me through the main body of the thesis. I have never previously written a report on a Hungarian doctoral thesis and I am writing the type of report that I would write for a thesis here in Ireland. I do no see any point in a more complete technical explanation of his results.

Recommendation

I have been aware of the mathematical work of Stacho as it unfolded over the last thirty years. I have always found his research original. Moreover, he is well regarded by the community of mathematicians, especially those who work in or close to his research area. It always added to my own understanding of the subject and now that I have the opportunity of reading this thesis I have got an overall impression of Stacho's contributions to mathematics. Stacho is very careful in attributing results to the correct sources and having searched the literature I can confirm that the results as presented in his thesis are his own original contributions to mathematics. I consider them substantial and ones that will stand the test of time. This is a good thesis, above the standard that would normally be, in my experience, required so my recommendation is strongly in favor of the doctoral degree being awarded to **LÁSZLÓ L. STACHÓ**. I outline my reasons in the following sections.

General Assessment

The research area of Stacho is different in a significant way to many other areas of mathematical research and I do believe that a preliminary explanation is in order. It is different because it draws on so many different area of mathematics and indeed certain motivations came from Physics (Jordan Algebras) and there is a continuing connection with Quantum Mechanics through the work of Upmeier (ref [102]).[1] These areas within mathematics include: *Banach Space Theory, C^* Algebras, Spectral Theory, Several Complex Variables*, (**Analysis**), *Complex Differential Geometry, Invariant metrics*, (**Geometry**), *Lie Groups, Lie Algebras, Jordan Algebras and Triples, Grids*, (**Algebra**) and *Differential Equations* and *Non-commutative Probability Theory*. All these areas are significant and researchers in different countries and with different backgrounds have brought different perspectives to the subject: e.g. Kaup and his students in Germany came from Several Complex Variables (the influence of Stein, Remmert, etc), Vigue, from France, came with the influence of H. Cartan, P.Lelong and M.Herve, there were others in the USA, Russo, Friedman, Harris (via Operator Theory and the Geometry of Banach spaces) and mathematicians in Spain, Ireland and Brazil were influenced mainly by Complex Analysis over Infinite Dimensional Spaces. Stacho studied in Pisa where I presume he was

[1] My references are to those given in the thesis.

influenced by the diverse research topics of E. Vesentini. All these make it quite difficult to enter or even appreciate the area. I have found that Stacho's approach required less background, than that needed for the work of say Kaup or Russo or Vigue. This for me was an advantage. However, it should be said that Stacho's methods are extremely technical and that a more polished presentation and a more suitable choice of notation would help both his readers and give his results a wider audience.

Stacho's joint book with Isidro is not only a good introduction to the study of known results but it contains useful[2] original material.

Complete Vector Fields

The topic of complete vector field arises in a number of the publications of Stacho and this is only natural as the Lie algebra of the Lie Group of all biholomorphic automorphisms of a bounded symmetric domain can be realised as the set of all complete holomorphic vector fields on the domain. The following is one of Stacho's main results. It contains one formulation of many of the concepts that have influenced his research over the years. It has also been generalized by Stacho and others and provides a bridge between the linear theory (that is Operator Theory, Banach Space Theory, Hilbert Spaces, Projections) and the holomorphic theory (Several Complex Variables, Invariant Metrics, Vector Fields). One could easily take one aspect of this result and spend a whole career fruitfully developing it. Stacho did not confine himself and he has gone on, over the last thirty years, to examine in great detail the algebraic, geometric and analytic potentialities inherent in this result.

Theorem 1 *If a domain D in a Banach space over $\mathbb{C}$ is equipped with a pseudometric locally equivalent to the norm so that all holomorphic self mappings are contractions and P is a holomorphic contraction then the projection of any semi-complete vector field is again semi-complete.*

[2] At least I could not find it elsewhere.

This result has many corollaries including one of my own showing that the bidual of a JB^* triple is also a JB^*. But also, as Stacho, has pointed out it has immediate applications showing that certain classical Banach spaces only admit linear biholomorphic self mappings of the unit ball. Moreover, its concrete application shows how closely related complete holomorphic vector fields are with numerical range and the classification of Hermitian operators on complex Banach spaces. Example 2.4.8 in the thesis is a very elaborate extension of this idea which first appeared in reference [77,79].

One Parameter Subgroups

In view of the natural connection between smooth, in some sense, vector fields and germs of one parameter groups it is rather natural that Stacho would turn his attention to one parameter semi-groups. Here again he was in a position to apply his projection principle. His main result here is a characterization, of the strongly continuous one parameter subgroups of $\mathbf{Aut}\big(\mathcal{L}(\mathbf{H}^1, \ldots, \mathbf{H}^2)\big)$, the automorphism group of set of n linear functionals where each $\mathbf{H}^i$ is a Hilbert Space. His proof is very very technical and involve concepts from a number of areas. Indeed it is a feature of his work in general that he does not hesitate to attack problems that are highly technical and that many others find very daunting. Hopefully some day, now that Stacho has found the results, less technical proof may be found.

Partial Triples

Again a natural development in the work of Stacho was to investigate Partial Jordan Triple Systems and Bounded Circular Doamins. Here he employs spectral theory, linear and quadratic complete holomorphic vector fields, other tools from topology and functional analysis, e.g. ultrafilters and non-commutative formal power series calculations, and methods from Lie Group and Lie Algebra theory and Differential Equations and once more his Projection Principle to attains his goals. Here again as elsewhere in his work he constructs very intricate and imaginative examples of low finite dimensional spaces to show, by counterexample, the natural boundaries to his results. His main result in this direction, a classification theorem, shows that geometric partial J^*, that is those defined by complete holomorphic vector fields, are linearly equivalent to a weakly commutative hermitian partial J^* triple. An unexpected biproduct is a new proof of Kaup's fundamental spectral estimate. His methods for this work are much more algebraic than previously but on this

Banach Stone type Theorems

Stacho proceeds to study Banach-Stone type theorems for complex lattice norms on $\mathcal{C}_0(\Omega)$ using Hermitian operators. This generalizes to continuous products the classical results about finite dimensional Reinhardt domains due to Thullen and Sunada ([97,98,99]) and which was extended to countable products in [2]. To obtain his results he first obtains a characterization of lattice norms on $\mathcal{C}_0(\Omega)$. He shows that these are essentially obtained from partitions $\mathcal{S}$of Ω into finite sets, on each of which lives a (finite dimensional) Hilbert space, that is if S is a set from the partition then there exists a (unique) inner product $\langle\cdot\rangle_S$ such that

$$\|f|_S = \sum_{s\in S}\alpha_s \mathbf{1}_s\|^2 = \langle f|_S, f|_S\rangle_S = \sum_{s\in S}\alpha_s\overline{\alpha_s} \text{ and } \|f\| = \sup_{S\in\mathcal{S}}\|f|_S\|_S.$$

The classification property is then obtained by reducing the problem to the each of these spaces and putting them together using, among other things, properties of Hermitian Operators. The details are non-trivial.

From the above Stacho proceeds in a logical fashion to examine the triple structure of Partial JB^* Triples and shows (see for Example 7.2.7) that it is the expected but complicated mixture of local Hilbert spaces with a global sup norm and furthermore he shows that the bidual inherits structure from the original partial triple structure.

Weighted Grids

This in to all intents complete Algebra but now involves tripotents and builds on the deep work summarized in [68] by Neher.

Conclusion

Stacho has proved original and significant mathematical results, he has produced a substantial body of work, the results claimed by him are definitely his. his results are widely known and appreciated by the international mathematical community. He should be awarded the doctoral degree.

Yours sincerely,

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