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Adaptive Control of Smooth Nonlinear Systems Based on Lucid Geometric Interpretation

By

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Abstract

The objective of this dissertation is to give a summary of a research work aiming at the use of *simple, geometrically well interpretable mathematical means in the adaptive control of partially and imperfectly modeled nonlinear systems*. Such systems may have dynamic coupling with hidden subsystems and may also be under a priori unknown external disturbances.

The novelty in this research consists in the fact that it did not want to proceed in the well established ruts of using Lyapunov functions. Lyapunov's 2nd or "direct" method seems to dominate contemporary nonlinear control worldwide. Though the fundamentals of this technique have lucid geometric interpretation, finding a proper Lyapunov function candidate for a given problem is a kind of "art". Furthermore, guaranteeing its non-positive time-derivative needs intricate mathematical manipulations that need great technical skills. Normally, these parts of the proofs take whole pages in the papers, and they usually result in special conditions that have to be met for the stability of the controllers. As it will be emphasized in this dissertation, the so obtained controllers may contain too much more or less arbitrary parameters. Furthermore, they do not result in optimal tuning. Certain adaptive solutions that try to exactly learn the analytical model of the system under control are vulnerable by the effects of unknown external disturbances and hidden, coupled subsystems.

To avoid the difficulties related to the application of Lyapunov's 2nd method I tried to utilize very simple and lucid geometric structures and convergent iterations obtained from contractive maps to construct adaptive controllers.

The basic philosophy of this approach is similar to that of the prevailing soft computing techniques. However, it does not apply the typical uniform structures of the modern soft computing that are related to Kolmogorov's approximation theorem proved in 1957. *Instead approximating continuous functions my approach approximates a far better behaving set of smooth functions by utilizing uniform structures of small sizes taken from various Lie groups.*

After giving a brief historical review on the advantages of "geometric way of thinking" the *Computed Torque Control* in Robotics and Lyapunov's 2nd Method in general and its illustrative applications in Robotics are critically studied and modified. Following that the subject area of soft computing as a special application of universal approximators is critically studied. The emphasis is on the sizing and scalability problems that generate difficulties in parameter tuning.

Instead using "universal approximators" various special elements of special Lie groups are suggested to the realization of partial, temporal, and situation-dependent system identification. The first approach is based on the phenomenological basis of Classical Mechanics in the control of Classical Mechanical Systems. The second one uses these structures at higher level of abstraction. *It is shown that these structures have limited number of tuneable parameters and they can be used for the approximation of the observed behavior of the system under control.*

In the next research phase various parametric Fixed Point Transformations were proposed for adaptive control to further release the problem of the complexity of system-identification. The geometric interpretation of the Singular Value Decomposition (SVD) of real matrices is also utilized in these approaches. *In contrast to Lyapunov's 2nd method that normally guarantees global stability of the control, in the new approach the convergence of the iteration that is necessary for stable control is guaranteed within a local region. However, it is shown that in many*

cases the basin of convergence is wide enough for practical applicability of the proposed novel methods. Furthermore, it is shown that the novel approach using "Robust Fixed Point Transformations" can be completed by various parameter tuning methods that are able to keep the controller nearby the center of the basin of attraction of the necessary iteration. This approach works only with three adaptive control parameters of which only one parameter has to be tuned. It also is shown that this tuning is not drastically coupled with the dynamics of the tuning-free controller. It slightly affects the speed of convergence of the iteration and the tracking precision of the tuned adaptive controller.

It also is shown that by the use of this novel adaptive approach a new branch of the "*Model Reference Adaptive Controllers*" can be developed in the design of which the Lyapunov function can be replaced by the simple Robust Fixed Point Transformations.

Finally, *a simple parametric numerical approximation of Caputo's fractional order derivatives is presented and applied in nonlinear control for smoothing purposes*.

The dissertation contains an "*Appendix*" that summarizing the most important geometric and group theoretical analogies that are utilized in the Thesis. To maintain the page limitation formally prescribed for the "core" material certain mathematical details and numerical computational results are presented in the Appendix, too.

The dissertation separately contains the author's own publications strongly related to the results given in the thesis, and the "References" that refer to other researchers' results and own publications that are not so strictly related to results of the present Thesis.

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Preliminary Remarks

In the present dissertation the results of research efforts of many years are summarized. Certain less elaborated and completed achievements are referred to as "antecedents", while the more matured and crystallized ones are included in the "theses". Therefore certain cited works in which I am a co-author myself occur amongst the "*References*" denoted by the "prefix" "R" in the numbered lists. The results that more strictly belong to the theses of this dissertation are marked by prefix "B" if they are book excerpts, prefix "J" if they are journal publications, and by prefix "C" if they were published in conference or workshop proceedings. The citations are arranged according to their first appearance in the Thesis. (The so obtained sequence considerably differs from the chronological one.)

The diversity and variety of "ad hoc" notations in the various publications related to these results do not justify any effort for developing a unified "system of notations" that is valid for the whole dissertation. Instead of that I tried to develop a consistent system of notations within each chapter only.

Since the dissertation partly is built on the use of more or less well known mathematical theorems, I give only the proofs of those ones that have significant details from the point of view of the present dissertation. The other fundamental statements are cited or referred to without their proofs.

The present subject area of control technology has a huge literature on the *linear methodologies* that are mainly useful for *linear systems*. Similar considerations or even notations frequently occur in control applications developed for *nonlinear systems*, too. It was not my aim to make any survey on these methods. I concentrated mainly on *smooth nonlinear systems in which certain non-smooth nonlinearities (e.g. friction) may also be present*. On this reason I mention and analyze in details only certain fundamental methods that are relevant for this dissertation, for making comparisons only.

The "comparative analysis" in this context can be understood in a very cautious manner. Since as alternatives to *Analytical Modeling (AM) Soft Computing (SC)* approaches based on various universal approximators having a huge number of parameters came into use in our days any effort for obtaining simple and decisive statement as e.g. "*method A is superior to method B*" seems to lose its sense. For instance in the field of *Evolutionary Computation (EC)* in which attempts are made for efficient setting of a huge number of parameters similar conditions prevail: "*A broad spectrum of representation techniques makes new results in EC almost incomparable. Sentences like 'This experiment was repeated ten times to obtain significant results' or 'We have proven that algorithm A is better than algorithm B' can still be found in current EC publications. …Evolutionary Computation shares these problems with other scientific disciplines such as simulation, artificial intelligence, numerical analysis, or industrial optimization.*" [R2], [R3]. In connection with such statements Eiben and Jelasity listed four typical problems as *a) the lack of standardized test-functions or benchmark problems, b) the usage of different performance measures, c) the impreciseness of results, and therefore no clearly specified conclusions, and d) the lack of reproducibility of experiments* especially when stochastic elements are applied in the methods [R4].

I definitely would like to evade such errors so in the comparisons I restrict myself only to certain fundamental points as "*simplicity*", "*lucidity*", "*reduced computational burden*", and "*simple realizability*", "*scalability*", "*smoothness of the results*". I have also been content with giving the relevant mathematical proofs and providing illustrative numerical simulations to exemplify the potential applicability of the novel control methods proposed in the dissertation. The particular examples used in these "illustrations" can also serve as "*typical paradigms*" of classes of physical systems for the control of which the novel approaches can be proposed.

Chapter 1: The Aims of the Dissertation

The main goal of the research efforts partly summarized in the present dissertation was finding simple, geometrically interpreted adaptive methods in the control of partially modeled and/or imprecisely known nonlinear physical systems. The conditions prevailing in the relatively small segment of control technology that I was able to study urged me to step ahead in this direction. More specifically the following observations gave the most important impetus:

- A typical class of control papers tackles the problems on the basis of the use of classical analytical models of the physical systems to be controlled. The main deficiency of such approaches is that they have very limited circle of applications: a detailed analytical model is valid only for a particular system. In analytical models quite little numerical contributions sometimes can be obtained by huge computational efforts. (A typical example is. the increasing order of the contributions in perturbation calculus.) In many cases it is very difficult or even impossible to identify the parameters of the analytical models of the systems as e.g. robots [R5], [R6]. For instance, in coding the precise dynamic model of a 6 Degree of Freedom (DOF) PUMA robot three persons worked for 5 weeks [R7]. In various publications the measured parameters of PUMA robot has considerable diversities, too [R8]. Identification of other parameters as that of a friction model is not very easy, too [R9], [R10].
- Even adaptive approaches that are based on some analytical model utilize very special properties of certain matrices as e.g. *Slotine's and Li's adative robot control* [R11], and assume the lack of unknown external perturbations and coupled hidden subsystems. The model-based approaches (e.g. the *Adaptive Inverse Dynamics*) also assume that the external disturbances are zeros, or at least temporal and almost negligible.
- The great majority of the control papers use *Lyapunov's ingenious 2nd Method* that itself has a lucid geometric interpretation, too. However, its application is not too easy, needs lot of invention in forming the candidate functions, and frequently leads to the introduction of ample number of almost arbitrary control parameters (for details see e.g. [C106]). My definite aim was to find far simpler methods that can guarantee the stability of the new control methods elaborated.
- Other popular and modern approaches instead of the analytical models use various means of *Soft Computing* that correspond to the "hidden application" of universal approximators [R12] being either *Artificial Neural Networks (ANN)* [R13] or *Fuzzy Systems (FS)* [R14]. Essentially the same can be stated for the use of *Tensor Product Models* [R15], [R49]. As it will be discussed later such "universal models" may have a huge number of parameters, suffer from *bad scalability* ("*curse of dimensionality*") and setting their parameters needs considerable computational efforts.

In spite of the difficulties of the traditional SC approaches their important features as "uniformity" of the model structures and the parameter tuning/setting procedures *remained an attractive property. It has challenged me to construct similar approaches that are free of the scalability problems or the curse of dimensionality.*

A search for the cause of scalability problem revealed that the problem roots in the fact that Kolmogorov's approximation theorem [R16] is valid for the very wide class of *continuous functions* that contains even very "*extreme*" elements at least from the point of view of the technical applications. (The first example of a function that everywhere is continuous but nowhere is differentiable was given by Weierstraß in 1872 [R17] on the inspiration by Riemann who formerly failed with constructing such a function [R18].) Intuitively it was expected that restricting our models to the far better behaving "*everywhere differentiable*" functions the problems with the dimensionality *ab ovo* could be evaded or at least reduced. It was also assumed that such a problem class is still wide enough for practical technical applications.

Later it was understood that other resource of complexity was the unnecessary effort for developing "*complete*", "*everlasting*", "*everywhere applicable*" models of the system to be controlled. In principle such efforts are correct and can be understood since the so obtained models (being expressed either by analytically or by the use of the means of universal approximators) can be inserted and used in various control and application environments. *However, if we restrict ourselves to the use of uniform structures determined by the degree of freedom of the "modeled part" of the whole system then simple model structures can be obtained that may be satisfactory for developing "partial", "temporal", and "situation-dependent" models. Such models need continuous maintenance. In this manner a significant source of complexity can be eliminated. In this case the cost of complexity reduction is the continuous need of observing the behavior of the system under control.*

In contrast to the traditional ideas the novel approaches partly use Lie groups the size of which is determined by the number of the modeled/directly controlled *Degrees of Freedom (DOF)* of the system. Therefore the number of the independent parameters is determined by the linearly independent generators of the Lie group chosen. Consequently this number is relatively very small and allows the use of simple tuning/setting procedures.

In the sequel, following the section in which the scientific methods of the research are summarized, in connection with the "*antecedents*" as well as the appropriate theses these solutions will be detailed together with the appropriate "ancillary" algebraic and group theoretical considerations.

Chapter 2: On the Scientific Methods of the Research

In the field of noninear control two typical methodologies can be chosen.

A typical possibility is assuming "ideal controllers and sensors" of extremely fast response. In this case the equations of motion of the controlled system can mathematically be approximated by a set of differential equations. A considerable segment of the control literature using Lyapunov's direct method (e.g. [R11]) proceeds along this rut. However, *it must be emphasised that the great majority of the practical problems results in differential equations that do not have solutions in closed analytical form. If we wish to see numerical details on the operation of the controllers the stability of which has been mathematically proved we have to develop numerical simulations.*

To achieve more realistic results it is expedient to take into account the limitations of our digital controllers and sensors of finite time-resolution. In this case the system originally described by differential equations must be completed by the insertion of *event clocks* and *sample holders* that represent the "cyclic" nature of the controllers. In this manner the "cycle time of the controller" can be distinguished from the time-resolution of the numerical simulations.

It must be emphasized that besides the discrete time-resolution applied various numerical simulators may apply different numerical integration methods and also allow setting certain numerical parameters that evidently concern the "results" of the numerical simulations. In the lights of the "believabilty considerations" expounded in the sequel I applied the following methods.

As the simplest and fastest approach, by the use of INRIA's SCILAB programming environment I developed numerical programs applying simple Euler integration with fixed time resolution. It was found that for stable control rough approximate results can be obtained for making the assumed cycle time of the controller (1 ms) identical with the time-resolution of the numerical integration. For checking consistency this time step was halved and if the results did not show significant modification they have been accepted for illustrating the operation of the proposed controller.

A further step towards more reliable results the fixed time-resolution was distinguished from the controller's cycle time and a control cycle was divided into 10 segments for numerical simulations. For such calculations I used the same simple SCILAB program language.

To make more professional simulations I applied the SCILAB's numerical co-simulator, SCICOS, that gave a convenient graphical interface for calling more professional numerical integrators. For simulating the discrete nature of digital controllers sample holders and event clocks were built in these simulations.

Another aspect concerning the methodology of research is the fact that the question of "*believability of the numerical results*" arises in each of the above mentioned numerical solutions. Following the pioneering work by Lorenz who made numerical computations on simple meteorological model of Earth using the computer technology of the sixties it became evident that there are "*stable*" and "*unstable*" systems in which the consequences of the initial errors remain finite or grow exponentially with time, respectively [R24]. Though for certain special systems of differential equations there are theoretical results for the proper application of finite element methods in general this problem cannot be tackled. In certain cases they can be tackled or understood by using the concepts of *Riemannian Geometry* if the solution of the equations corresponds to some geodesic line of a given geometry.

Using the concept of "*parallel translation of vectors and tensors*" two geodesic lines starting from neighboring points with identical initial velocity can be considered as it was discussed by Arnold [R25].

In my investigations I assumed that

- The successful adaptive control corresponds to a *stable system*;
- The actual numerical results obtained naturally depend on the time resolution applied but only in a slight extent;
- For a finite duration of motion the stable numerical results were declared to be believable if halving the finite time-step in the simulation did not lead to observable differences in them. This attitude is right since the *convergence*, or at least the *possibility of the convergence within a region of attraction* were *theoretically proved* before running the simulations that *only illustrated but proved* the stability and usability of the proposed methods.
- In certain cases I also used the *ODE Solver* of INRIA's SCILAB and SCICOS software that generally applies various, quite sophisticated numerical integration methods, depending on the stiffness of the problem considered. (Its use is especially convenient when graphical programming can be applied to build up the appropriate environment in which the *ODE Solver* can be called.) It also modifies the density of the discrete time-resolution automatically to meet the prescribed precision requirements. By carefully prescribing the *allowable maximal time step* and the *relative and absolute tolerance* consistent results were obtained for the stable systems to ***illustrate*** the operation of the stable controller
- If the results were divergent their details were not "believed". Such runs only illustrated the possibility of leaving the range of convergence of the applied method.

Another relevant point is the "*believability or realistic nature of the models*" applied in the simulations. While in general it can be accepted that no any given model can fully and completely describe the reality, a good model can be regarded at least as a "cubist picture" that contains *significant features of the reality*, therefore it can be used as a "*paradigm*" i.e. as characteristic representative of a whole set or class of problems. *In this sense the simulation results obtained cannot be regarded completely worthless or improper means of illustration, though it has to be admitted that any particular practical application of the proposed method needs further detailed investigations.*

To technically realize the proposed novel approaches the observation of the behavior of the controlled system was necessary. For this purpose the "*Expected – Realized Response Scheme*" was introduced. According to that scheme a considerable part of the control tasks could be formulated by using the concepts of the appropriate "*excitation*" Q of the controlled system to which it is expected to respond by some prescribed or "*desired response*" r^d. (The physical meaning of the appropriate excitation and response depend on the phenomenology of the system under consideration. In the case of Classical Mechanical Systems the excitation physically can be force and/or torque, while the response can be linear or angular acceleration, etc.) The appropriate excitation can be computed by the use of some available approximate "*inverse dynamic model*" as $Q=\varphi(r^d)$. Since normally this inverse model is neither complete nor exact, the actual response determined by the system's dynamics, ψ, results in a "*realized response*" r^r that differs from the desired one: $r^r=\psi(\varphi(r^d))\neq r^d$. It is worth noting that the functions $\varphi()$ and $\psi()$ may contain various hidden parameters that partly correspond to the dynamic model of the system, and partly pertain to unknown external dynamic forces acting on it. Due to phenomenological reasons the controller can manipulate or "*deform*" the input value from r^d to some r_*^d so that $r^d=\psi(\varphi(r_*^d))$. Other possibility is the manipulation of the output of the rough model.

The above structure evidently indicated that using the pairs of the "*desired*" response *known and set by the controller* and comparing it to the observed "*realized*" response mathematically can be formulated as seeking the solution of a *Fixed Point Problem*. From this point on the main direction of the research was seeking various deformations or fixed point transformations that were able to generate appropriate sequences of responses that can converge to the fixed point. In this approach in each control cycle one iterative step can be done with the actually available updated "desired response", and in the next cycle the deformation applied can be updated on the basis of the "observed response". If the dynamics of the adaptive iteration is considerably faster than that of the control task such solution may result in practically acceptable tracking. (This idea is in strict analyogy with the use of *Cellular Neural Networks* in picture processing based on the concept of *Complete Stability* [R19].) Similar "*dynamic approaches*" were also applied in the literature as e.g. dynamic inversion of nonlinear maps by Getz, Getz and Marsden [R20], [R21], but these considerations extensively used the technique of the *Lyapunov Functions*.

In contrast to *Lyapunov's 2nd Method* [R22], [R23] that normally can generate quadratic expressions with absolute minima in wide environments that can act as basins of attraction of convergent solutions, in the novel approach convergence can be achieved by applying contractive maps in *Banach Spaces.* In this manner iterative sequences converging to the fixed point of the appropriate map can be obtained. This latter solution can be more "*fragile*", but in the same time far simpler than the application of some Lyapunov function. Furthermore, its realization may need far less complicated computations.

Chapter 3: Introduction

In order to substantiate the main aim of the dissertation i.e. the "*systematic use of geometric way of thinking*" in control technology first I would like to give a very brief historical survey to show how fruitful and profitable it was in the field of the natural sciences. Since the *historical background* of these methods normally are not mentioned (neither in the standard university-education of Mathematics nor in the more specific scientific papers), for collecting this information (*rigorously only for this purpose*) I intensively used the materials available on the Web at the pages of *Wikipedia*, the free encyclopedia [R26]. The result of this brief historical research was quite surprising and shocking for me because it revealed that Mankind has clear, precise, and well generalized concepts of this subject area practically only from the middle of the 19th Century.

3.1. Certain Representative Examples of Uneven Development

From a historical point of view it can be stated that the main concepts had crystallized only "recently" that has the interesting consequences that certain fundamental mathematical methods widely used in Technical Sciences obtained rigorous mathematical explanation only after their invention. To mention only a few significant examples: when Euler invented one of the fundamental equations of *Fluid Dynamics* in 1755 no systematic concepts of *vectors*, *tensors*, or other *directed quantities* were available [R27]. When Maxwell published his famous "*Treatise on Electricity and Magnetism*" in 1892 [R28] both Hamilton's "*quaternions*" [R29] as well as Grassmann's "*vectors*" already existed [R30] (he worked on this idea from 1832), however, the latter concept became widely available only a few years after issuing the "*Treatise*", therefore Maxwell used quaternions for the quantitative description of electromagnetic phenomena. This observation highlights the "*incidental nature*" of the development in sciences. As is well known the later issues of the "*Treatise*" already used the concept of vectors and tensors instead of quaternions. It was an interesting and inspiring question to look after what kind of Electrodynamics we could have now if the "custom" of using quaternion prevailed. For instance, in a common work with Iván Abonyi and János F. Bitó we found that the two invariants in *Electrodynamics* could be more easily explored by using the complex extension of *Quaternion Algebra* than by using tensors. It was also found that the significant components of the relativistic tensor formulation of Electrodynamics could be also identified in the quaternion representation [R31]. On this reason in the next part I present a very brief historical summary of the fundamental concepts.

3.2. Historical Antecedents of Geometric Way of Thinking

Until the 1st half of the 20th Century the development of Mathematics aimed at serving the needs of natural and technical sciences. In the history of the "*quantitative sciences*" geometric way of thinking always played a pioneering role.

The principles of geometry first were reduced to a small set of axioms by Euclid of Alexandria, a Greek mathematician who worked during the reign of Ptolemy I (323-283 BC) in Egypt. His method of proving mathematical theorems by logical reasoning from accepted first principles remained the backbone of mathematics even in our days, and is responsible for that field's characteristic rigor [R32].

Following the pioneering work clarifying the phenomenology of Classical Mechanics by *Galilei* and *Newton*, in his fundamental work entitled "*Mécanique Analytique*" [R33] *Joseph-Louis Lagrange* (1736-1813) solved various optimization problems under constraints, introduced the concept of "*Reduced Gradient*" and that of what we refer to nowadays as "*Lagrange Multipliers*" [R34]. It has to be noted that at that time the concept of "*linear vector spaces*" was not clarified at all.

The first mathematical means of describing quantities with direction, i.e. the quaternions introduced by Sir William Rowan Hamilton (1805-1865) appeared not very long time after Lagrange's death [R29]. In the 19^{th} Century quaternions were generally used for such purposes. For instance, in the first edition of Maxwell's famous "*Treatise on Electricity and Magnetism*" quaternions were used for describing the "directed" magnetic and electric fields.

The first known appearance of what are now called "*linear algebra*" and the notion of a "*vector space*" is related to *Hermann Günther Grassmann* (1809-1877), who started to work on the concept from 1832. In 1844, Grassmann published his masterpiece [R30] that commonly is referred to as the "*Ausdehnungslehre*", ("*theory of extension*" or "*theory of extensive magnitudes*"). This work was mainly inspired by Lagrange's "*Mécanique analytique*" [R33]. *Grassmann showed that once geometry is put into the algebraic form he advocated, then the number three has no privileged role as the number of spatial dimensions: the number of possible dimensions is in fact unbounded* [R35].

The close relationship between geometry and algebra was realized and strongly utilized by *William Kingdon Clifford* (1845-1879) who introduced various "*associative algebras*", the so called "*Clifford Algebras*" [R36]. As special cases Clifford Algebras contain the algebra of the real, the complex, the dual numbers, the quaternion algebra, and the algebra of octonions (biquaternions) [R37]. His "*Geometric Algebra*" is widely used in technical sciences as e.g. in computer graphics, robotics, etc.

Equipped with the concepts of linear vector spaces *Marius Sophus Lie* (1842-1899) in his PhD dissertation studied the properties of geometric symmetry transformations [R38]. One of his greatest achievements was the discovery that continuous transformation groups (now called after him Lie groups) could be better understood by studying the properties of the tangent space of the group elements, that form linear vector spaces (the vector space of the so-called infinitesimal generators), and with the commutator as multiplication also form algebras, the so called "*Lie Algebras*".

In the very fertile period of Mathematics, in the 19^{th} Century *Georg Friedrich Bernhard Riemann* (1826-1866) elaborated the *geometry of curved spaces* in a special form that made it possible to study physical quantities as tensors even if the geometry of the space differs from the *Euclidean Geometry* [R39]. This concept was very fruitfully used in the *General Theory of Relativity*.

David Hilbert (1862-1943) [R40] extended the concept of the *Euclidean Geometry* to *linear, normed, complete metric spaces in which the norm originates from a scalar product*.

Stefan Banach (1892-1945) [R41] introduced the more general concept, the concept of *Banach Spaces* that are *linear, normed, complete metric spaces in which the norm not necessarily originates from a scalar product*. The great practical advantage of Banach's invention is that by adding various norms to the same mathematical set various complete, linear, normed metric spaces can be obtained that offer a wide basis for elaborating diverse practical variants and solutions pertaining to the essentially same basic idea.

Vladimir Igorevich Arnold (1937-) [R42] studied the *Symplectic Geometry* and *Symplectic Topology* that are extremely useful means of studying the behavior of various mechanical and other physical systems.

The geometric way of thinking outlined above appeared in one of the best textbooks used for teaching functional analysis, too (the excellent book by *László Máté* [R43]).

By the middle of the eighties of the past century certain elements of the sophisticated geometric concepts were systematically utilized in control technology. The first edition of Isidori's book in 1985 [R44] contained cahpeters as "Geometric Theory of State Feedback" and "Geometric Theory of Nonlinear Systems". An even more systematic surveay and application of Group Theory and Differentiable Manifolds can be found in Jurdjevic's book from 1997 [R45].

Another, very important mathematical tool that makes it easy to apply geometric way of thinking is the *Singular Value Decomposition (SVD)*. The history of matrix decomposition goes back to the 1850s. During the last 150 years several mathematicians — *Eugenio Beltrami* (1835–1899), *Camille Jordan* (1838–1921), *James Joseph Sylvester* (1814–1897), *Erhard Schmidt* (1876–1959), and *Hermann Weyl* (1885–1955), who were perhaps the most important ones, contributed to establishing the existence of the singular value decomposition and developing its theory [R46]. Thanks to the pioneering efforts of *Gene Golub*, there exist efficient, stable algorithms to compute the singular value decomposition [R47]. Certain realization of *SVD* is available in Hungarian for a long time in the excellent book by *Pál Rózsa* [R48]. In our days *SVD* is a standard service (function) of software designed for the use in research, as e.g. INRIA's SCILAB.

More recently, *SVD,* and its novel variant, the so called *Higher Order Singular Value Decomposition (HOSVD)* (e.g. [R49], [R50]) started to play an important role in several scientific fields as signal processing (e.g. [R51], [R52], [R53]), control applications in dealing with system models of *Tensor Product (TP)* form (e.g., the very interesting PhD Thesis by *Zoltán Petres* [R54] can be referred to in this context). The *real variant of SVD* was extensively used in the present Thesis, too.

My aim with providing this brief historical survey was to show that geometric way of thinking is a very useful and fruitful mode of problem-tackling in various fields. The use of the inventions by Hamilton, Grassmann, Hilbert, Banach, and Clifford in Physics and technical fields makes it possible

- To apply a "geometric way of thinking" with which we became familiar in our childhood in our playing house. Then we daily experienced the *Euclidean Geometry* of the reality around us. Selection and use of adequate associations with simple pictures as vectors or directed quantities, linear combinations, basis vectors, orthogonality, orthogonal subspaces, tangents and tangent space of a surface in a given point, the notion of surfaces or hypersurfaces embedded in higher dimensional spaces became instinctive, hidden practice of our early years;
- To strengthen the above, almost "instinctive" associations with the aid of lucid, simple, aesthetic equations of algebraic relationships.

In the sequel its advantages will be shown in the field of nonlinear control. For this purpose I try to give a brief survey on the prevailing, from certain point of view "classic" approaches.

Chapter 4: Brief Survey on the Prevailing Approaches Based on the Use and Learning of Exact Analytical Models

A plausible approach to solving control tasks would be to elaborate and use the “exact dynamic model” of the system to be controlled. In the case of the control of mechanical systems as robots this approach can be referred to as “*Computed Torque Control*” since in this case the mechanical model establishes mathematical relationships between the joint coordinate accelerations and the torques or forces acting on the system partly by its own drives and/or by its environment with which the system may be in dynamic coupling. In the case of other systems as e.g. chemical reactions considered in [R55] the notion of “*Globally Linearizing Controllers (GLC)*” can be mentioned in which certain order time-derivative of the state variable of the system to be controlled or that of a well-defined function of the state variables can instantaneously be set by the control signal. In the sequel these typical cases are considered. I intentionally do not mention the classical “canonical forms” concerning controllability and observability issues the use of which already became a standard approach for a wide set of systems and also has a huge literature. The same holds for the various parameter estimation techniques using some Kalman filters and typical assumptions regarding the statistical nature of the noises characteristic to the problem under consideration. My aim was to develop and use different techniques for system identification.

4.1. Computed Torque Control (CTC) in Robotics

Before going into details it has to be noted that involving the model of the operation of the drives of a *Classical Mechanical System* may considerably increase the complexity of the problem. However, even modeling the mechanical behavior itself is a very complex task. As a result of such efforts the *Euler-Lagrange Equations of Motion* can be obtained for an open kinematic chain as follows:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{Q} \tag{4.1.1}$$

in which $\mathbf{H}(\mathbf{q})$ describes the configuration-dependent “*inertia matrix*” of the system, a part of $\mathbf{h}(\mathbf{q},d\mathbf{q}/dt)$ is quadratic in $d\mathbf{q}/dt$ and describes e.g. the Coriolis terms, while its other part depending only on $\mathbf{q}$ is responsible for the gravitational effects. It is worth noting that due to physical reasons $\mathbf{H}$ is always *symmetric and positive definite*, though *it may be badly conditioned, too*. The term $\mathbf{Q}$ stands for the generalized forces that partly originate from the robot's own drives or from the environment. (This equation is valid only if the kinetic energy of the system is given with respect to *an inertial frame* of reference in which case the components of $\mathbf{Q}$ can be interpreted as *forces for the prismatic generalized coordinates*, and *torques for the rotational axes*.) In the possession of this "exact" model on the basis purely kinematic considerations some *desired* $d^2\mathbf{q}^d/dt^2$ can be computed in each control cycle to exert the necessary $\mathbf{Q}^d$. This part of the controller is often referred to as “feedforward” control. For more precise tracking the “feedforward part” generally has to be completed by PID-type feedback terms basec on the tracking error.

However, an important practical problem related to the application of CTC control is the fact that in many cases it is very difficult or even impossible to identify the parameters of the analytical models of the systems as e.g. robots [R5], [R6]. In the classical example in which Armstrong et al. developed the dynamic model of a six degree of freedom PUMA robot arm three persons worked for five weeks [R7]. This work involved the measurement of the appropriate data besides coding the model in software blocks. In various publications the measured parameters of PUMA robot has considerable diversities, too [R8].

Another practical problem in the application of this method is that normally there are no sensors available that could exactly measure the external parts of **Q**. Their effects can be observed only as their consequences in the actual motion of the system and in general cannot efficiently be compensated by simply prescribing some feedback correction in $d^2\mathbf{q}^d/dt^2$. Such kind of feedback correction can work only if the unknown external perturbations are

- generally insignificant, or, if they are significant,
- they can be only instantaneous but permanent.

It is worth noting that the kinematic structure of the robot arm itself determines the main mathematical "skeleton" of (4.1.1): normally a *parameter vector* can be introduced that contains the unknown dynamical information, while the elements of this vector in (4.1.1) are multiplied by known kinematic functions. This fact serves as a basis for developing the analytical model based controllerss toward adaptive solutions in order to correct the imprecisions in the parameters of the available dynamic model. Representative examples are the "*Adaptive Inverse Dynamics*" or the "*Adaptive Slotine-Li Controller*" approaches. Since these methods are based on analytical modeling and the use of Lyapunov functions in the sequel Lyapunov's 2nd Method will be studied.

4.2. On Lyapunov's 2nd Method in General

Lyapunov's 2nd Method is a widely used technique in the analysis of the stability of the motion of the *non-autonomous dynamic systems* of equation of motion as $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$. Since in the prevailing literature this method normally is referred to, in the sequel, for the purposes of making comparisons between this method and the proposed novel one, I would like to pay some attention to its background and deeper details.

The typical stability proofs provided by Lyapunov's original method published in 1892 [R22] ***(and later on e.g. in*** [R23]***) have the great advantage that they do not require to solve the equations of motion.*** Instead of that the uniformly continuous nature and *non-positive time-derivative of a positive definite Lyapunov-function V* constructed of the tracking errors and the modeling errors of the system's parameters are assumed in the $t\in[0,\infty]$ domain from which the convergence $dV/dt\to 0$ can be concluded according to Barbalat's lemma [R56]. This lemma states that if the integral of a uniformly continuous function (in this case the integral of dV/dt i.e. V) in $[0,\infty)$ is bounded then this function has to converge to zero [R11]. The uniform continuity of dV/dt used to be guaranteed by showing that d^2V/dt^2 is bounded. Due to the positive definite nature of V from that it normally follows that the tracking errors have to remain bounded, or in certain special cases, have to converge to 0.

An alternative possibility for utilizing Lyapunov's theorem is the use of the so-called special "*function class κ*" certain elements of which can serve as upper and lower bounds of V so evading the direct application of Barbalat's lemma to show uniform stability of the system.

By definition a function $\kappa : [0,k) \to [0,\infty)$ is of class κ if $\kappa(0)=0$ and $\kappa(t)$ is strictly increasing (normally $k<\infty$ but $k=\infty$ may happen in special cases, now we restrict ourselves to the $k<\infty$ case). In the forthcoming considerations **x** denotes some tracking error, therefore the desired stable equilibrium point $\mathbf{x}=\mathbf{0}$ is sought for.

By definition the state $\mathbf{x}^*$ is an *equilibrium state* if $\forall\, t \in [t_0,\infty)\quad \mathbf{f}(\mathbf{x}^*,t) = \mathbf{0}$.

The $\mathbf{x}^*$ state is a *stable equilibrium in* $t=t_0$ if for $\forall\rho>0$ there exists $r(\rho,t_0)>0$ such that $\|\mathbf{x}(t_0)-\mathbf{x}^*\|<r(\rho,t_0)\Rightarrow\|\mathbf{x}(t)-\mathbf{x}^*\|<\rho \quad \forall t>t_0$.

Uniformly stable states can be defined if in the above definitions in $r(\rho,t_0)>0$ t_0 does not play significant role: $r(\rho)>0$.

The $\mathbf{x}^*$ *equilibrium state is asymptotically stable at* $t=t_0$ if it is stable and there exists $r(t_0)>0$ such that $\|\mathbf{x}(t_0)-\mathbf{x}^*\|<r(t_0)\Rightarrow\|\mathbf{x}(t)-\mathbf{x}^*\|\to 0 \; for \quad t\to\infty$.

The $\mathbf{x}^*$ *equilibrium state is globally asymptotically stable* if $x(t)\to x^*$ as $t\to\infty \quad for \quad \forall x(t_0)$ (its basin of attraction is the whole space).

According to Fig. 4.1., by the use of the above definitions the following statements can be done. Let the $\alpha(\|\mathbf{x}\|)$, $\beta(\|\mathbf{x}\|)$, $\gamma(\|\mathbf{x}\|)$ functions belong to function class κ!

- If $V(0,t)=0 \quad and \quad V(\mathbf{x},t)\geq\alpha(\|\mathbf{x}\|)>0$ and $\dot{V}(\mathbf{x},t)\leq 0$ then the equilibrium point **x=0** is stable.

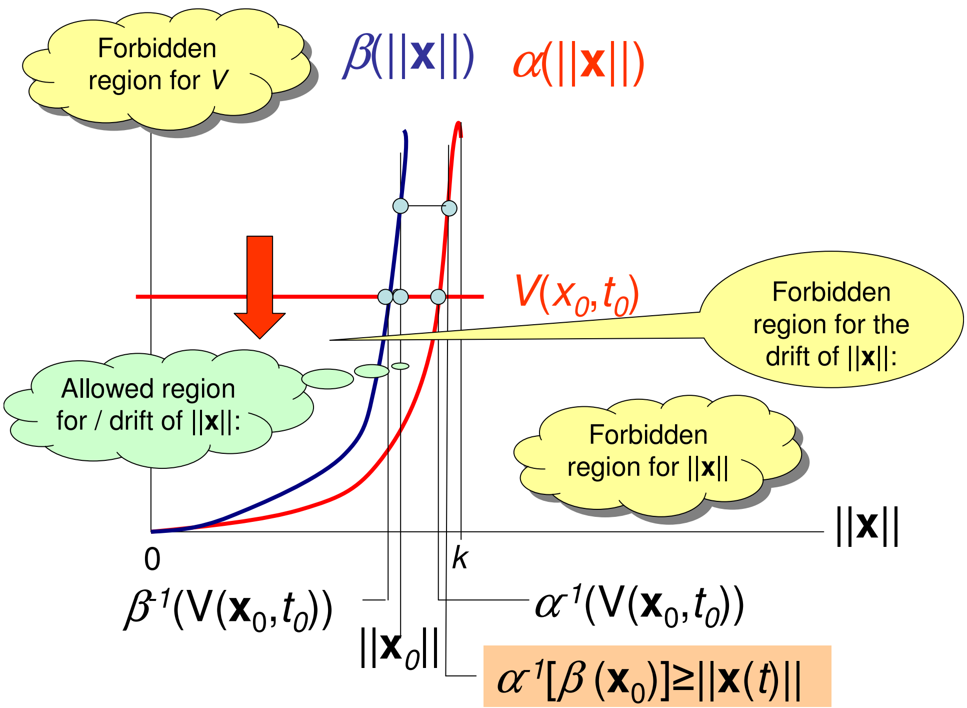


Figure 4.1. The geometric interpretation of Lyapunov's 2nd Method

To prove that it is enough to consider the limit $\|\mathbf{x}(t)\|\leq\alpha^{-1}(V(\mathbf{x}_0,t_0)) \quad for \quad t>t_0$ in Fig. 4.1. Here the initial error norm in t_0 has significance! In this case the allowable range in V and $\|\mathbf{x}\|$ is bounded by the graph of $\alpha(\|\mathbf{x}\|)$ from the right side, and by the $V(\mathbf{x}_0,t_0)$ line from the top.

- If $V(0,t)=0 \quad and \quad V(\mathbf{x},t)\geq\alpha(\|\mathbf{x}\|)>0$ and $\dot{V}(\mathbf{x},t)\leq 0$, and $V(\mathbf{x},t)\leq\beta(\|\mathbf{x}\|)>0$ then the equilibrium point **x=0** is *uniformly stable*.

To prove this statement it is enough to consider Fig. 4.1. again. Evidently $\beta^{-1}(V(\mathbf{x}_0,t_0))\leq\|\mathbf{x}(t)\|\leq\alpha^{-1}(V(\mathbf{x}_0,t_0))$ *for* $t>t_0$ and $\alpha^{-1}[\beta(\mathbf{x}_0)]\geq\|\mathbf{x}(t)\|$. This estimation is independent of t_0!

- If $V(0,t)=0$ *and* $V(\mathbf{x},t)\geq\alpha(\|\mathbf{x}\|)>0$, and $\dot{V}(\mathbf{x},t)\leq 0$, and $V(\mathbf{x},t)\leq\beta(\|\mathbf{x}\|)>0$, and $\dot{V}\leq-\gamma(\|\mathbf{x}\|)<0$ then the equilibrium point **x=0** is *uniformly asymptotically stable*.

For proving that consider the Fig. 4.1. again! Evidently V cannot be stopped at finite $\|\mathbf{x}\|$. It can be stopped only in $\|\mathbf{x}\|=0$. The allowed range is shrunk to $\|\mathbf{x}\|=0$ as the level of V sinks down to 0.

4.3. Globally Linearizing Controllers

The concept of "*Globally Linearizing Controllers*" as introduced e.g. by Khalil [R57], Goodwine & Stepan [R58], are designed for the following more or less "canonical" form of equations of motion:

$$\dot{\mathbf{x}}=\mathbf{f}(\mathbf{x})+\mathbf{g}(\mathbf{x})\mathbf{u},\quad \mathbf{y}=\mathbf{h}(\mathbf{x}) \tag{4.3.1}$$

in which $\mathbf{x}\in\Re^n$ denotes the state variable of the system, $\mathbf{y}\in\Re^m$ denotes its observable output, $\mathbf{u}\in\Re^k$ means the manipulated input (control signal). By applying the *chain rule of derivation* the time-derivative of $\mathbf{y}$ can be obtained from (4.3.1) as

$$\frac{dy_i}{dt}=\sum_{s=1}^{n}\frac{\partial h_i}{\partial x_s}\left[f_s(\mathbf{x})+\sum_{z=1}^{k}g_{sz}(\mathbf{x})u_z\right]\equiv L_f h_i+(L_g h_i)u\,. \tag{4.3.2}$$

in which the very condensed notation of the *Lie-derivatives* is applied as $L_f h_i$, etc. If the lucky situation occurs in which $L_g h_i\neq 0$ then $d\mathbf{y}/dt$ can simply be expressed as an *affine function* of $\mathbf{u}$. In this case the matrix $M_{iz}(\mathbf{x}):=\sum_{s=1}^{n}\frac{\partial h_i(\mathbf{x})}{\partial x_s}g_{sz}(\mathbf{x})$ and the single index array $b_i:=\frac{dy_i}{dt}-\sum_{s=1}^{n}\frac{\partial h_i}{\partial x_s}f_s(\mathbf{x})$ can be defined. If on the basis of some kinematic considerations we idea on the *desired value of* dy_i/dt, in principle the **Mu=b** equation may be soluble and the necessary control signal can be computed, of course, only in the possession of the analytic form of the model coded in functions **f** and **g**. If $L_g\mathbf{h}\equiv\mathbf{0}$ then d^2y_i/dt^2 can be expressed by repeating the use of the chain rule, etc. In general if we have $j>0$ so that $L_gL_f^s\equiv 0$ if $s=0,1,2,...,j-1$, but $L_gL_f^j\neq 0$ the dependence of the j^{th} time-derivative of $\mathbf{y}$ on $\mathbf{u}$ has an *affine form* as

$$\mathbf{y}^{(j)}=L_f^j\mathbf{h}+L_gL_f^{j-1}\mathbf{h}u\,. \tag{4.3.3}$$

In this case j is referred to as the *relative degree* of the nonlinear system. In the possession of the exact system model the appropriate Lie-derivatives in (4.3.3) can be computed. Whenever (4.3.3) is able to uniquely determine the appropriate value of \$u\$ that is needed for achieving a desired j^{th} derivative of the observable output $\mathbf{y}^{(j)d}$ determined on the basis of some "kinematic" consideration, this formalism can evidently be successfully used for the control. The control signal **u** evidently can be fed back in the form of $\mathbf{u}=\mathbf{p}(\mathbf{x})+\mathbf{q}(\mathbf{x})\,\mathbf{y}^{(j)d}$ from which the name of the controller, i.e. the notion of "*Globally Linearizing Control*" originates. It is worth noting that in spite of the very "special form" of the suppositions concerning the identically zero values of certain Lie-derivatives in the practice various physical systems meet these conditions. In [R55] e.g. the temperature control of a "*Jacketed Continuous Stirred Tank Rector (JCSTR)*" is considered in which the heat released in an exothermic reaction has to be extracted by the cooling system in the jacket, while in e.g. [C90] a 4^{th} order Classical Mechanical system is considered.

In general it has to be noted that the elegant form of (4.3.3) in the case of a higher relative order *j* covers quite complicated computations due to the repeated application of the chain rule. In the case of a complex process this may mean quite considerable computational burden while developing the analytical model for the control.

It has to be noted again that even if we are not in the possession of the exact model, the analytical form of (4.3.3) still can be a good basis for developing a novel type adaptive controllers as e.g. in the case of the control of a polymerization process we applied in [C95]. It is also worthy of note that the novel controller can be developed even in the cases in which the conditions for developing a *GLC* do not prevail as e.g. in the case of a convoy of coupled vehicles [C99]. In the sequel the most sophisticated classical adaptive controllers based on analytical modeling in Robotics will be studied and modified.

4.4. Adaptive Inverse Dynamics Control of Robots

Before going into any detail we note that in the forthcoming considerations we use the Lyapunov function technique in a special case in which the eigenvalues of positive definite and negative definite matrices can be used for estimation purposes. (More systematic and general analysis of this method will be given later.) This approach is based on a more detailed form of (4.1.1) and *assumes that at least the kinematic model of the system is precisely known*. On this basis a parameter vector **p** representing the *dynamical parameters* and an array built up of well known kinematic functions $\mathbf{Y}(\mathbf{q},d\mathbf{q}/dt,d^2\mathbf{q}^d/dt^2)$ can be introduced in the dynamic model as follows:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})=\mathbf{Q}=\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\mathbf{p} \tag{4.4.1}$$

It is also supposed that some approximate model built up of the functions $\hat{\mathbf{H}}(\mathbf{q})$, $\hat{\mathbf{h}}(\mathbf{q},\dot{\mathbf{q}})$ also is available with the model parameters $\hat{\mathbf{p}}$ on the basis of which the generalized forces are calculated and exerted. The exerted forces *ab ovo* contain feedback-correction depending on the tracking error and its derivatives $\mathbf{e}:=\mathbf{q}^N-\mathbf{q},\dot{\mathbf{e}}:=\dot{\mathbf{q}}^N-\dot{\mathbf{q}},\ddot{\mathbf{e}}:=\ddot{\mathbf{q}}^N-\ddot{\mathbf{q}}$ with symmetric positive definite gain matrices $\mathbf{K}_0$ and $\mathbf{K}_1$ as

$$\hat{\mathbf{H}}(\mathbf{q})\left(\ddot{\mathbf{q}}^N+\mathbf{K}_0\mathbf{e}+\mathbf{K}_1\dot{\mathbf{e}}\right)+\hat{\mathbf{h}}(\mathbf{q},\dot{\mathbf{q}})=\mathbf{Q}=\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) \tag{4.4.2}$$

It is worth noting that in this method it is a supposition of crucial importance that the validity of (4.4.2) *is supposed*, i.e. *it is assumed that* **Q** *originates from the drives and does not contain unknown external components*. On the basis of this assumption (4.4.2) can be subtracted from (4.4.1) to obtain $\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})-\hat{\mathbf{H}}(\mathbf{q})\left(\ddot{\mathbf{q}}^N+\mathbf{K}_0\mathbf{e}+\mathbf{K}_1\dot{\mathbf{e}}\right)-\hat{\mathbf{h}}(\mathbf{q},\dot{\mathbf{q}})=0$. By subtracting and adding $\hat{\mathbf{H}}(\mathbf{q})\ddot{\mathbf{q}}$ at the left hand side and keeping only the modeling errors at this side it is obtained that

$$\hat{\mathbf{H}}(\mathbf{q})[\ddot{\mathbf{e}}+\mathbf{K}_0\mathbf{e}+\mathbf{K}_1\dot{\mathbf{e}}]=$$
$$=\left[\underbrace{\mathbf{H}(\mathbf{q})-\hat{\mathbf{H}}(\mathbf{q})}_{-\tilde{\mathbf{H}}}\right]\ddot{\mathbf{q}}+\left[\underbrace{\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})-\hat{\mathbf{h}}(\mathbf{q},\dot{\mathbf{q}})}_{-\tilde{\mathbf{h}}}\right]=\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\left(\underbrace{\mathbf{p}-\hat{\mathbf{p}}}_{:=\tilde{\mathbf{p}}}\right) \tag{4.4.3}$$

in which one side contains the *model data*, while the other side contains the *modeling errors* defined by the quantities denoted by the *tilde (~)* symbol. Via multiplying both sides of (4.4.3) with the inverse of the known model and formally introducing the array $\mathbf{x}:=\begin{bmatrix}\mathbf{e}\\ \dot{\mathbf{e}}\end{bmatrix}$, $\dot{\mathbf{x}}:=\begin{bmatrix}\dot{\mathbf{e}}\\ \ddot{\mathbf{e}}\end{bmatrix}$ an equation of motion can be obtained for the system with error-feedback that corresponds to the "standardized form" of that of the non-autonomous dynamic systems:

$$\begin{bmatrix}\dot{\mathbf{e}}\\ \ddot{\mathbf{e}}\end{bmatrix}-\underbrace{\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{K}_0 & -\mathbf{K}_1\end{bmatrix}}_{\mathbf{A}}\begin{bmatrix}\mathbf{e}\\ \dot{\mathbf{e}}\end{bmatrix}=\begin{bmatrix}\mathbf{0}\\ \mathbf{\Phi}\tilde{\mathbf{p}}\end{bmatrix} \tag{4.4.4}$$

or

$$\dot{\mathbf{x}}-\mathbf{A}\mathbf{x}=\mathbf{B}\mathbf{\Phi}\tilde{\mathbf{p}}\ ,\ \mathbf{B}:=\begin{bmatrix}\mathbf{0}\\ \mathbf{I}\end{bmatrix} \tag{4.4.5}$$

Now let us try to construct a Lyapunov function of the tracking error and its 1st time-derivative and of $\tilde{\mathbf{p}}$ as $V:=\mathbf{x}^T\mathbf{P}\mathbf{x}+\tilde{\mathbf{p}}^T\mathbf{R}\tilde{\mathbf{p}}$ where $\mathbf{P}$ and $\mathbf{R}$ are constant, symmetric positive definite matrices of proper dimensions! Then evidently

$$\dot{V}:=\dot{\mathbf{x}}^T\mathbf{P}\mathbf{x}+\mathbf{x}^T\mathbf{P}\dot{\mathbf{x}}+\dot{\tilde{\mathbf{p}}}^T\mathbf{R}\tilde{\mathbf{p}}+\tilde{\mathbf{p}}^T\mathbf{R}\dot{\tilde{\mathbf{p}}}<0\,. \tag{4.4.6}$$

From (4.4.5) it follows that

$$\dot{V}:=\mathbf{x}^T\left(\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}+\tilde{\mathbf{p}}^T\mathbf{\Phi}^T\mathbf{B}^T\mathbf{P}\mathbf{x}+\mathbf{x}^T\mathbf{P}\mathbf{B}\mathbf{\Phi}\tilde{\mathbf{p}}+\dot{\tilde{\mathbf{p}}}\mathbf{R}\tilde{\mathbf{p}}+\tilde{\mathbf{p}}^T\mathbf{R}\dot{\tilde{\mathbf{p}}}<0 \tag{4.4.7}$$

Due to the symmetry of matrices $\mathbf{P}$ and $\mathbf{R}$ (4.4.7) can be simplified as

$$\dot{V}:=\mathbf{x}^T\left(\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x}+2\tilde{\mathbf{p}}^T\mathbf{\Phi}^T\mathbf{B}^T\mathbf{P}\mathbf{x}+2\tilde{\mathbf{p}}^T\mathbf{R}\dot{\tilde{\mathbf{p}}}<0 \tag{4.4.8}$$

To guarantee *dV/dt*<0 for finite $\mathbf{x}$ the following restrictions can be prescribed: let $\mathbf{U}$ be a negative definite symmetric matrix, and let

$$\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}=\mathbf{U} \tag{4.4.9}$$

and

$$\tilde{\mathbf{p}}^T\left(\mathbf{\Phi}^T\mathbf{B}^T\mathbf{P}\mathbf{x}+\mathbf{R}\dot{\tilde{\mathbf{p}}}\right)=0\Rightarrow\dot{\tilde{\mathbf{p}}}=-\mathbf{R}^{-1}\mathbf{\Phi}^T\mathbf{B}^T\mathbf{P}\mathbf{x} \tag{4.4.10}$$

Equation (4.4.9) is referred to as the "*Lyapunov Equation*". Normally an appropriate **U** is prescribed and the task is to find a proper **P** for this **U** by solving the Lyapunov Equation that equation evidently sets linear functional connection between the elements of **P** and **U** that may or may not have solution. (For the existence of a solution the real part of each eigenvalue of **A** must be negative.) Since **A**=const. the Lyapunov Equation has to be solved only one times in order to find a proper **P** for the prescribed **U**. (Each common software package as e.g. INRIA's SCILAB or Wolfram Research's MATLAB immediately yields the solution of this equation in a single command.) To satisfy the second important equation (4.4.10), its right hand side has to be expressed from its definition through **B** and **Φ**. It is obtained that

$$\dot{\tilde{\mathbf{p}}} = \dot{\mathbf{p}} - \dot{\hat{\mathbf{p}}} = \mathbf{0} - \dot{\hat{\mathbf{p}}} = -\mathbf{R}^{-1}\mathbf{Y}\hat{\mathbf{H}}^{-1}[\mathbf{0},\mathbf{I}]\mathbf{P}\mathbf{x} \tag{4.4.11}$$

in which the computational burden mainly consists in the need for inverting the model inertia matrix that must have the exact, intricate form determined by the particular kinematic model of the given robot arm.

If the adaptation rule is applied by the controller for the convergence of this method the following cases can be imagined.

- A possibility is the case of $\|\mathbf{x}\|\to 0$ and $\|\tilde{\mathbf{p}}\| > F > 0$, i.e. exponential trajectory tracking in principle may be achieved without exactly learning the system model. That may happen if the nominal and realized (controlled) trajectories do not yield satisfactory information on the complete dynamic model.
- $\|\mathbf{x}\|\to 0$ and $\|\tilde{\mathbf{p}}\| \to 0$ i.e. exponential trajectory tracking with exactly learned dynamic model may also happen.
- It is impossible to have $\|\mathbf{x}\|>E>0$ for arbitrarily long time because *dV/dt<0* can be estimated as $\dot{V} < 0, \left|\dot{V}\right| = -\mathbf{x}^T\mathbf{U}\mathbf{x} \geq \left|U^{Eig}\right|_{\min}\mathbf{x}^T\mathbf{x} \geq \left|U^{Eig}\right|_{\min} E^2 > 0$ for finite $\mathbf{x}$, while $V \geq P_{\min}^{Eig}E^2 + \tilde{\mathbf{p}}^T\Gamma\tilde{\mathbf{p}} \geq P_{\min}^{Eig}E_p^2 > 0$ that is a contradiction since an initially finite positive value $V(0)$ with at least constant speed of decrease has to achieve 0 during finite time.
- Similar observations can be done if we use Barbalat's lemma for *dV/dt*: since *V* is a quadratic function of the errors constructed of positive definite terms, for finite *V* these errors must be bounded in the future since *dV/dt*≤0; due to the bounded errors d^2V/dt^2 remains bounded that means that *dV/dt* is uniformly continuous in time; in this case its finite integral $0\leq V(\infty)<\infty$ means that $dV/dt\to 0$ as $t\to\infty$, i.e. $\dot{V}(\infty) := \mathbf{x}^T(\infty)\left(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{x}(\infty) = 0$ since the parameter tuning in (4.4.11) always guarantees that the additions to the quadratic term in (4.4.8) take zero; since $\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} = \mathbf{U}$ is negative definite it is concluded that $\mathbf{x}(\infty)=\mathbf{0}$.

To sum up the main features of this method the following criticism can be done:

- The great advantage is that the under the relatively clear conditions of applicability it guarantees asymptotically zero error according to the above considerations.
- The details of error relaxation are prescribed by the construction of *V* and (4.4.3), and cannot be further manipulated.
- Besides that a lot of tedious computations have to be done by the direct use of the *exact form* of the normally quite complicated kinematic model, and real-time inversion of a positive definite model inertia matrix is needed in a cycle, too. We have to note that in spite of its positive definite nature this matrix can be badly conditioned as it was pointed out in connection with the adaptive control of a cart plus double pendulum system in one of our works [C63]. Another consequence of the presence of this inverted matrix is the relatively limited acceptable speed of parameter tuning: in a

finite element approach too big step in the estimation of $\hat{\mathbf{H}}(\mathbf{q})$ may lead to singularity that can stop the numerical learning algorithm.

- According to (4.4.2) it is assumed that the generalized force $\mathbf{Q}$ is fully known and correspond to that exerted by the drives on the basis of the available model. *Therefore, the external perturbations must be only temporal and insignificant* otherwise the method tends to compensate their effects on the basis of false assumption (by modifying the model parameters instead of observing/identifying the external perturbations).
- Furthermore, the present form is exempt of any feedback of the integrated tracking error that usually considerably can improve the quality of control by making small and slowly varying errors relax, too.

In the sequel two step modifications of the *Adaptive Inverse Dynamics Controller* will be proposed. It will be shown that the slow tuning process of the original approach can be replaced by a far more efficient one if we do not insist on the use of a single Lyapunov function for deriving the tuning rule. In the next step the original method will be completed by the use of an integrated feedback that also allows the more conventional parameter tuning via using a Lyapunov function, as well as the improved tuning in which the Lyapunov function is dropped.

4.4.1. Modification of the Tuning Rule of the Adaptive Inverse Dynamics Controller

The proposed modification is based on the observations as follows:

- Let us exert the driving force/torque values exactly as it was proposed in (4.4.2) by using the actual approximate values of the model parameters;
- Consider (4.4.3) in its original form and do not use the inverse of the actual estimation of the inertia matrix since this step may be critical and may lead to ill-conditioned estimation the inverse of which may cause numerical problems:

$$\hat{\mathbf{H}}(\mathbf{q})[\ddot{\mathbf{e}}+\mathbf{K}_0\mathbf{e}+\mathbf{K}_1\dot{\mathbf{e}}]=\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\left(\underbrace{\mathbf{p}-\hat{\mathbf{p}}}_{:=\tilde{\mathbf{p}}}\right) \tag{4.4.12}$$

- Since the LHS of (4.4.12) consists of known and measurable terms, and the same holds for matrix $\mathbf{Y}$ at the RHS, *observe that (4.4.12) contains all the actual information that is available for the parameter estimation error*. Instead manipulating with the inverse of the estimated inertia for the sake of using some Lyapunov function take the following observation: if the parameters are already properly estimated, the RHS becomes zero, and since $\hat{\mathbf{H}}(\mathbf{q})$ in principle must be positive definite, for precise parameter estimation it holds that $\ddot{\mathbf{e}}+\mathbf{K}_0\mathbf{e}+\mathbf{K}_1\dot{\mathbf{e}}=\mathbf{0}$. With properly chosen feedback parameters from this equation it follows that $\mathbf{e}\to\mathbf{0}$ as $t\to 0$. From that it follows that the tracking error can increase only during the tuning process while the estimation error at the RHS means some perturbation. To estimate the significance of this possible "meandering" of the tracking error consider the following equation that utilizes (4.4.3):

$$\frac{d}{dt}\left(\overbrace{\mathbf{x}^T\mathbf{P}\mathbf{x}}^{\geq 0}\right) := \mathbf{x}^T\left(\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}\right)\mathbf{x} + 2\tilde{\mathbf{p}}^T\mathbf{\Phi}^T\mathbf{B}^T\mathbf{P}\mathbf{x}. \qquad (4.4.13)$$

- It is evident that if a symmetric positive definite matrix $\mathbf{P}$ is properly chosen, i.e. $\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}$ is negative definite, the LHS of (4.4.13) corresponds to the time-derivative of a positive error metrics, the dominating quadratic term for large $\mathbf{x}$ values at the RHS is negative and the disturbance term that is only linear in $\mathbf{x}$ yields negligible contribution. That means that during the tuning process the tracking error is kept at bay even if the tuning itself is not based on the use of a Lyapunov function and it is yet imperfect.
- So we can utilize this possibility by applying the *Singular Value Decomposition* (*SVD*) for $\mathbf{Y}^T$ to obtain information on the appropriate orthogonal directions of the parameter estimations that significantly influence the actual value at the LHS of (4.4.12). By replacing the too small singular values with zero, a proper generalized inverse of $\mathbf{Y}^T$ containing the reciprocal of the significant singular values can be introduced for a quick exponential tuning with a positive γ parameter

$$-\gamma\mathbf{Y}^{T+}\hat{\mathbf{H}}(\mathbf{q})\left[\ddot{\mathbf{e}}+\mathbf{K}_0\mathbf{e}+\mathbf{K}_1\dot{\mathbf{e}}\right] = -\dot{\hat{\mathbf{p}}}. \qquad (4.4.14)$$

This approach is evidently free of the "critical step" of computing the inverse of the model inertia, evidently allows more efficient parameter tuning by properly utilizing the actual information available for the parameter estimation error. However, this control still does not contain any integrated feedback that practically used to be very efficient. In the next step the feedback terms in the original form of the *Adaptive Inverse Dynamics Controller* will be modified in order to introduce the integrated error in the feedback.

4.4.2. Introduction of Integrating Term in the Adaptive Inverse Dynamics Controller

For the seek of simplicity let us have only a single positive definite matrix $\mathbf{\Lambda}$ and consider the time-derivative of the integrated tracking error in the following form:

$$\begin{gathered}\xi(t) := \int_0^t \mathbf{e}(\tau)d\tau, \\ \mathbf{S} := \left(\frac{d}{dt}+\mathbf{\Lambda}\right)^3\xi(t) = \overbrace{\dddot{\xi}(t)}^{\ddot{\mathbf{e}}} + 3\mathbf{\Lambda}\overbrace{\ddot{\xi}(t)}^{\dot{\mathbf{e}}} + 3\mathbf{\Lambda}^2\overbrace{\dot{\xi}(t)}^{\mathbf{e}} + \mathbf{\Lambda}^3\xi(t)\end{gathered} \qquad (4.4.15)$$

The term $\mathbf{S}$ is similar to the "error metrics" usually used in the *Variable Structure / Sliding Mode* (*VS/SM*) controllers, and from $\mathbf{S}\equiv\mathbf{0}$ it follows that $\boldsymbol{\xi}\to\mathbf{0}$ as $t\to\infty$. So modify the exerted force/torque components in (4.4.2) as follows:

$$\hat{\mathbf{H}}(\mathbf{q})\left(\ddot{\mathbf{q}}^N+\boldsymbol{\Lambda}^3\xi+3\boldsymbol{\Lambda}^2\mathbf{e}+3\boldsymbol{\Lambda}\dot{\mathbf{e}}\right)+\hat{\mathbf{h}}(\mathbf{q},\dot{\mathbf{q}})=\mathbf{Q}=\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{h}(\mathbf{q},\dot{\mathbf{q}}). \quad (4.4.16)$$

Evidently (4.4.16) is a counterpart of (4.4.2) and via similar manipulations it yields the counterpart of (4.4.3) as

$$\hat{\mathbf{H}}(\mathbf{q})\left[\ddot{\mathbf{e}}+3\boldsymbol{\Lambda}\dot{\mathbf{e}}+3\boldsymbol{\Lambda}^2\mathbf{e}+\boldsymbol{\Lambda}^3\xi\right]=\mathbf{Y}^T(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\tilde{\mathbf{p}} \quad (4.4.17)$$

that justifies the introduction of the array $\breve{\mathbf{x}}=\left[\xi^T,\mathbf{e}^T,\dot{\mathbf{e}}^T\right]^T$ as "state variable" of the formal dynamic system in Lyapunov's theory, and leads to the differential equation

$$\underbrace{\begin{bmatrix}\mathbf{e}\\ \dot{\mathbf{e}}\\ \ddot{\mathbf{e}}\end{bmatrix}}_{\dot{\breve{\mathbf{x}}}}-\underbrace{\begin{bmatrix}\mathbf{0} & \mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I}\\ -\boldsymbol{\Lambda}^3 & -3\boldsymbol{\Lambda}^2 & -3\boldsymbol{\Lambda}\end{bmatrix}}_{\breve{\mathbf{A}}}\underbrace{\begin{bmatrix}\xi\\ \mathbf{e}\\ \dot{\mathbf{e}}\end{bmatrix}}_{\breve{\mathbf{x}}}=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \boldsymbol{\Phi}\tilde{\mathbf{p}}\end{bmatrix} \quad (4.4.18)$$

that is a strict analogy of (4.4.5). On this basis now a new Lyapunov function similar to the original one as $\breve{V}:=\breve{\mathbf{x}}^T\breve{\mathbf{P}}\breve{\mathbf{x}}+\tilde{\mathbf{p}}^T\mathbf{R}\tilde{\mathbf{p}}$ can be introduced in which the positive definite symmetric matrix $\breve{\mathbf{P}}$ contains much more independent elements than the original matrix **P**. It is evident that exactly the same manipulations can be done with the time-derivative of this new function that lead to the "orthodox" tuning rule:

$$-\dot{\hat{\mathbf{p}}}=-\mathbf{R}^{-1}\boldsymbol{\Phi}^T\mathbf{B}^T\breve{\mathbf{P}}\breve{\mathbf{x}}. \quad (4.4.19)$$

It is evident again that (4.4.17) contains all the available information on the parameter estimation error therefore the more "brave" tuning can be applied even in this case, too:

$$-\gamma\mathbf{Y}^{T+}\hat{\mathbf{H}}(\mathbf{q})\left[\ddot{\mathbf{e}}+3\boldsymbol{\Lambda}\dot{\mathbf{e}}+3\boldsymbol{\Lambda}^2\mathbf{e}+\boldsymbol{\Lambda}^3\xi\right]=-\dot{\hat{\mathbf{p}}}. \quad (4.4.20)$$

Again, (4.4.17) guarantees that in the case of proper parameter estimation the tracking error and its integral must converge to zero. In similar manner, for the stage of imperfect tuning the following equation is valid

$$\frac{d}{dt}\left(\overbrace{\breve{\mathbf{x}}^T\breve{\mathbf{P}}\breve{\mathbf{x}}}^{\geq 0}\right):=\breve{\mathbf{x}}^T\left(\breve{\mathbf{A}}^T\breve{\mathbf{P}}+\breve{\mathbf{P}}\breve{\mathbf{A}}\right)\breve{\mathbf{x}}+2\tilde{\mathbf{p}}^T\boldsymbol{\Phi}^T\mathbf{B}^T\breve{\mathbf{P}}\breve{\mathbf{x}} \quad (4.4.21)$$

From which it follows that if a symmetric positive definite matrix $\breve{\mathbf{P}}$ is properly chosen, i.e. $\breve{\mathbf{A}}^T\breve{\mathbf{P}}+\breve{\mathbf{P}}\breve{\mathbf{A}}$ is negative definite, the LHS corresponds to the time-derivative of a positive error metrics, the dominating quadratic term for large $\breve{\mathbf{x}}$ at the RHS is negative and the disturbance term that is only linear in $\breve{\mathbf{x}}$ yields negligible contribution. That means that during the tuning process the tracking error is kept at bay even if the tuning itself is not based on the use of a Lyapunov function and it is yet imperfect. In general similar observations can be done in connection with the original and the adaptive variants of Slotine's and Li's control method [R11] as it will be analyzed in details in the next section.

4.5. Adaptive Slotine-Li Controller for Robots

This controller utilizes subtle details of the equation of motion of the robots (more generally Classical Mechanical Systems) that are not observed and used in the

Adaptive Inverse Dynamics approach, namely the terms quadratic in the time-derivatives of the generalized coordinates are not independent of the inertia matrix. Really, the Euler-Lagrange equations in details are as follows

$$L=\frac{1}{2}\sum_{i,j}H_{ij}\dot{q}_i\dot{q}_j-V(\mathbf{q}),\quad Q_i=\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i}-\frac{\partial L}{\partial q_i}$$
$$Q_i=\sum_j H_{ij}\ddot{q}_j+\sum_{s,j}\frac{\partial H_{ij}}{\partial q_s}\dot{q}_s\dot{q}_j-\frac{1}{2}\sum_{s,j}\frac{\partial H_{sj}}{\partial q_i}\dot{q}_s\dot{q}_j+\frac{\partial V(\mathbf{q})}{\partial q_i}. \tag{4.5.1}$$

It can be observed that the since the quadratic term $\dot{q}_s\dot{q}_j$ is symmetric in the indices (*s,j*) those part of its coefficient that is skew-symmetric in this indices does give contribution in the sum according to *j* and *s*. Therefore, though it is seemingly more complicated because containing more terms, it is enough to keep the symmetric part of this coefficient the symmetry of which later can be conveniently utilized. Since in the symmetrized term the components of $\dot{q}_s\dot{q}_j$ are in equal position, one of them can be included in a matrix **C** that yields the following, generally valid equations of motion:

$$Q_i=\sum_j H_{ij}\ddot{q}_j+\underbrace{\frac{1}{2}\sum_{s,j}\frac{\partial H_{is}}{\partial q_j}\dot{q}_s\dot{q}_j+\frac{1}{2}\sum_{s,j}\frac{\partial H_{ij}}{\partial q_s}\dot{q}_s\dot{q}_j-\frac{1}{2}\sum_{s,j}\frac{\partial H_{sj}}{\partial q_i}\dot{q}_s\dot{q}_j}_{\sum_j C_{ij}(\mathbf{q},\dot{\mathbf{q}})\dot{q}_j}+\underbrace{\frac{\partial V(\mathbf{q})}{\partial q_i}}_{g_i}. \tag{4.5.2}$$

Assuming that neither unknown external disturbances, nor dynamically coupled subsystem unknown by the controller exist, in the possession of an approximate dynamic model in this control method the following generalized forces can be exerted / equations of motion can be obtained for a symmetric positive definite matrix $\mathbf{K}_D$:

$$\mathbf{Q}=\hat{\mathbf{H}}(\mathbf{q})\left(\underbrace{\ddot{\mathbf{q}}^N+\Lambda\dot{\mathbf{e}}}_{\dot{\mathbf{v}}}\right)+\hat{\mathbf{C}}\left(\underbrace{\dot{\mathbf{q}}^N+\Lambda\mathbf{e}}_{\mathbf{v}}\right)+\hat{\mathbf{g}}+\mathbf{K}_D\left(\underbrace{\dot{\mathbf{e}}+\Lambda\mathbf{e}}_{:=\mathbf{r}}\right)=\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}\dot{\mathbf{q}}+\mathbf{g} \tag{4.5.3}$$

in which the Coriolis and the gravitational terms are separately dealt with, $\mathbf{q}^N$ denotes the nominal trajectory, $\mathbf{e}:=\mathbf{q}^N-\mathbf{q}$ denotes the tracking error. It can be observed that the term denoted by **r** corresponds to some error metrics used in the Variable Structure / Sliding Mode controllers. In order express the modeling errors and keep the quantity **v** in the equations $\mathbf{H}d\mathbf{v}/dt$, **g**, $\mathbf{K}_D\mathbf{r}$ and $\mathbf{C}\mathbf{v}$ is subtracted from both sides, and it is utilized again that the array of the dynamic parameters **p** can be separated in a multiplicative form. The result is

$$\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\mathbf{v},\dot{\mathbf{v}})\left(\overbrace{\hat{\mathbf{p}}-\mathbf{p}}^{:=\tilde{\mathbf{p}}}\right)=\left(\hat{\mathbf{H}}-\mathbf{H}\right)\dot{\mathbf{v}}+\left(\hat{\mathbf{C}}-\mathbf{C}\right)\mathbf{v}+\hat{\mathbf{g}}-\mathbf{g}=-\mathbf{K}_D\mathbf{r}-\mathbf{H}\dot{\mathbf{r}}-\mathbf{C}\mathbf{r} \tag{4.5.4}$$

The Lyapunov function chosen by Slotine and Li and its time-derivative is

$$V=\frac{1}{2}\mathbf{r}^T\mathbf{H}\mathbf{r}+\frac{1}{2}\tilde{\mathbf{p}}^T\mathbf{\Gamma}\tilde{\mathbf{p}} \tag{4.5.5}$$

$$\dot{V}=\mathbf{r}^T\mathbf{H}\dot{\mathbf{r}}+\frac{1}{2}\mathbf{r}^T\dot{\mathbf{H}}\mathbf{r}+\dot{\tilde{\mathbf{p}}}^T\mathbf{\Gamma}\tilde{\mathbf{p}} \tag{4.5.6}$$

in which $\mathbf{\Gamma}$ is symmetric positive definite matrix. From (4.5.4) $\mathbf{H}\dot{\mathbf{r}}$ can be expressed and substituted into (4.5.6). By selecting the quadratic terms in $\mathbf{r}$ we obtain that

$$\dot{V} = -\mathbf{r}^T \mathbf{K}_D \mathbf{r} + \mathbf{r}^T \left(\frac{1}{2} \dot{\mathbf{H}} - \mathbf{C} \right) \mathbf{r} + \dot{\tilde{\mathbf{p}}}^T \mathbf{\Gamma} \tilde{\mathbf{p}} - \mathbf{Y} \tilde{\mathbf{p}} \tag{4.5.7}$$

in which the 1st term in the LHS is negative, the 2nd one is zero on symmetry reasons (for this purpose was symmetrized the term containing the quadratic $\dot{q}_s \dot{q}_j$ products), and making the remnant terms zero yields the parameter tuning rule as

$$\mathbf{0} = \left(\dot{\tilde{\mathbf{p}}}^T \mathbf{\Gamma} - \mathbf{Y} \right) \underbrace{\tilde{\mathbf{p}}}_{\neq 0} \Rightarrow \dot{\hat{\mathbf{p}}} = \mathbf{\Gamma}^{-1} \mathbf{Y}^T . \tag{4.5.8}$$

This tuning is much better than that of the *Adaptive Inverse Dynamics Controller*, since it does not require the use of the inverse of the model inertia, and does not require a symmetric positive definite matrix $\mathbf{P}$ with its large number of arbitrary matrix elements. However, it still contains a lot of arbitrary elements in the matrix $\mathbf{\Gamma}$, and does not contain integrated feedback.

4.5.1. Modification of the Parameter Tuning Process in the Adaptive Slotine-Li Controller

Regarding the tuning rule it applies the same observations can be done as in connection with the *Adaptive Inverse Dynamics Controller*:

- Equation (4.5.4) as $\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{v}, \dot{\mathbf{v}})(\hat{\mathbf{p}} - \mathbf{p}) = -\mathbf{K}_D \mathbf{r} - \mathbf{H}\dot{\mathbf{r}} - \mathbf{C}\mathbf{r}$ does not contain all the actually available information on the actual parameter estimation error since the exact matrix $\mathbf{H}$ is unknown. Therefore, for tuning purposes the use of some $\mathbf{\Gamma}$ matrix containing a lot of arbitrary control parameters is needed on formal reasons in the present construction. Its presence is the consequence of insisting on the use of some Lyapunov function.
- To release this difficulty let us go back to (4.5.3), and instead of $\mathbf{H}d\mathbf{v}/dt$, $\mathbf{g}$ and $\mathbf{C}\mathbf{v}$ subtract form both sides $\hat{\mathbf{H}}\ddot{\mathbf{q}}, \hat{\mathbf{C}}\dot{\mathbf{q}}$! This again leads to the appearance of the modeling errors and to the appearance of a well known matrix $\mathbf{\Xi}$ serving as a coefficient of the modeling errors as follows:

$$\hat{\mathbf{H}}\dot{\mathbf{r}} + \hat{\mathbf{C}}\mathbf{r} + \mathbf{K}_D \mathbf{r} = \left(\mathbf{H} - \hat{\mathbf{H}} \right) \ddot{\mathbf{q}} + \left(\mathbf{C} - \hat{\mathbf{C}} \right) \dot{\mathbf{q}} + \mathbf{g} - \hat{\mathbf{g}} = \mathbf{\Xi}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})(\mathbf{p} - \hat{\mathbf{p}}) . \tag{4.5.9}$$

- Since each term in the LHS of (4.5.9) is either known or measurable, and the same holds to the component of $\mathbf{\Xi}$ at the RHS, by the application of the *SVD* on $\mathbf{\Xi}$ in (4.5.9) fast and efficient tuning can be achieved.
- Regarding the behavior of the tracking errors during the tuning process consider the time-derivative of the following quantity that can serve as a kind of metrics for the tracking error, independently of the fact that $\mathbf{H}$ in it is actually unknown:

$$\frac{d}{dt}\left(\mathbf{r}^T \mathbf{H} \mathbf{r} \right) = \mathbf{r}^T \mathbf{H} \dot{\mathbf{r}} + \frac{1}{2} \mathbf{r}^T \dot{\mathbf{H}} \mathbf{r} \tag{4.5.10}$$

- Since (4.5.4) and (4.5.9) are simultaneously valid quite independently of the actual parameter tuning applied, the term $\mathbf{H}\dot{\mathbf{r}}$

can be substituted from (4.5.9) into (4.5.10) yielding $\frac{d}{dt}\left(\mathbf{r}^T\mathbf{H}\mathbf{r}\right)=-\mathbf{r}^T\mathbf{K}_D\mathbf{r}-\mathbf{Y}\tilde{\mathbf{p}}$ from which it immediately follows that in the case of exact parameter estimation $\mathbf{r}\to\mathbf{0}$ as $t\to 0$ (consequently the tracking error also converges to zero), and for improper estimation, i.e. during the process of tuning it is kept at bay by the quadratic negative term $-\mathbf{r}^T\mathbf{K}_D\mathbf{r}$. This observation does not exclude the tuning on the basis of (4.5.9).

The possibility for the introduction of integrated feedback will be considered in the next section.

4.5.2. Introduction of Integrating Feedback in the Adaptive Slotine-Li Controller

To obtain only 2nd order time-derivatives consider the following modification of the error metrics to be used instead of $\mathbf{r}$:

$$\xi(t):=\int_0^t \mathbf{e}(\tau)d\tau,\quad \mathbf{S}:=\left(\frac{d}{dt}+\Lambda\right)^2\xi(t)=\overbrace{\ddot{\xi}(t)}^{\dot{\mathbf{e}}}+2\Lambda\overbrace{\dot{\xi}(t)}^{\mathbf{e}}+\Lambda^2\xi(t),\qquad (4.5.11)$$

by the use of which the exerted forces force / torque components and the equations of motion can be modified as

$$\begin{gathered}\hat{\mathbf{H}}(\mathbf{q})\left(\underbrace{\ddot{\mathbf{q}}^N+2\Lambda\dot{\mathbf{e}}+\Lambda^2\dot{\xi}}_{\dot{\breve{\mathbf{v}}}}\right)+\hat{\mathbf{C}}\left(\underbrace{\dot{\mathbf{q}}^N+2\Lambda\mathbf{e}+\Lambda^2\xi}_{\breve{\mathbf{v}}}\right)+\hat{\mathbf{g}}+\mathbf{K}_D\left(\underbrace{\dot{\mathbf{e}}+2\Lambda\mathbf{e}+\Lambda^2\xi}_{:=\mathbf{S}}\right)=\\ =\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}\dot{\mathbf{q}}+\mathbf{g}\end{gathered}\qquad (4.5.12)$$

[strict counterpart of (4.5.3)]. As it was originally done, in order express the modeling errors and keep the quantity $\breve{\mathbf{v}}$ in the equations, $\mathbf{H}\dot{\breve{\mathbf{v}}}$, $\mathbf{g}$, $\mathbf{K}_D\mathbf{S}$ and $\mathbf{C}\breve{\mathbf{v}}$ can be subtracted from both sides, and it is utilized again that the array of the dynamic parameters $\mathbf{p}$ can be separated in a multiplicative form we can obtain that

$$\breve{\mathbf{Y}}\left(\mathbf{q},\dot{\mathbf{q}},\breve{\mathbf{v}},\dot{\breve{\mathbf{v}}}\right)\left(\overbrace{\hat{\mathbf{p}}-\mathbf{p}}^{\tilde{\mathbf{p}}}\right)=\left(\hat{\mathbf{H}}-\mathbf{H}\right)\dot{\breve{\mathbf{v}}}+\left(\hat{\mathbf{C}}-\mathbf{C}\right)\breve{\mathbf{v}}+\hat{\mathbf{g}}-\mathbf{g}=-\mathbf{K}_D\mathbf{S}-\mathbf{H}\dot{\mathbf{S}}-\mathbf{C}\mathbf{S}\qquad (4.5.13)$$

The modified Lyapunov Function can be constructed as

$$\breve{V}=\frac{1}{2}\mathbf{S}^T\mathbf{H}\mathbf{S}+\frac{1}{2}\tilde{\mathbf{p}}^T\mathbf{\Gamma}\tilde{\mathbf{p}}\,,\qquad (4.5.14)$$

its derivative is

$$\dot{\breve{V}}=\mathbf{S}^T\mathbf{H}\dot{\mathbf{S}}+\frac{1}{2}\mathbf{S}^T\dot{\mathbf{H}}\mathbf{S}+\dot{\tilde{\mathbf{p}}}^T\mathbf{\Gamma}\tilde{\mathbf{p}}\,,\qquad (4.5.15)$$

from (4.5.13) $\mathbf{H}\dot{\mathbf{S}}$ can be substituted to obtain the structure for negative time-derivative

$$\dot{\breve{V}}=-\overbrace{\mathbf{S}^T\mathbf{K}_D\mathbf{S}}^{\geq 0}+\overbrace{\mathbf{S}^T\left(\frac{1}{2}\dot{\mathbf{H}}-\mathbf{C}\right)\mathbf{S}}^{=0}+\left(\dot{\tilde{\mathbf{p}}}^T\mathbf{\Gamma}-\breve{\mathbf{Y}}\right)\overbrace{\tilde{\mathbf{p}}}^{\neq\mathbf{0}}<0\qquad (4.5.16)$$

that leds to the "orthodox tuning rule"

$$\dot{\tilde{\mathbf{p}}}^T = \breve{\mathbf{Y}}\mathbf{\Gamma}^{-1}. \tag{4.5.17}$$

As it was previously done, the "non-orthodox tuning" can be introduced in the following manner: manipulate (4.5.12) by subtracting $\hat{\mathbf{H}}(\mathbf{q})\ddot{\mathbf{q}} + \hat{\mathbf{C}}\dot{\mathbf{q}} + \hat{\mathbf{g}}$ from both sides leading to

$$\hat{\mathbf{H}}\dot{\mathbf{S}} + \hat{\mathbf{C}}\mathbf{S} + \mathbf{K}_D\mathbf{S} = \left(\mathbf{H} - \hat{\mathbf{H}}\right)\ddot{\mathbf{q}} + \left(\mathbf{C} - \hat{\mathbf{C}}\right)\dot{\mathbf{q}} + \mathbf{g} - \hat{\mathbf{g}} = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})(\mathbf{p} - \hat{\mathbf{p}}) \tag{4.5.18}$$

where the LHS and $\mathbf{Y}$ is known, therefore this equation contains all the available information on the parameter estimation error. Therefore, via using *SVD* for $\mathbf{Y}$, replacing the negligible singular values by 0, and computing the "generalized inverse" we arrive at the tuning rule

$$-\gamma\mathbf{Y}^+(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\left[\hat{\mathbf{H}}\dot{\mathbf{S}} + \hat{\mathbf{C}}\mathbf{S} + \mathbf{K}_D\mathbf{S}\right] = -\dot{\hat{\mathbf{p}}}. \tag{4.5.19}$$

Since (4.5.13) still is valid independently of the parameter tuning, the time-derivative of a positive definite error metrics is

$$\begin{aligned} &\frac{d}{dt}\left(\frac{1}{2}\mathbf{S}^T\mathbf{H}\mathbf{S}\right) = \mathbf{S}^T\mathbf{H}\dot{\mathbf{S}} + \frac{1}{2}\mathbf{S}^T\dot{\mathbf{H}}\mathbf{S} = \\ &= \underbrace{\mathbf{S}^T\left(\frac{1}{2}\dot{\mathbf{H}} - \mathbf{C}\right)\mathbf{S}}_{0\text{ due to symmetry}} - \underbrace{\mathbf{S}^T\mathbf{K}_D\mathbf{S}}_{>0} - \underbrace{\mathbf{S}^T\breve{\mathbf{Y}}\tilde{\mathbf{p}}}_{\text{"disturbance"}}, \end{aligned} \tag{4.5.20}$$

that for the case of "perfect estimation" (i.e. when $\tilde{\mathbf{p}} = \mathbf{0}$) guarantees the $\mathbf{S} \to \mathbf{0}$ as $t \to 0$, and for imperfect estimation the negative, quadratic contribution to the time-derivative of the positive number, $-\mathbf{S}^T\mathbf{K}_D\mathbf{S}$ keeps at bay the error-metrics since the disturbance is only its linear function.

4.5.3. Simulation Examples for Adaptive Inverse Dynamics Controller and the Adaptive Slotine-Li Controller

To illustrate the operation of the original and modified controllers simulation results are detailed in Appendix A.1. for the Adaptive Inverse Dynamics Controller, and in Appendix A.2. for the Adaptive Slotine-Li Controller, and their modifications.

On the basis of the simulation results *it can be stated and must be stressed that both the original and the modified versions of the above considered adaptive controllers mathematically are based on the fundamental assumption that the generalized forces are exactly known by the controller, i.e. the controlled system cannot be under permanent effects of external perturbations, and cannot contain not modeled, dynamically coupled subsystems*. Such phenomena as friction mean significant difficulties in this context since the friction models normally are strongly nonlinear and their parameters cannot be separated into a single array within a matrix product structure. Therefore the significant segment of reality does not meet the formal requirements needed for the application of these otherwise very sophisticated methods.

4.6. Thesis 1: Analysis, Criticism, and Improvement of the Classical “Adaptive Inverse Dynamics Controller” and “Adaptive Slotine-Li Controller” (Summary of the Results of Chapter 4)

In this Thesis I gave an analysis of the most sophisticated classical model-based adaptive control approaches as the “*Adaptive Inverse Dynamics Controller*” and the “*Adaptive Slotine-Li Controller*”. These controllers are designed on the basis of appropriate Lyapunov functions and parameter tuning guaranteeing asymptotic stability of the control by assuring negative time-derivative of the Lyapunov functions. I have shown

- that both methods can be completed by the inclusion of integrated tracking error in their feedback loops;
- this completion can be treated by essentially similar Lyapunov functions and parameter tuning strtategies as that of the original methods; (I developed the new Lyapunov functions and the appropriate modification of the original parameter tuning rules;)
- that by dropping the use of the original Lyapunov functions and tuning strategies more efficient parameter adaptation can be developed; the novel tuning proposed directly utilizes all the information available on the actual parameter estimation error by using the same feedback terms and equations of motion as the original methods; this novel tuning is not deduced from the original Lyapunov functions;
- that the tracking errors have to asymptotically converge to zero independently of any Lyapunov function, following the accomplishment of the process of parameter identification;
- that during the novel tuning processes, independently of the details of these processes the tracking errors are kept at bay;
- that in the case of the “*Adaptive Inverse Dynamics Controller*” the critical step, i.e. calculation and use of the inverse of the actual estimation of the inertia matrix can be avoided;
- that both the original and the modified versions are very sensitive to unknown external disturbances and dynamic interactions of the controlled subsystem with not modeled ones;
- that the present form of these controllers cannot be applied whenever the dynamic parameters cannot be separated into a parameter array in a matrix product structure.

All the above statements were substantiated/illustrated by simulation examples. The following publications are related to this Thesis: [J14], [C104], [C106], and [C116]. The subject area was concerned in the oral presentation in [C115].

The general difficulties in the construction of Lyapunov functions, the problems related to the identification of system parameters embedded in nonlinear models of various forms made me seek alternative approaches. For this purpose in the sequel the Soft Computing based approaches will be briefly analyzed.

Chapter 5: Soft Computing as the Use of Universal Approximators

To be correct in briefly and properly evaluating the *method of analytical modeling in technical applications* one has to take into account historical issues regarding the technological background actually available by the researchers.

Historically analytical modeling is strictly related to Euclid's Geometry from the timed 300 BC [R32]. These early steps in the history of mankind made it necessary to study and understand the necessity for introducing the set of *Real Numbers* in order to make certain geometric tasks soluble, and in the same time to study the properties of certain particular functions as x^2, $x^{1/2}$, the trigonometric functions, the exponential and the logarithm functions, etc. The first iterative and numerical techniques were elaborated for calculating the values of these special functions and the first numerical tables were created for these functions only.

Together with the need of the physical interpretation (phenomenology) of the modeled concept for a very long time *analytical modeling* was the only practically viable way for scientists to create quantitative models of reality. With the development of the theory of integrals in the 16th Century it became clear that this set of special functions is not satisfactory for describing everything. For instance the integrals of several special functions cannot be expressed by closed analytical form by using the same set of special functions. In spite of that there was a strict insistence on using these functions together with integral tables even for approximate modeling purposes purely due to the lack of computing power and other technological possibilities for making calculations.

As theoretical possibility the use of function sequences and series to approximate "non-special" functions with "special and well known ones" was a possibility extensively used even in early calculations in Quantum Mechanics in the 1st half of the 20th Century, obtaining precise numerical values was possible only for rich institutions having expensive equipment of high computational power.

Though the mathematical background of using universal approximators for continuous functions appeared in the late fifties [R16] and in the sixties [R62] and [R61], preliminary stage of computer technology at that time did not allow real practical applications. It can be stated that in the beginning of the 21st Century the price of a common PC or laptop with considerable computational power together with available software achieved the level for which it can generally be stated that cheap and efficient computational power became commonly available for everybody for making numerical computations.

The mathematical foundation of the modern *Soft Computing (SC)* techniques goes back to the middle of the 20th Century, namely to the first rebuttal of David Hilbert's 13th conjecture [R59] that was delivered by Arnold [R60] (considering continuous functions of 3 variables), and Kolmogorov [R16] in 1957. Hilbert supposed that there exist such continuous multi-variable functions that cannot be decomposed as the finite superposition of continuous functions of fewer variables. Kolmogorov provided a constructive proof stating that arbitrary continuous function on a compact domain can be approximated with arbitrary accuracy by the composition of single-variable continuous functions. Though the construction of Kolmogorov's functions as well as that of the later refinements of the essentially same idea in the sixties as e.g. by Sprecher [R61], and Lorentz [R62] that are used in this theorem is difficult, his theorem later was found to be the mathematical basis of the present *SC* techniques.

From the late eighties several authors proved that different types of neural networks possessed the universal approximation property [R12], [R13], [R63], [R64]. Similar results have been published from the early nineties in fuzzy theory claiming that different fuzzy reasoning methods are related to universal approximators, too [R14], [R65], [R66]. As it will be highlighted in the sequel, the practical applications of these firm theoretical results always have to cope with sizing and tuning problems.

5.1. Observations on Sizing and Scalability Problems of Classic SC

In spite of these theoretically inspiring and promising conclusions, from the point of view of the practical applicability of these methods various theoretical doubts emerged. The most significant problem was, and remained important problem even in our days, the "*curse of dimensionality*" that means that the approximating models have exponential complexity in terms of the number of components i.e. the number of components grows exponentially as the approximation error tends to zero. *If the number of the components is bounded, the resulting set of models is nowhere dense in the space of the approximated functions.* These observations frequently were formulated in a negatory style, as e.g. in [R67] stating that "*Sugeno controllers with a bounded number of rules are nowhere dense*", and initiated various investigations on the nowhere denseness of certain fuzzy controllers containing pre-restricted number of rules e.g. in [R68], [R69].

In general similar problems arise with the application of the *Tensor Product (TP)* representation of multiple variable continuous functions that were also extended to *Linear Parameter-Varying (LPV)* models [R70]. The *TP representation* can be used for achieving *polytopic decomposition* of *LPV* models i.e. obtaining a linear combination of *Linear Time-Invariant (LTI)* models in which the coefficients of the linear combination depend on time. The application of the *Higher Order Singular Value Decomposition (HOSVD)* provides this result in an especially convenient form [R49], [R50]. Such a preparation or preprocessing of the initial model is very attractive from practical point of view since due to it the Lyapunov-functions based stability criteria generally used in the control of nonlinear systems can be reformulated in the form of *Linear Matrix Inequalities (LMI)*. Due to the pioneering work by Gahinet, Apkarian, Chilai [R71], Boyd [R72], and Bokor e.g. [R15], [R73], the feasibility problem of Lyapunov-based criteria was reinterpreted as a *Convex Optimization Problem*. J. Bokor and his research group gave a very lucid geometrical interpretation of this new representation and methodology that was found to be very fruitful in solving optimization problems, too, beyond stability issues. When polytopic model decomposition is realized and the appropriate control is designed by the use of commercially available software as e.g. MATLAB as in [R74] the available finite computational capacity always seems to be a "*bottleneck*". Possible complexity reduction techniques as e.g. HOSVD have to be applied in order to remain within treatable problem sizes. This technique reduces modeling accuracy in a "controlled" or at least well interpreted manner [R54].

In the case of the use of "traditional" universal approximators various approaches were elaborated to cope with the *sizing problem*. For instance, a *Feedforward Artificial Neural Network* (also referred to as *Multilayer Perceptron*) generally must have only a well defined number of layers (i.e. the input layer, the layer hyperplanes halving the input space, the layer of convex objects, the layer of concave objects, and some output weighting and output layer), the number of the necessary neurons depends on the particular problem under consideration, and can be quite big within the frames of the universal approximators elaborated for multivariable continuous functions. Consequently, for a huge number of "independent parameters" complicated or computational power consuming tuning methods have to be applied.

The "first phase" of using *SC methods*, that is identification of the problem class and finding the appropriate structure, normally is relatively easy. The following phase, i.e. determining the necessary structure-size and fitting it is far less easy. Even in the nineties considerable improvements were achieved in the "learning methods". For neural networks certain solutions start from a quite big initial network and apply *dynamic pruning* for getting rid of the "dead" nodes (e.g. Reed in 1993 [R75]). An alternative method starts with small network, and the number of nodes is increased step by step (see e.g. in Fahlmann & Lebiere 1990 [R76], and Nabhan & Zomaya 1994 [R77]). Due to the possible existence of "local optima" in "backpropagation training" inadequacy of a given number of neurons cannot be concluded simply. Alternative learning methods also including stochastic elements were seriously improved in the nineties and to some extent released this problem (see e.g. in Magoulas et al. 1997 [R78], Chen & Chang 1996 [R79], Kinnenbrock 1994 [R80], Kanarachos & Geramanis 1998 [R81]). However, the generally big size of the classic universal approximators, i.e. the great number of the parameters necessary for accurate modeling generates tuning or learning problems, too, that are briefly considered in the next section.

5.2. Observations on Parameter Tuning Problems in Classic SC

Classic Soft Computing in my view is based on three essential pillars: on certain universal structures representing universal approximators having either *Artificial Neural Networks* or *Fuzzy Systems* based implementation or their combination, and some efficient parameter tuning/setting method that may be based on the traditional causal *Gradient Descent* (often called "*Backpropagation*" in the *ANN* literature in connection with teaching perceptrons) or its close relatives as the *Newton*, *Gauss-Newton* and *Levenberg-Marquardt Algorithms* (this latter was the result of two independent researches [R82], [R83]) or *Simplex or Complex Algorithms*, semi-causal and semi-stochastic tuning like *Simulated Annealing (SA)*, or *Particle Swarm Optimization (PSO)* [R84], or any stochastic or semi-stochastic *Genetic Algorithm (GA)*, or other *Evolutionary Computation (EC)* methods.

It can generally be stated that due to the huge number of parameters to be set in the case of any universal approximators based model the tuning task itself needs considerable computational burden so these approaches are rather fit to offline development of models. The main problem with the gradient descent like methods and the simplex or complex algorithms is that they are apt to converge to a local optimum depending on the surroundings of the normally stochastically chosen initial values. Non-satisfactory operation of this optimum does not automatically mean the necessity of modifying/resizing the structure itself. From different initial values appropriate solution may be achieved by using the same structure. The old method of *SA* (e.g. [R85]) to some extent solves this problem by adding stochastic noise to the gradients so increasing the probability of jumping out of the basin of attraction of local optima that are far from the global one(s). For this purpose various cooling techniques are in use.

The concept of the simple *GA* was invented by Holland and his colleagues in the 1960s and 1970s [R86]. It is especially appropriate for searching in the space of a huge number of parameters for minimizing a single cost function (e.g. [R87], [R88]) that *ab ovo* means a stochastic approach in which the “repetitive search” of the gradient descent like methods is replaced by dealing with the numerous members of great populations. According to [R89] it can be stated that the main problems related to the application of *GA* based methods that is the selection of a proper set of parameters as number of generations, population size, crossover probability, mutation rate, etc. surprisingly are not the subject of ample systematic research. Mainly “rules of thumb” obtained on the basis simulation experience are available for this purpose (e.g. [R90], [R91]). Statistics based approaches are relatively rare and they are restricted to specific problems as e.g. [R92] in which the effect of 17 GP parameters on three binary classification problems were investigated.

The *Multi Objective Genetic Algorithms (MOGA)* try to find the limits of the set of feasible solutions (the so called *Edgeworth-Pareto optimum* [R93]) by describing this set in the system of coordinates of the non negative cost functions. According to this definition a solution is Pareto optimal if no feasible vector of decision variables exists that would decrease some criterion without causing a simultaneous increase in at least one other criterion. That means that if we wish to decrease one of the cost components by moving towards the (desired) zero value along an axis, along some other axis we must increase the appropriate cost component. According to that the multi objective optimum forms some hypersurface in the embedding space (the Pareto front), and its points correspond to various compromises between the different goals, therefore the designer can choose an appropriate point of this geometric object. The result of the basic algorithm that contains dominated (i.e. “optimal”) and non-optimal solutions must be filtered for obtaining the elements of the front (e.g. [R94]). The improvement of this filtering technique recently obtained considerable attention (e.g. [R95]). The basic algorithm suffered various modifications to more evenly cover the Pareto front (e.g. Nondominated Sorting Genetic Algorithm (NSGA) [R96], Fast Non-dominated Sorting Genetic Algorithm (NSGA-II) [R97], etc.) and now they are implemented in the publicly available software package SCILAB 5.1.1. by INRIA.

In spite of this development for strongly coupled non-linear multivariable systems SC still has considerable drawbacks. The number of the necessary fuzzy rules, as well as that of the necessary neurons in a neural network strongly increases with the degree of freedom and the intricacy of the problem. External dynamic interactions on which no satisfactory information is available for the controller influences the system's behavior in dynamic manner. The big structure-sizes and the huge number of tunable parameters, as well as the time-varying “goal” still mean serious problem. These sophisticated approaches need ample computations and do not correspond to our main purposes.

In contrast to these observations *SC techniques* obtained very wide range of real practical applications. As examples implementation of backward identification methods [R99], the control of a furnace testing various features of plastic threads by Schuster [R100], [R101], sensor data fusion by Hermann [R102], building up control mechanisms for *Expert Systems* by Bucko and Madarász [R103], linearization of sensor signals by Kováčová et al. [R104], can be mentioned. The methodology of the *SC techniques*, partly concerning control applications, had fast theoretical development in recent years, too. Various operators concerning the operation of the fuzzy inference processes were investigated by Tick and Fodor [R105], [R106], minimum and maximum fuzziness generalized operators were invented by Rudas and Kaynak [R107], and new parametric operator families were introduced by Rudas [R108], etc.

To resolve the seemingly “antagonistic” contradiction between the successful practical applications and the theoretically proved “nowhere denseness properties” of *SC methods* one became apt to arrive at the conclusion that the problem roots in the fact that Kolmogorov's approximation theorem is valid for the *very wide class of continuous functions* that contains even very “extreme” elements at least from the point of view of the technical applications.

The “extremities” in the class of continuous functions inspire me to seek the possibilities for working with the approximation of models using less “intricate” functions. These efforts are summarized in the next chapter.

Chapter 6: Introduction of Uniform Model Structures for Partial, Temporal, and Situation-Dependent Identification on Phenomenological Basis

The relationship between the proposed novel method and "*Classic Soft Computing*" is symbolized by Fig. 6.1. It indicates that *Classic Soft Computing* in my view is based on three essential pillars: on certain universal structures representing universal approximators having either *Artificial Neural Networks* or *Fuzzy Systems* based implementation or their combination, and some efficient parameter tuning/setting method that can be used for fitting the given structure to the particular problem considered.

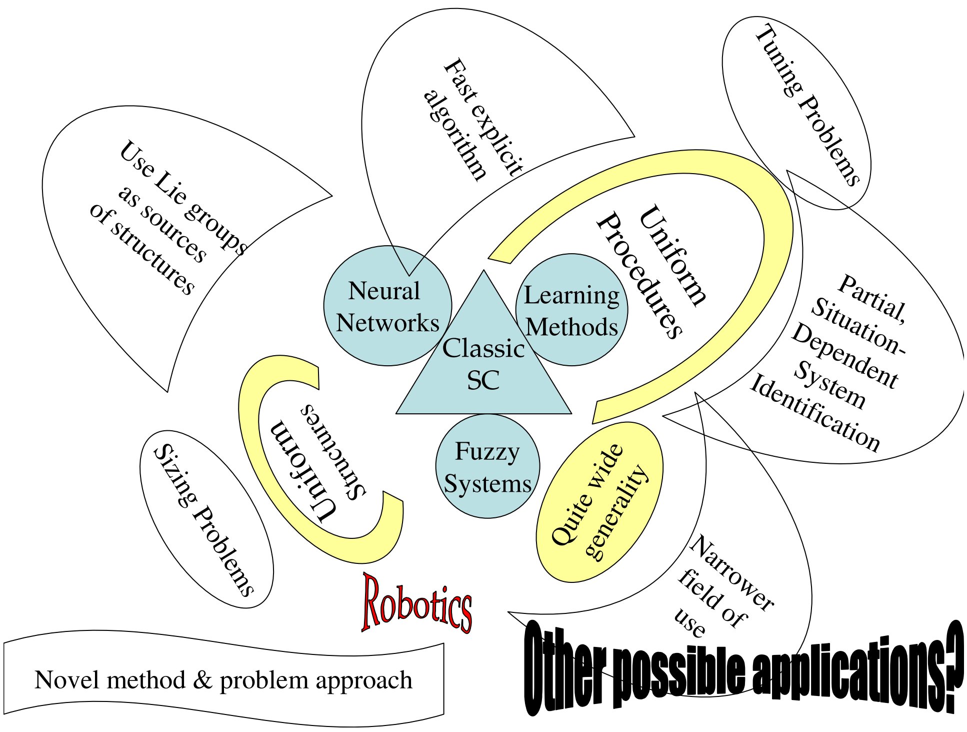


Figure 6.1. The relationships between the proposed novel method and "Classic Soft Computing"

To illustrate the "extremity" of the class of continuous functions for which the nowhere denseness of uniform *SC* structures of limited number of components is valid may be the first example of a function that everywhere is continuous but nowhere is differentiable given by Weierstraß in 1872 [R17].

As it was already mentioned the important features of the traditional SC approaches as "*uniformity*" of the model structures and the parameter tuning/setting procedures sometimes also referred to as "*machine learning*" remained an attractive property that generated a "*challenge*" to construct similar approaches that are free of the scalability problems or the *curse of dimensionality.*

The first steps in this direction were done by considering the phenomenological and formal mathematical structure of *Classical Mechanics (CM)*. It was observed that *Classical Mechanics* in the present control literature can be tackled in essentially two different manners. The direct use of the Euler-Lagrange equations is strictly related to the phenomenological foundations of *CM*: at first a

Lagrange function has to be constructed by choosing an inertial frame of coordinates and expressing the *kinetic energy* of the system considered minus the *potential energy* i.e. that part of the interaction between the system's components and between the system components and the external world that can be originated from some potential. The kinetic energy can be expressed by using the time-derivatives of the Cartesian coordinates of the elementary mass points of the bodies constituting this system by using some generalized coordinates. As a consequence, in the time derivatives appearing in the Euler-Lagrange equations of motion the second derivatives of the Cartesian coordinates occur multiplied with the masses of the elementary points. With respect to an inertial frame such quantities can be interpreted as forces, therefore the generalized forces of these equations obtain the physical interpretation as force or torque quantities for prismatic or rotary axles. *This phenomenology-close formulation does not have too much mathematical simplicity and lucidity because it does not assume direct and simple geometric interpretation.*

A more abstract level of mathematical tackling of *CM* can be achieved by using *Legendre Transform* due to which in the equations of motion *only the first time-derivatives of the independent variables appear*. The mathematical form of the so obtained *Canonical Equations of Motion* is in close relationship with the *Symplectic Geometry* [R25].

Taking into account that due to the phenomenology of the *CM* systems the primary agents immediately determining the second time-derivatives of the generalized coordinates according to the Euler-Lagrange equations in general and in a more specific case of robots [(4.1.1)] are the *generalized forces* related to the second time-derivatives through the *inertia matrix*, *the positive definite nature of the inertia matrix* offered a formal possibility to model the *CM* system by using uniform structures instead of the given, particular analytical dynamic model constructed by the use of e.g. the Denavit-Hartenberg conventions [R6]. Similar statement is true for the used of *Symplectic Geometry* as alternative source of uniform structures for modeling.

Both kinds of "uniformities" were treatable by the systematic use of appropriate mathematical tools (diagonalization of symmetric matrices by orthogonal transformations leading to studying the *Orthogonal Group*), and directly studying the properties of the *Symplectic Group*.

In the sequel at first the approaches based on the *Orthogonal Group* and the *Symplectic* Group will be briefly considered.

6.1. The Orthogonal Group as Source of Uniform Structures in CM

This approach was based on the observation that *the inertia matrix* of a Classical Mechanical system [$\mathbf{H}(\mathbf{q})$ in (4.1.1)] is *symmetric positive definite* and each symmetric positive definite matrix can be diagonalized by an appropriate *Orthogonal Transformation* as

$$\mathbf{H}(\mathbf{q}) = \mathbf{O}(\mathbf{q})\mathbf{D}(\mathbf{q})\mathbf{O}^T(\mathbf{q}) \tag{6.1.1}$$

in which $\mathbf{D}$ is a *diagonal matrix with positive main diagonal elements*, and $\mathbf{O}$ is an *orthogonal matrix*. If besides this in (4.1.1) [$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{Q}$] we consider that the $\mathbf{q}$ and the $d\mathbf{q}/dt$ quantities only slowly can vary due to the inertia of the system, but d^2q/dt^2 can abruptly be modified by exerting appropriate generalized force $\mathbf{Q}$ by the drives of the system, (4.1.1) practically corresponds to an *affine function of* d^2q/dt^2 i.e. it consists of a *linear* and a *constant* part in this variable. If we concentrate on the linear part of (4.1.1) (6.1.1) serves a mathematical means to give a *uniform formulation of* $\mathbf{H}(\mathbf{q})$ if we apply group theoretical considerations.

More specifically it is well known that the *Orthogonal Matrices of Unit Determinant* form a Lie group. The elements of any Lie group can conveniently be parameterized by using matrix exponential functions (see more generally e.g. in [R109]) of the form

$$\mathbf{O}(\mathbf{G},\xi)=\exp(\xi\mathbf{G}) \tag{6.1.2}$$

in which the matrix $\mathbf{G}$ is a *generator* (i.e. an element of the *tangent space* of the group at the unit matrix also belonging to the group), and ξ is a continuous parameter. Since the generators of a Lie group form a linear space (in our case of finite dimensions), and in the particular case of the *Orthogonal Group* the generators are the *skew symmetric matrices* several different variants of (6.1.2) can be invented. For instance, in three dimensional case three linearly independent skew symmetric generators "mixing" the (1,2), (2,3), and (3,1) dimensions can be invented (in higher dimensional cases in similar manner two dimensional subspaces can be chosen for "mixing" the components while the remaining sub-spaces of the whole space remain invariant during the transformations generated by the appropriate generators) generating the elements as

$$\begin{bmatrix}\cos\xi_{12} & -\sin\xi_{12} & 0\\ \sin\xi_{12} & \cos\xi_{12} & 0\\ 0 & 0 & 0\end{bmatrix},\begin{bmatrix}\cos\xi_{13} & 0 & -\sin\xi_{13}\\ 0 & 0 & 0\\ \sin\xi_{13} & 0 & \cos\xi_{13}\end{bmatrix},\begin{bmatrix}0 & 0 & 0\\ 0 & \cos\xi_{23} & -\sin\xi_{23}\\ 0 & \sin\xi_{23} & \cos\xi_{23}\end{bmatrix}. \tag{6.1.3}$$

It is also evident that the *diagonal matrices of positive main diagonal elements* also form a Lie group and can be expressed with their generators in the diagonal form as

$$\langle\exp(\xi_{11}) \quad \cdots \quad \exp(\xi_{nn})\rangle. \tag{6.1.4}$$

Furthermore, taking into account, too, that an even more detailed mathematical form of (4.1.1) can be obtained on phenomenological considerations of *Classical Mechanics* as

$$\sum_j H_{ij}(\mathbf{q})\ddot{q}_j+\sum_{js}\frac{\partial H_{ij}}{\partial q_s}\dot{q}_s\dot{q}_j-\sum_{sj}\frac{\partial H_{sj}}{\partial q_i}\dot{q}_s\dot{q}_j+\frac{\partial V(\mathbf{q})}{\partial q_i}=Q_i \tag{6.1.5}$$

that makes it possible to express the parts originating from matrix $\mathbf{H}$ of (6.1.5) by using the *uniform structures* in (6.1.3) and (6.1.4). Certain particular forms as e.g.

$$\begin{aligned}&\begin{bmatrix}\cos\xi_{12} & -\sin\xi_{12} & 0\\ \sin\xi_{12} & \cos\xi_{12} & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\exp(\xi_{11}) & 0 & 0\\ 0 & \exp(\xi_{22}) & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\cos\xi_{12} & \sin\xi_{12} & 0\\ -\sin\xi_{12} & \cos\xi_{12} & 0\\ 0 & 0 & 0\end{bmatrix}+\\ &+\begin{bmatrix}\cos\xi_{13} & 0 & -\sin\xi_{13}\\ 0 & 0 & 0\\ \sin\xi_{13} & 0 & \cos\xi_{13}\end{bmatrix}\begin{bmatrix}\exp(\xi_{11}) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \exp(\xi_{33})\end{bmatrix}\begin{bmatrix}\cos\xi_{13} & 0 & \sin\xi_{13}\\ 0 & 0 & 0\\ -\sin\xi_{13} & 0 & \cos\xi_{13}\end{bmatrix}+(6.1.6)\\ &+\begin{bmatrix}0 & 0 & 0\\ 0 & \cos\xi_{23} & -\sin\xi_{23}\\ 0 & \sin\xi_{23} & \cos\xi_{23}\end{bmatrix}\begin{bmatrix}0 & 0 & 0\\ 0 & \exp(\xi_{22}) & 0\\ 0 & 0 & \exp(\xi_{33})\end{bmatrix}\begin{bmatrix}0 & 0 & 0\\ 0 & \cos\xi_{23} & \sin\xi_{23}\\ 0 & -\sin\xi_{23} & \cos\xi_{23}\end{bmatrix}\end{aligned}$$

and

$$\frac{\partial H_{ij}}{\partial q_k} = \sum_s \frac{\partial H_{ij}}{\partial \xi_s} \frac{\partial \xi_{st}}{\partial q_k} \tag{6.1.7}$$

were introduced e.g. in [C24], in which the appropriate tunable parameters were adjusted by the *Simplex Algorithm.*.

This tuning is a rough analogy of controlling a bowl rolling on the surface of a plane in a gravitational field: small variation in tilting the plane can keep the bowl on the appropriate trajectory. (In an n dimensional real linear vector space the set of n+1 vertices $\{\boldsymbol{x}^{(i)}|i=0, 1,\ldots,n\}$ forms a simplex if the vectors $\{\boldsymbol{x}^{(i)}-\boldsymbol{x}^{(0)}|i=1,\ldots,n\}$ are linearly independent. The essence of the *Simplex Algorithm* is finding the worst vertex and mirroring it to the center of mass of the remaining n vertices. In this manner the whole simplex proceeds towards a local optimum. Stretching the simplex can improve the speed of the procedure, shrinking the simplex improves its precision. The *Complex Algorithm* is similar to the *Simplex Algorithm:* instead of the *simplex* it applies some *complex* that consists of more vertices than a *simplex.* While in the case of using a simplex only linearly independent "directions" are available, the *Complex Algorithm* is similar to animals having many pieces of hair in their whisker, therefore they have very fine direction-resolution.) The number of the free parameters was found to be large enough to cope with the problem of environmental interactions. The need for small and fast changes in the directly tuned parameters made it possible to use a fast, dynamic, "*incomplete or partial system identification*" with time-varying "*identified*" parameters in (6.1.6).

The form of (6.1.6) has the following advantages with respect to the "traditional" and Artificial Neural Network -based descriptions:

- The tedious work of constructing a dynamic model on the basis of the Denavit-Hartenberg conventions can be avoided.
- The number, the characteristic range and the proper role of each tuned parameter is completely independent of the particular dynamic properties of the robot, is clearly set ($[-\pi,\pi]$ for the rotational ones, and for the exponential terms it is trivial that exp(-10) is very small and exp(10) is very big). This number is quite limited in comparison with the possibly required number of neurons in the case of a multilayer perceptron.

Further ancillary tool to back learning i.e. regression analysis to deal with the distinction between the **H**-dependent and the remaining parts in (6.1.5) was applied in [C25] (later extended to [J3]). To make the initial phase of learning more efficient an "Additional Generalized Force" term based on a simple approximation assuming that for the few previous control steps the change in the joint acceleration must be proportional with the change in the observable generalized forces exerted by the robot drives:

$$\begin{gathered}\mathbf{M}\Delta\ddot{\mathbf{q}} \cong \Delta\mathbf{Q}, \\ \mathbf{H}\sum_{s=1}^{10}\Delta\ddot{\mathbf{q}}(t-s)\Delta\ddot{\mathbf{q}}^T(t-s) \cong \sum_{s=1}^{10}\Delta\mathbf{Q}(t-s)\Delta\ddot{\mathbf{q}}^T(t-s)\end{gathered} \tag{6.1.8}$$

This led to simple version of *Regression Analysis* needing the inversion of a symmetric matrix at the right hand side. Since this relation is significantly concerned by the external dynamic interactions, the matrix **H** so obtained cannot be considered as the estimation of the robot's inertia matrix. It is worth noting that instead calculating the sum of finite number of terms in (6.1.8) computationally it is more efficient to apply the sum of infinite number of terms combined with a *forgetting factor* $0<\alpha<1$: in this case in the control cycle a buffer's content can be multiplied by α and the new term must be added to it.

$$\mathbf{H}\sum_{s=1}^{\infty}\alpha^s\Delta\ddot{\mathbf{q}}(t-s)\Delta\ddot{\mathbf{q}}^T(t-s)\cong\sum_{s=1}^{10}\alpha^s\Delta\mathbf{Q}(t-s)\Delta\ddot{\mathbf{q}}^T(t-s) \tag{6.1.9}$$

The basic idea of the above approach was investigated in various contexts (e.g. in [C22], [C23], [C26], [C28], [C40]). In the sequel a polishing application is considered in details using the results published in [C24] [C23]. Following that an alternative approach is detailed that considers the quasi-diagonalization of the inertia matrix in an alternative manner.

6.1.1. Application Example for the Use of the Orthogonal Matrices as Sources of Uniform Structures in Classical Mechanics

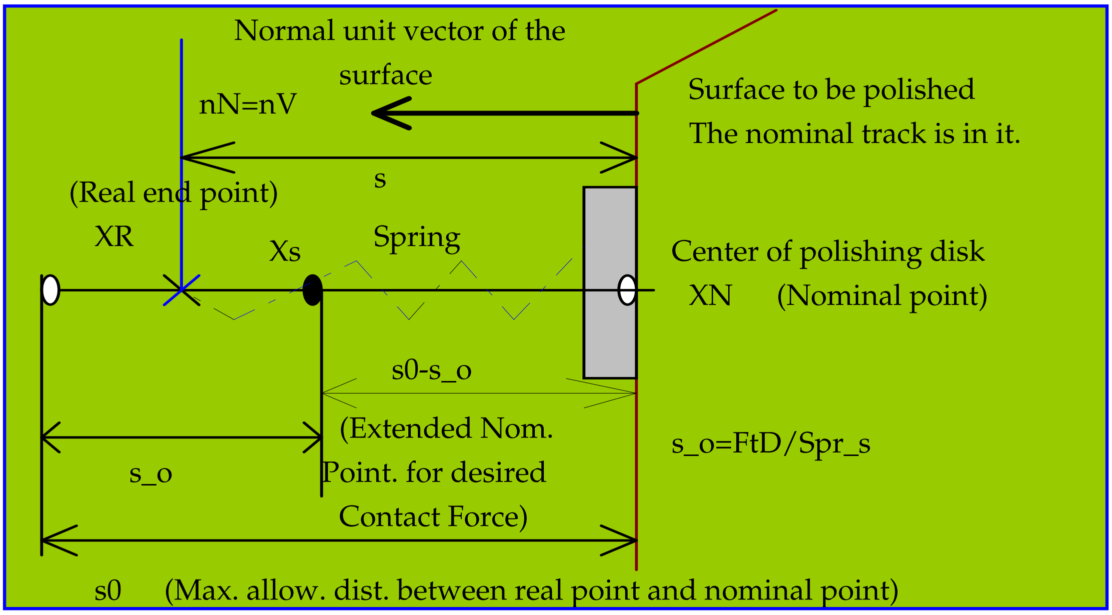


Figure 6.1.1.1. The idea of transforming the force/position/velocity task into pure kinematic problem by using a passive compliance and the proposed control

The here presented figures and conclusions are taken from [C23]. In the present method a 3 DOF SCARA arm having a translational and two rotary joints was completed by a third “link” in the form of a “pipe” parallel with the telescopic shaft and rigidly attached to the end of the second rotary link. The pipe contains a passive elastic component, a spring of a not very large, a priori known stiffness and negligible viscous damping. Consequently, from the purely kinematic data of the spring's deformation its force depending on the contact force prescribed for polishing as a technology requirement can be determined. By knowing the location of the surface to be polished the required contact force can be transformed into the desired location of the endpoint of the last rotary link which otherwise may have arbitrary velocity with respect to the workshop's system of reference. Via applying a cardan link for fixing the polishing disc in the case of mechanical contact the disc will always be located in the tangent plane of the surface of the work-piece at the given point. In the case of a relatively precise location of the disk the errors in the positioning of the disk will be transformed into a minor error in the contact force originally prescribed (see Fig. 6.1.1.1.).

Due to the flexibility of the cardan shaft in the model of dynamic interaction of polishing -used in the simulation only- an even pressure distribution "p" over the disc's surface was supposed. The small surface element of the disc "d**S**" gives the following contribution into the torque:

$$d\mathbf{M} = \mu p dS \left(\frac{\mathbf{v}^{TR} + \Omega \times \mathbf{r}}{\left| \mathbf{v}^{TR} + \Omega \times \mathbf{r} \right|} \right) \times \mathbf{r} \tag{6.1.10}$$

The polishing disc was supposed to have a fast rotation therefore for the great majority of the surface of the disk the velocity component originating from the rotation far exceeded the translational component $\mathbf{v}^{TR}$. By neglecting the effect of the central part of the disk this yields the normal component of

$$M_{norm} \approx \int_0^{2\pi} d\varphi \int_0^R dr \mu p r^2 = \frac{\mu p 2\pi R^3}{3} = \frac{2\mu R}{3} F \tag{6.1.11}$$

in which "F" is the absolute value of the contact force pressing the disc against the work-piece, and "μ" denotes the friction coefficient. The net force of friction from the small surface element has an expression to (6.1.10)

$$d\mathbf{F} = \mu p dS \left(\frac{\mathbf{v}^{TR} + \Omega \times \mathbf{r}}{\left| \mathbf{v}^{TR} + \Omega \times \mathbf{r} \right|} \right) \tag{6.1.12}$$

Again, by neglecting the effect of the small central part of the disc the term in the parentheses in (6.1.12) corresponds to a rotating unit vector resulting in zero in the integral. Therefore the effect of the contact forces was modeled according to (6.1.11).

The directly tuned parameters were the "g_{ijk}" parameters defined as $g_{ijk} \equiv \frac{\partial \xi_{ij}}{\partial q_k}$ in (6.1.6) and in (6.1.7). The estimated inertia was integrated according to these ever varying coefficients. In principle such decomposition can describe the Coriolis forces and other terms quadratic in the angular velocities in (6.1.5). The initial model was a pure diagonal matrix proportional to the identity operator. This was improved step by step by tuning the "g_{ijk}" parameters according to the *Simplex Algorithm* in which the optimum i.e. the difference between the desired and the achieved joint accelerations was minimized. To support this process the following ancillary tools were applied:

- an "*Additional Generalized Force*" term based on a simple version of regression analysis in which the prediction is "qualified" and suppressed according to the noisiness of the environment it originates from;
- a tuned *PID* term described in details e.g. in [C24] (also detailed below);
- a truncation in the angular velocities at a lover limit when calculating the inertia matrix according to (6.1.6) to achieve good adaptivity for slow motion, too;

- a slower external loop simultaneously tuning a "slope" in the PID/ST term and a parameter qualifying the "noisiness" of the regression correction also optimizing a longer term integral of the acceleration error.

Regarding the details, the initial model was a pure diagonal matrix proportional to the identity operator. This model was improved step by step by tuning the "g_{ijk}" parameters according to the Simplex Algorithm in which a function of the difference between the desired ("D") and the realized ("R") joint accelerations is minimized:

$$Cost = \frac{\left|\ddot{\mathbf{q}}^D - \ddot{\mathbf{q}}^R\right|}{1 + \frac{\left|\ddot{\mathbf{q}}^D\right|}{KRel}} \cong \begin{cases} \left|\ddot{\mathbf{q}}^D - \ddot{\mathbf{q}}^R\right| if \left|\ddot{\mathbf{q}}^D\right| << KRel \\ KRel \frac{\left|\ddot{\mathbf{q}}^D - \ddot{\mathbf{q}}^R\right|}{\left|\ddot{\mathbf{q}}^D\right|} if \left|\ddot{\mathbf{q}}^D\right| >> KRel \end{cases} \tag{6.1.13}$$

This cost function is proportional to the relative error in the acceleration for "large" desired acceleration, and it approximates the absolute error for "small" desired ones (terms *large* and *small* are to be understood in comparison with a constant *KRel*). To support this process further ancillary tools were applied:

- a $sigmoid(x) = x/(1+|x|) \in (-1,1)$ function used in stabilization against the effect of extreme noises in the terms
$\sin(\xi_{ij}) \to \sin(\pi \times sigmoid(\xi_{ij}/\pi))$
$\cos(\xi_{ij}) \to \cos(\pi \times sigmoid(\xi_{ij}/\pi))$
$\exp(\xi_{ii}) \to \exp(2.3 \times sigmoid(\xi_{ii}/2.3))$
(for reducing computational complexity this saturated nature is not taken into account in the calculation of the partial derivatives of **H**);
- an "additional generalized force" term based on a simple version of regression analysis in which the prediction is "qualified" and suppressed according to the noisiness of the environment it originates from [C23];
- a *PID* term in which the coefficients of the proportional, derivative and integrated term are tuned as the function of the integrated error in order to keep a prescribed pole-structure in the desired damping of the coordinate errors fixed (described in details e.g. in [9]); in the present version this approach is improved by allowing this feedback increase if the overall torques of the drives are smooth functions of time, and it is decreased in the more "noisy" phases of the motion; here "noisiness" is determined by the *forgetting sum* $c_{\text{int}}(t+1) = \alpha \times c_{\text{int}}(t) + \left|\mathbf{Q}(t) - \mathbf{Q}(t-1)\right|$ and a fuzzy membership function describing the "smoothness" of the torque signal in comparison with a reference value *cCoeff*.

$$c = c_0\left(1 + 2\frac{cCoeff}{cCoeff + (1-\alpha)c_{\text{int}}}\right) \tag{6.1.14}$$

$$\ddot{\varepsilon} = -b'\varepsilon - c'\dot{\varepsilon} - k\int_{-\infty}^{t} \varepsilon(t')dt' \tag{6.1.15}$$

$$\kappa = \frac{c}{2}\left[1 + sigmoid\left(50\left|\int_{-\infty}^{t}\varepsilon(t')dt'\right|\right)\right] \qquad (6.1.16)$$

$$c' = c + \kappa, b' = \frac{c^2}{4} + c\kappa, k = \frac{c^2}{4}\kappa \qquad (6.1.17)$$

The fraction in '*c*' can be also interpreted as a fuzzy set describing the "*smoothness*" of the control: *for small torque derivatives* it approaches 1, while *for too fast changes in the momentum it converges to zero*; this rigid rule means that for strongly varying momentum it is not reasonable to require too strong feedback in order to avoid instabilities and overshoots, but in the "stable phase" of the control an increase in the feedback may improve accuracy.

- "external loop parameters" of slow tuning used as reference values -built in certain fuzzy membership functions- in the "assessment" of several properties of the control; their appropriate value can be set roughly "experimentally"; further slow real-time tuning can help in finding their optimum value; since the optimum setting can change in time, it is expedient to keep them adjusted in real-time.

All the above ancillary tools required minor computational power and also were independent of the particular characteristics of the control problem to be solved.

6.1.2. Simulation Results for the Use of Diagonalization of the Inertia Matrix

In the simulations the robot had the task of polishing a strip on a bell-shaped surface. The strip was located at constant distance from the telescopic axis of the robot. The force with which the polishing disk was requested to press the surface was 1200 [*N*], the spring in the elastic component had the stiffness of *Spr*=400 [*N/m*]. Detailed figures are given in Appendix A.3.

As a conclusion of this section it can be stated that it was illustrated via simulations that the proposed method combining an improved version of the classic *PID/ST and simple uniform structures with free parameters* adjusted by the *Simplex Algorithm* and with the ancillary tool of regression analysis can co-operate successfully. The synthesis of the individually quite limited methods leads to an efficient control in which the significance of the different components remains comparable and changes according to the task to be executed. The method is free from scaling problems. It can be regarded as a compromise between the traditional *Soft Computing* and *Hard Computing*. The introduction of the passive compliant element makes was successful for technological operations.

6.2. The Symplectic Group as Source of Uniform Structures in CM

The phenomenological foundation of any analytical description in *Classical Analytical Mechanics* is the *Lagrangian Model*, by the use of which the kinetic energy of the mechanical systems can be formulated in an inertial system of coordinates in the Newtonian sense. The generalized coordinates and generalized forces in the Lagrangian model normally *in principle are directly measurable (observable) quantities* as rotational angles, angular or linear velocities, and force or torque components. Via introducing the *generalized momentums* the *Hamiltonian Model* can be "built up" on the basis of the Lagrangian one for conservative mechanical systems by using the Legendre Transformation. *This model considers the set of the possible physical states of the system to be a differentiable manifold for the* *description of which different "maps" or abstract systems of coordinates can be applied. Nature distinguishes those maps by the use of which the mathematical form of the state propagation gains the possible simplest form.* The coordinates of these special maps are referred to as *canonical coordinates*, by the use of which the equations of motion take the form of

$$\dot{x}_i = \sum_j \Im_{ij} \frac{\partial H(\mathbf{x})}{\partial \mathbf{x}_j} + \tilde{Q}_i^{Free}, \ \Im \equiv \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix} \tag{6.2.1}$$

in which in which $H(\mathbf{x})$ describes the Hamiltonian (that is the full energy) of the conservative system as the function of its physical state $\mathbf{x}$. This physical state is represented by the canonical coordinates consisting of the generalized coordinates $\mathbf{q}$ in the first, and the generalized momentums $\mathbf{p}$ in the second block of DOF dimensions (*DOF = Degrees of Freedom*) $[\mathbf{q}^T,\mathbf{p}^T]^T$, and the block of the generalized forces $\tilde{\mathbf{Q}}^{Free} = [\mathbf{0}^{\mathbf{T}}, \mathbf{Q}^{\mathbf{Free}^T}]^T$. *It is important to note that the first DOF components of $\tilde{\mathbf{Q}}^{Free}$ must be zero, this directly follows from the Legendre Transformation and from the Euler-Lagrange equations of motion.* From physical point of view the nonzero components of the generalized forces have force dimension [*N*] for linear, and torque dimensions [*Nm*] for the rotary joints. *They represent the appropriate projections of the external free forces exerted by the environment on the robot or by its own drives.* By applying some different map for describing the same physical system another canonical coordinates $\mathbf{x}'(\mathbf{x})$ can be introduced leaving the form of the equation of motion exactly the same as in (6.2.1). These transformations are defined by the restriction that their Jacobian must be *Symplectic*, that is

$$T_{ij} \equiv \frac{\partial x_i'}{\partial x_j}, \ \sum_{s,t} T_{is} \Im_{st} T_{jt} = \Im_{ij}, \det \mathbf{T} = 1 \tag{6.2.2}$$

and lead to the "transformation rule" for the generalized momentums as

$$\tilde{Q}_i'^{Free} = \sum_s T_{is} \tilde{Q}_s'^{Free} \,. \tag{6.2.3}$$

As in the case of the *Euclidean Geometry* the internal symmetry of which is described by the *Orthogonal Transformations* leaving the scalar product of two arbitrary vectors unchanged both in form and in numerical value, the *Symplectic Transformations* can also be considered as mathematical tools describing the internal symmetry of the *Classical Mechanical Systems*. They leave both the numerical value and the form of the quadratic Symplectic structure $\sum_{i,j} u_i \Im_{ij} v_j$ invariant. (Here $\mathbf{u}$ and $\mathbf{v}$ are two arbitrary vectors of the tangent space of the system's physical states. The geometry based on the Symplectic structure is called the *Symplectic Geometry* (see details in Appendix A.10. for the analogies between various geometries occurring in Natural Sciences).

In close analogy with the idea applied by Lajos Jánossy the *Symplectic Transformations* given in (6.2.2) can be also interpreted in an alternative manner that offers the possibility for using them in modeling Classical Mechanical Systems for control purposes. Jánossy studied the *Lorentz Transformations* for the four-component space-time and other physically important vectors $\mathbf{x}'(\mathbf{x})=\mathbf{\Lambda x}$ being the internal symmetry of Maxwell's Electrodynamics (see details in Appendix A.10.) and applied the following observation: a given Lorentz transformation may have two kinds of physical interpretation: *a) the* **x**' *coordinates serve as new coordinates on the same physical system for the description of which the original coordinates* **x** *were used; b) the* **x**' *coordinates may be interpreted as the coordinates of a different physical system (the "deformed" version of the original one) that behaves similar manner as the original physical system since it obeys the same symmetry restriction*. Jánossy called this latter interpretation as the "*Deformation Principle*".

According to the *Deformation Principle* the *Canonical Transformations* may be also interpreted as deformations of the original mechanical system, and on this basis local canonical transformations may be regarded as mathematical possibilities for describing adaptive control by modifying the free parameters of these local transformations. Geometric, group-theoretic and algorithmic aspects of the method were analyzed in details in [J2]. However, for correct phenomenological interpretability the restriction guaranteeing the structure $\tilde{\mathbf{Q}}^{Free} = [\mathbf{0}^{\mathbf{T}}, \mathbf{Q}^{\mathbf{Free}^T}]^T$ in (6.2.3) has to be kept in mind, too.

As it will be discussed in details in paragraph "6.2.2. Complementary Tuning Possibilities in the Cumulative Control", for keeping the first n (n is equal to the *Degree of Freedom* of the mechanical system) components of $\tilde{\mathbf{Q}}^{Free}$ zero even in the case of external perturbations in general block-diagonal symplectic transformations are needed. (Though these special block-diagonal transformations do not "mix" the **q** and **p** components, some coupling between them still remains: if **q** is shrunk/stretched then **p** has to suffer stretching/shrinking. The dimension of the Symplectic transformations not mixing the **q** and **p** components is n^2.)

If we apply a very rough approximate dynamic system model, it can be characterized by a constant inertia matrix **M**, a constant gravitational term **h**, and a Lagrangian defined as

$$L(\mathbf{q},\dot{\mathbf{q}}) \equiv \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}\dot{\mathbf{q}} + \mathbf{h}^T\mathbf{q} \tag{6.2.4}$$

The appropriate restrictions to be imposed for the purposes of the deformation principle are as follows: In the canonical map directly deduced from the Lagrangian model the generalized force vectors have only 1×DOF non-zero components. Since a general canonical transformation can combine all the 2×DOF components of the transformed generalized force vectors, a considerable part of the canonical transformations cannot be applied for deformation purposes. Only those solutions can be accepted for which the necessarily "truncated", phenomenologically non-interpretable components of the generalized force vector are negligible in comparison with the interpretable parts.

On the basis of simple geometric and algebraic considerations (using the *Symplectizing Algorithm* operating with the concept of the *Antiorthogonal Subspaces* detailed in Table A.10.1. of Appendix A.10.) in the first step of the control a "drastic" Symplectic transformation can be introduced.

The effect of this transformation can either be "refined" by applying further Symplectic deformations in the consecutive control steps, or it can be started from the initial rough model immediately in each step.

In the first case, due to the group properties of the symplectic matrices the expected result is a symplectic matrix expressed as a product of the first matrix and many other, near unity symplectic transformations, assuming that the method converges. Since the effects of the step by step deformations are "accumulated" in this product, this solution is referred to as the "*cumulative approach*". The latter one using drastic deformations in each single step is called the "*non-cumulative approach*". Of course, when the result of the cumulative approach starts to become "extreme", casually the identification can be started the onslaught again.

Further possibility independent of this cumulative/non-cumulative approach is *tuning the parameters of the Symplectic Group*. For each *Lie Group* in a manner similar to that of the *Orthogonal Matrices* uniform structures can be introduced for describing its elements. In the sequel the use of these two kinds of transformations will be discussed.

6.2.1. Simple Cumulative Control based on the Symplectizing Algorithm

The essence of this control consists in the difference in the phase currents generated by H' and H in the same point of the differentiable manifold. The idea of *partial system identification* is related to this interpretation: starting with a very rough initial model of constant **M** in the Lagrangian and with a constant **h** the model establishes a connection between the exerted local generalized forces and the propagation of the state-vector $\dot{\mathbf{y}}^{Mod}$. In the reality the encoders measure a different propagation $\dot{\mathbf{y}}^{\mathrm{Re}al} \neq \dot{\mathbf{y}}^{Mod}$. It is expected that the difference can be eliminated by some deformation of the initial model in the form of $H'(\mathbf{z})=H(\mathbf{y}(\mathbf{z}))$. According to the original canonical formulation, an appropriate Symplectic matrix is to be found for which $\mathbf{a} = \dot{\mathbf{x}}^{\mathrm{Re}al} = \mathbf{S}\dot{\mathbf{x}}^{Mod} = \mathbf{S}\mathbf{b}$ is valid. This can be done e.g. in the following way: by making two quadratic matrices of the column vectors **a** and **b** (**A** and **B**, respectively) via "putting near them" further linearly independent vectors the matrix relation **A=SB** can be prescribed. Due to the group properties of the Symplectic matrices this can be satisfied if both **A** and **B** are Symplectic and their first column is equal to **a** and **b**, respectively. This can be achieved by the use of the symplectizing algorithm (Table A.10.1. of Appendix A.10.). The solution is simply $\mathbf{S}=\mathbf{A}\mathbf{B}^{-1}=\mathbf{A}\Im\mathbf{B}^T\Im^T$. in connection with it is worth noting that calculating the inverse of the matrices belonging to Lie Groups defined by nonsingular fundamental quadratic expressions computationally is very cost-efficient: can be solved by two matrix production that is far simpler than inverting a general quadratic matrix. In these investigations step by step "refining" this drastic initial deformation in a "*cumulative*" way was applied.

Regarding stability and convergence issues, for certain conventional control methods as e.g. Model Reference Adaptive Control, on the basis of well defined mathematical restrictions concerning the model of the system to be controlled asymptotic stability can be proved in closed analytical way. In the case of the method here presented such elegant step cannot be done due to the following reasons:

- In general, no any assumption was made regarding the nature of the external perturbations influencing the system.
- Instead such modeling assumptions in each control point the controller can find an appropriate Symplectic matrix which deforms the actual model according to the observed behavior without trying to "explain" the reason of the difference between them.
- Consequently the behavior of such a control can be tested via simulation instead of closed analytical calculations.

However, insisting on the use of the elements of the *Symplectic Group* already contains inherent "brakes" serving the stability and keeping the errors finite. Each Symplectic matrix is unimodular and is invertible via transposition and multiplication by other unimodular matrices. Consequently, all the numerical operations used by the controller are far away from the possibility of singularities ("*inherent brake*"). Besides the "inherent brakes" the method gives ample possibilities for additional or "built in brakes" for serving stability. Any appropriate motion planning guarantees finite error if the desired joint accelerations in it are well approximated. In connection with this the following problems may arise. Though these matrices can never be singular, it may occur that the unimodularity is guaranteed by the occurrence of very big and very small matrix elements. In principle this may cause overflow problems in the digital representation of these matrices. However, such situations may occur only in special cases which can easily be identified by the controller's "built in brakes":

- In the equation **A=SB** the first column of **A** or **B** or both of them is close to zero: in this case the Symplectic identification based on the *Symplectizing Algorithm* can be switched off by the controller and a simple linear control can be followed;
- The extra columns in **A** and **B** or in both are so chosen that one of them is almost parallel with the first vector: this case can be evaded by choosing an initially well conditioned, 2×DOF pieces of linearly independent Symplectic set of vectors in the columns of the matrices, and try to replace the first one with the model and the measured data, respectively; this replacement can be preceded by a permutation of the columns, so the Symplectizing Algorithm will operate without numerical overflow problems;
- In the cumulative approach certain matrix elements may increase in a dangerous manner; this situation can easily be monitored and it can be evaded by starting the Symplectic identification from the beginning with a new, rough single step; after this step further cumulative corrections can be allowed again.

The *Symplectizing Algorithm* used in the cumulative control, too, from certain point of view may behave as a "drastic", non-continuous algorithm especially when the sequence of choosing the appropriate vector to be transformed for being the "Symplectic mate" of the previously chosen vector is concerned. For achieving "continuous" tuning the *Symplectic Group* can be considered as a set parameterized by continuous parameters. This mathematical approach and its complementary use are considered in the next paragraph.

6.2.2. Complementary Tuning Possibilities in the Cumulative Control

From a purely mathematical point of view Symplectic matrices form a *Lie Group* more or less similar to the *Orthogonal Group*. By the use of special generators of this group, each of its elements can be "parameterized" by continuous parameters in the form of simple closed analytical expressions describing *simultaneous exponential stretches and shrinks*, *conventional and hyperbolic rotations in the tangent space* [J2]. On the above basis the following model strategy can be elaborated. Instead using the original matrix relation **A=SB** its slight modification can be introduced in the form as

$$\mathbf{P^*A=SPB} \tag{6.2.5}$$

in which **P*** is a special Symplectic matrix leaving *the first column of* **A** *invariant.* In similar way, **P** also is a Symplectic matrix leaving the first column of **B** invariant. It is evident, that (6.2.5) corresponds to the same control requirement as the original equation **A=SB**, but the resulting Symplectic transformation **S** will differ from the original one. The difference between the two controls consists in different dealing with the antiorthogonal subspaces of the 1st columns of **A** ad **B**, respectively. Since the first columns of the **A**(n) and **B**(n) matrices of the n^{th} control step may contain components from the antiorthogonal subspaces of the 1st columns of the preceding **A**(n-1) and **B**(n-1) matrices, slight tuning of the **P*** and **P** matrices can improve the control quality since it can "reveal" and "trace" tendencies in the variation of the control task.

For constructing appropriate **P** and **P*** matrices their exponential series expression can be used. For instance, let **u** correspond to the first column of matrix **B**, and **G** be one of the generators of the *Symplectic Group*:

$$\mathbf{u} = \mathbf{P}\mathbf{u} = \exp(\mathbf{G}t)\mathbf{u} \equiv \sum_{s=0}^{\infty} \frac{t^s}{s!} \mathbf{G}^s \mathbf{u} \,. \tag{6.2.6}$$

Evidently, if **Gu=0**, that is if **G** maps **u** to zero, (6.2.6) is fulfilled, therefore it is expedient to systematically study the structure of the generators of the *Symplectic Group*.

For this purpose the standard technique for constructing the generators of a *Lie Group* considering almost unit transformations can conveniently be used:

$$(\mathbf{I} + \xi\mathbf{G})\mathfrak{I}(\mathbf{I} + \xi\mathbf{G})^T = \mathfrak{I} + \xi\left(\mathbf{G} + \mathbf{G}^T\right)\mathfrak{I} + \mathcal{O}\left(\xi^2\right) = \mathfrak{I} \,. \tag{6.2.7}$$

In the 1st order approximation according to $\xi \neq 0$ (6.2.7) satisfied if $\mathbf{G}\mathfrak{I} + \mathfrak{I}\mathbf{G}^T = \mathbf{0}$. Since $\mathfrak{I}^T = -\mathfrak{I}$ this means that $\mathfrak{I}\mathbf{G}^T = -\mathfrak{I}^T\mathbf{G}^T = -(\mathbf{G}\mathfrak{I})^T$, therefore the matrix $(\mathbf{G}\mathfrak{I}) = \mathbf{S}$ must be symmetric. This immediately reveals the dimension of the linear space of the generators: for an n DOF mechanical system **S** has the dimensions of $2n \times 2n$ that may have $2n+(4n^2-2n)/2=2n^2+n$ linearly independent elements. Considering **S** in a block-diagonal structure it must consist of the symmetric real **A** and **B** matrices, and an arbitrary **H** matrix as

$$\mathbf{S} = -\left[\begin{array}{c|c} \mathbf{A} & \mathbf{H}^T \\ \hline \mathbf{H} & \mathbf{B} \end{array}\right]. \tag{6.2.8}$$

in which the minus sign was introduced for later convenience, **A** and **B** together have $2[n+(n^2-n)/2]=n^2+n$ independent elements due to their symmetry, and **H** has n^2 independent elements that altogether is $2n^2+n$. Since $\mathfrak{I}^2 = -\mathbf{I}$, $\mathbf{G}\mathfrak{I}^2 = -\mathbf{G} = \mathbf{S}\mathfrak{I}$, so from (6.2.8) it is obtained that

$$\mathbf{G} = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{H}^T \\ \hline \mathbf{H} & \mathbf{B} \end{array}\right]\mathfrak{I} = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{H}^T \\ \hline \mathbf{H} & \mathbf{B} \end{array}\right]\left[\begin{array}{c|c} \mathbf{0} & \mathbf{I} \\ \hline -\mathbf{I} & \mathbf{0} \end{array}\right] = \left[\begin{array}{c|c} -\mathbf{H}^T & \mathbf{A} \\ \hline -\mathbf{B} & \mathbf{H} \end{array}\right]. \tag{6.2.9}$$

It s worth noting that for achieving block-diagonal transformations not mixing the **q** and **p** components, in (6.2.9) the condition **A=0**, **B=0** has to be met, and we have the n^2 linearly independent components of the arbitrary real **H**.

A relatively lucid description of the generators can be achieved if we observe that $\mathbf{G}^T$ in (6.2.9) essentially has the same structure as **G**, therefore if **G** is generator then $\mathbf{G}^T$ is a generator, too:

$$\mathbf{G}^T = \left[\begin{array}{c|c} -\mathbf{H} & -\mathbf{B}^T \\ \hline \mathbf{A}^T & \mathbf{H}^T \end{array}\right]. \tag{6.2.10}$$

Really, in the upper left block of $\mathbf{G}^T$ an arbitrary matrix stands, and in the lower right block minus one times its transpose can be found. In the upper right and lower left corners two independent symmetric matrices can be found, just as in **G**.

Utilizing the fact that the generators form a linear space, symmetric and skew symmetric generators can be constructed of (6.2.9) and (6.2.10) as

$$\frac{1}{2}\left(\mathbf{G}+\mathbf{G}^T\right)=\left[\begin{array}{c|c} -\frac{1}{2}\left(\mathbf{H}^T+\mathbf{H}\right) & \frac{1}{2}\left(\mathbf{A}-\mathbf{B}^T\right) \\ \hline \frac{1}{2}\left(\mathbf{A}^T-\mathbf{B}\right) & \frac{1}{2}\left(\mathbf{H}^T+\mathbf{H}\right) \end{array}\right], \tag{6.2.11}$$

$$\frac{1}{2}\left(\mathbf{G}-\mathbf{G}^T\right)=\left[\begin{array}{c|c} \frac{1}{2}\left(\mathbf{H}-\mathbf{H}^T\right) & \frac{1}{2}\left(\mathbf{B}^T+\mathbf{A}\right) \\ \hline -\frac{1}{2}\left(\mathbf{B}+\mathbf{A}^T\right) & \frac{1}{2}\left(\mathbf{H}-\mathbf{H}^T\right) \end{array}\right]. \tag{6.2.12}$$

It is worth noting again that for not mixing the **q** and **p** components in the symmetric generators (6.2.11) we have the n+(n^2-n)/2 linearly independent symmetric components of **H**, and in the skew symmetric part (6.2.12) we have the (n^2-n)/2 linearly independent skew symmetric components of **H**, that altogether is n^2.

It is very easy to find conveniently applicable basis vectors in the space of the generators that consist of 0 and ±1 matrix elements, and lead to simple exponential series that can be expressed in closed analytical form.

Consider at first the *symmetric* generators! For instance in the case of n=1 the generators that have elements only in the main diagonal of the symmetric matrices yield as e.g.

$$\exp\left\{t\left[\begin{array}{c|c} \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & 0 \end{array}\right] & \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right] \\ \hline \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right] & \left[\begin{array}{c|c} -1 & 0 \\ \hline 0 & 0 \end{array}\right] \end{array}\right]\right\}=\left[\begin{array}{c|c} \left[\begin{array}{c|c} \exp(t) & 0 \\ \hline 0 & 1 \end{array}\right] & \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right] \\ \hline \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right] & \left[\begin{array}{c|c} \exp(-t) & 0 \\ \hline 0 & 1 \end{array}\right] \end{array}\right]. \tag{6.2.13}$$

These generators generate simultaneous stretches and shrinks strictly in the main diagonals. In similar manner, if we have nonzero elements in the nondiagonal parts we easily obtain that

$$\exp\left\{t\left[\begin{array}{c|c} \left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array}\right] & \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right] \\ \hline \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right] & \left[\begin{array}{c|c} 0 & -1 \\ \hline -1 & 0 \end{array}\right] \end{array}\right]\right\}= \\ =\left[\begin{array}{c|c} \left[\begin{array}{c|c} \cosh(t) & \sinh(t) \\ \hline \sinh(t) & \cosh(t) \end{array}\right] & \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right] \\ \hline \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right] & \left[\begin{array}{c|c} \cosh(t) & -\sinh(t) \\ \hline -\sinh(t) & \cosh(t) \end{array}\right] \end{array}\right] \tag{6.2.14}$$

since the 2nd power of this generator just yields the unit matrix, therefore the proper powers of variable t can be recognized in the appropriate matrix elements. So these generators generate *hyperbolic rotations in the main block diagonals*. In close analogy with that hyperbolic rotations can be generated between the nondiagonals as e.g. by

$$\exp\left\{t\left[\begin{array}{cc|cc}0&0&1&0\\0&0&0&0\\\hline 1&0&0&0\\0&0&0&0\end{array}\right]\right\}=\left[\begin{array}{cc|cc}\cosh(t)&0&\sinh(t)&0\\0&1&0&0\\\hline \sinh(t)&0&\cosh(t)&0\\0&0&0&1\end{array}\right] \tag{6.2.15}$$

since the 3rd power of such generators is just identical with their 1st power therefore it is very easy to recognize the appropriate power series of t in the matrix elements.

Regarding the *skew symmetric generators* similar considerations can be done. Consider the main block diagonals in (6.2.12) for n=1:

$$\exp\left\{t\left[\begin{array}{cc|cc}0&-1&0&0\\1&0&0&0\\\hline 0&0&0&-1\\0&0&1&0\end{array}\right]\right\}=\left[\begin{array}{cc|cc}\cos(t)&-\sin(t)&0&0\\\sin(t)&\cos(t)&0&0\\\hline 0&0&\cos(t)&-\sin(t)\\0&0&\sin(t)&\cos(t)\end{array}\right] \tag{6.2.16}$$

since the 3rd power of such generators just are equal to minus one times their first power. Similar observation can be done for the transformations originating by the off-main diagonals as e.g. the example below, in which

$$\exp\left\{t\left[\begin{array}{cc|cc}0&0&-1&0\\0&0&0&0\\\hline 1&0&0&0\\0&0&0&0\end{array}\right]\right\}=\left[\begin{array}{cc|cc}\cos(t)&0&-\sin(t)&0\\0&1&0&0\\\hline \sin(t)&0&\cos(t)&0\\0&0&0&1\end{array}\right] \tag{6.2.17}$$

because the 3rd power of such generators yield their 1st power again, etc. In higher dimensional cases just the same considerations can be done with very similar generators and their power series.

Now let us return to the problem in (6.2.6) for finding appropriate generator for the condition $\mathbf{Gu}=\mathbf{0}$. For convenient utilization of the block structures in (6.2.11) and (6.2.12) let us decompose $\mathbf{u}$ into two sub-blocks of dimension n as $\mathbf{u}=[\mathbf{a}^T,\mathbf{b}^T]^T$ and consider the generally n-2 dimensional orthogonal subspace of $\mathbf{a}$ and $\mathbf{b}$. (For their specialties the $\mathbf{a}$ parallel to $\mathbf{b}$, $\mathbf{a}=\mathbf{0}$, $\mathbf{b}=\mathbf{0}$, and $\mathbf{a}=\mathbf{b}=\mathbf{0}$ cases are not considered since they are insignificant from the needs of the control as later it will be explained). Let the set of orthonormal basis vectors $\{\mathbf{c}^{(i)}|i=3,4,\dots,n\}$ in the orthogonal subset of $\mathbf{a}$ and $\mathbf{b}$! It is very easy to create *symmetric blocks* for (6.2.11) in the form of $S^{(ij)}_{kl}:=(c^{(i)}_k c^{(j)}_l+c^{(i)}_l c^{(j)}_k)/2$ and skew symmetric blocks for (6.2.12) as $A^{(ij)}_{kl}:=(c^{(i)}_k c^{(j)}_l-c^{(i)}_l c^{(j)}_k)/2$ making arbitrary linear combinations of these matrices according to their upper pair of indices (i,j) since the linear combination of symmetric and skew symmetric matrices remain symmetric and skew symmetric, respectively. If the appropriate blocks of generator $\mathbf{G}$ are built up of such linear combinations the $\mathbf{Gu}=\mathbf{0}$ restriction automatically and trivially holds.

The next question is for the goal of elaborating continuous parameterization: how can we calculate the power series of such generators in closed analytical form. The answer is very easy if the set of n+2 vectors is considered consisting of $\mathbf{a}$ and $\mathbf{b}$, and of the columns of the $n\times n$ dimensional unit matrix as $\{\mathbf{a}, \mathbf{b}, \mathbf{e}^{(i)}|i=1,2,\dots,n\}$, in which $e^{(i)}_j=\delta_{ij}$. Normally this set is linearly dependent. If we apply the Gram-Schmidt orthonormalization algorithm detailed in Table A.10.1. of Appendix A.10., n linearly independent, orthonormal unit vectors have to be obtained as $\{\mathbf{c}^{(i)}|i=1,2,\dots,n\}$, in which $\mathbf{c}^{(1)}$ is parallel with $\mathbf{a}$, $\mathbf{c}^{(2)}$ is parallel with the component of $\mathbf{b}$ that is orthogonal to **a**, while the remaining $\{\mathbf{c}^{(i)}|i=3,4,\ldots,n\}$ vectors span the n-2 dimensional subspace orthogonal to both **a** and **b**. By putting near each other the columns of the $\{\mathbf{c}^{(i)}|i=1,2,\ldots,n\}$ vectors, due to their orthogonality an orthogonal matrix **C** is obtained, that satisfying the relationships with the unit matrix **I** as **C=CI**, can be interpreted in the following manner:

$$\begin{aligned}\left[\mathbf{c}^{(1)} \quad \mathbf{c}^{(2)} \quad \cdots \quad \mathbf{c}^{(n)}\right] = \mathbf{CI} = \mathbf{C}\left[\mathbf{e}^{(1)} \quad \mathbf{e}^{(2)} \quad \cdots \quad \mathbf{e}^{(n)}\right] = \\ = \left[\mathbf{Ce}^{(1)} \quad \mathbf{Ce}^{(2)} \quad \cdots \quad \mathbf{Ce}^{(n)}\right]\end{aligned} \tag{6.2.18}$$

i.e. for obtaining the $\mathbf{c}^{(i)}$ vectors the columns of the $\mathbf{e}^{(i)}$ vectors have to be rotated by the orthogonal matrix **C**. Since the orthogonal matrices satisfy the simple relationship $\mathbf{C}^T\mathbf{C}=\mathbf{I}$, the appropriate blocks of **G** can be obtained from the $\mathbf{Ce}^{(i)}\mathbf{e}^{(j)T}\mathbf{C}^T$ matrices. Now consider arbitrary blocks **D**, **E**, **F**, **G**, **K**, **L**, **M**, **N** of the dimension $n\times n$, and consider the matrix product below

$$\begin{aligned}&\left[\begin{array}{c|c}\mathbf{CDC}^T & \mathbf{CEC}^T \\ \hline \mathbf{CFC}^T & \mathbf{CGC}^T\end{array}\right]\left[\begin{array}{c|c}\mathbf{CKC}^T & \mathbf{CLC}^T \\ \hline \mathbf{CMC}^T & \mathbf{CNC}^T\end{array}\right] = \\ &= \left[\begin{array}{c|c}\mathbf{CDKC}^T + \mathbf{CEMC}^T & \mathbf{CDLC}^T + \mathbf{CENC}^T \\ \hline \mathbf{CFKC}^T + \mathbf{CGMC}^T & \mathbf{CFLC}^T + \mathbf{CGNC}^T\end{array}\right] = \\ &= \left[\begin{array}{c|c}\mathbf{C}(\mathbf{DK}+\mathbf{EM})\mathbf{C}^T & \mathbf{C}(\mathbf{DL}+\mathbf{EN})\mathbf{C}^T \\ \hline \mathbf{C}(\mathbf{FK}+\mathbf{GM})\mathbf{C}^T & \mathbf{C}(\mathbf{FL}+\mathbf{GN})\mathbf{C}^T\end{array}\right]\end{aligned} \tag{6.2.19}$$

from which it follows that e.g. the appropriate blocks of the exponential series of the transformed generators can be obtained from the blocks of the exponential series of the original generators multiplied by the orthogonal matrix **C** from the left hand side, and by $\mathbf{C}^T$ from the right hand side. If the original generators are cleverly chosen their exponential series can easily be computed in closed analytical form as it was shown in the equations (6.2.13)-(6.2.17).

Now for the sake of a simple application example apply the above considerations for the case of n=3, in which the components of **u** as **a** and **b** allow only a single dimensional subspace. Since from a single vector no non-zero skew symmetric matrix can be produced, we can produce only the symmetric generators in (6.2.11) by using the unit vector $\mathbf{e}^{(3)}$ and **C** in the form as $\mathbf{K}=\mathbf{Ce}^{(3)}\mathbf{e}^{(3)T}\mathbf{C}^T=\mathbf{CK}_0^{(3)}\mathbf{C}^T=\mathbf{K}^T$, and this symmetric matrix can be placed either into the main diagonals and in the non main diagonals as

$$\left[\begin{array}{c|c}\mathbf{K} & \mathbf{0} \\ \hline \mathbf{0} & -\mathbf{K}\end{array}\right], \left[\begin{array}{c|c}\mathbf{0} & \mathbf{K} \\ \hline \mathbf{K} & \mathbf{0}\end{array}\right]. \tag{6.2.20}$$

The first generator evidently yields the power series as

$$\exp\left\{t\left[\begin{array}{c|c}\mathbf{K} & \mathbf{0}\\ \hline \mathbf{0} & -\mathbf{K}\end{array}\right]\right\}=\left[\begin{array}{c|c}\mathbf{C}\exp\left(t\mathbf{K}_0^{(3)}\right)\mathbf{C}^T & \mathbf{0}\\ \hline \mathbf{0} & \mathbf{C}\exp\left(-t\mathbf{K}_0^{(3)}\right)\mathbf{C}^T\end{array}\right]=$$

$$=\left[\begin{array}{c|c}\mathbf{C}\begin{bmatrix}1&0&0\\0&1&0\\0&0&\exp(t)\end{bmatrix}\mathbf{C}^T & \mathbf{0}\\ \hline \mathbf{0} & \mathbf{C}\begin{bmatrix}1&0&0\\0&1&0\\0&0&\exp(-t)\end{bmatrix}\mathbf{C}^T\end{array}\right]. \qquad (6.2.21)$$

Regarding the second one, its 2nd and 3rd powers can directly be considered as

$$\left[\begin{array}{c|c}\mathbf{0} & \mathbf{K}\\ \hline \mathbf{K} & \mathbf{0}\end{array}\right]\left[\begin{array}{c|c}\mathbf{0} & \mathbf{K}\\ \hline \mathbf{K} & \mathbf{0}\end{array}\right]=\left[\begin{array}{c|c}\mathbf{K}^2 & \mathbf{0}\\ \hline \mathbf{0} & \mathbf{K}^2\end{array}\right]=$$
$$=\left[\begin{array}{c|c}\mathbf{C}\mathbf{e}^{(3)}\mathbf{e}^{(3)T}\mathbf{C}^T\mathbf{C}\mathbf{e}^{(3)}\mathbf{e}^{(3)T}\mathbf{C}^T & \mathbf{0}\\ \hline \mathbf{0} & \mathbf{K}^2\end{array}\right]=. \qquad (6.2.22)$$
$$=\left[\begin{array}{c|c}\mathbf{C}\mathbf{e}^{(3)}\mathbf{e}^{(3)T}\mathbf{C}^T & \mathbf{0}\\ \hline \mathbf{0} & \mathbf{K}^2\end{array}\right]=\left[\begin{array}{c|c}\mathbf{K} & \mathbf{0}\\ \hline \mathbf{0} & \mathbf{K}\end{array}\right]$$

$$\left[\begin{array}{c|c}\mathbf{0} & \mathbf{K}\\ \hline \mathbf{K} & \mathbf{0}\end{array}\right]^3=\left[\begin{array}{c|c}\mathbf{0} & \mathbf{K}\\ \hline \mathbf{K} & \mathbf{0}\end{array}\right]^2\left[\begin{array}{c|c}\mathbf{0} & \mathbf{K}\\ \hline \mathbf{K} & \mathbf{0}\end{array}\right]=$$
$$=\left[\begin{array}{c|c}\mathbf{K} & \mathbf{0}\\ \hline \mathbf{0} & \mathbf{K}\end{array}\right]\left[\begin{array}{c|c}\mathbf{0} & \mathbf{K}\\ \hline \mathbf{K} & \mathbf{0}\end{array}\right]= \quad . \qquad (6.2.23)$$
$$=\left[\begin{array}{c|c}\mathbf{0} & \mathbf{K}\\ \hline \mathbf{K} & \mathbf{0}\end{array}\right]$$

therefore *even* and the *odd* powers can be separately accumulated as

$$\exp\left\{t\left[\begin{array}{c|c}\mathbf{0} & \mathbf{K}\\ \hline \mathbf{K} & \mathbf{0}\end{array}\right]\right\}=\left[\begin{array}{c|c}\cosh(\mathbf{K}) & \sinh(\mathbf{K})\\ \hline \sinh(\mathbf{K}) & \cosh(\mathbf{K})\end{array}\right]=$$
$$=\left[\begin{array}{c|c}\mathbf{C}\begin{bmatrix}1&0&0\\0&1&0\\0&0&\cosh(t)\end{bmatrix}\mathbf{C}^T & \mathbf{C}\begin{bmatrix}1&0&0\\0&1&0\\0&0&\sinh(t)\end{bmatrix}\mathbf{C}^T\\ \hline \mathbf{C}\begin{bmatrix}1&0&0\\0&1&0\\0&0&\sinh(t)\end{bmatrix}\mathbf{C}^T & \mathbf{C}\begin{bmatrix}1&0&0\\0&1&0\\0&0&\cosh(t)\end{bmatrix}\mathbf{C}^T\end{array}\right]. \qquad (6.2.24)$$

It is evident that (6.2.21) and (6.2.24) well and easily programmable, as well as the Gram-Schmidt Algorithm that can be used for a given **u** to construct **C**. In the sequel an application example will be given for the control of a robot arm the endpoint of which is connected to a dashpot producing elastic spring forces and viscous damping as external perturbations.

6.2.3. Application Example for the Use of Symplectic Transformations as the Sources of Uniform Structures in Classical Mechanics

Detailed simulation results obtained for a possible application example are given in Appendix A.4. It can be stated that these results well illustrate and substantiate the applicability of the above theoretical considerations.

6.3. Thesis 2: Introduction of Uniform Model Structures for Partial and Temporal, Situation-Dependent Identification on Phenomenological Basis by Uniform Procedures (Summary of the Results of Chapter 6)

Following a brief critical analysis of the *Classic Soft Computing* based approaches I realized that they have mathematical difficulties of general nature. I have shown that its main theoretical problem as "*bad scalability*" ("*curse of dimensionality*" or the "*nowhere denseness property*" of the uniform structures containing only finite number of elements) originate from the fact that these "models" are related to Kolmogorov's Approximation Theorem. According to that multiple variable continuous functions are approximated by single variable ones over a compact domain.

Keeping in mind the famous example by Weierstraß from 1872 [R17] I realized that the class of "*continuous functions*" is too wide for the great majority of *technical applications* in which mainly "*smooth*" functions occur. I also realized that this simple fact can explain the historically long observation that in many applications *Classical Soft Computing* works very well in spite of the "*nowhere denseness property*" of the practically useful approximations.

To utilize the attractive idea of "*uniformity of structures and procedures*" I proposed the introduction of "*uniform structures and procedures*" *that do not suffer from the curse of dimensionality.* For this purpose I restricted the modeling domain to the description of *Classical Mechanical Systems* that were considered in two different levels of abstraction. The first attempt considered the level of the *Euler-Lagrange Equations of Motion* directly related to the phenomenological foundations of *Classical Mechanics.* The second attempt considered a higher level of abstraction, in the form of the *Canonical Equations of Motion.* On this basis the following modeling structures have been elaborated and investigated via simulations:

- I have shown that *uniform structures can be constructed for the mathematical description of the inertia matrices of Classical Mechanical Systems* independently of the intricate traditional procedure (i.e. the use of *Homogeneous Matrices* and the *Denavit-Hartenberg Conventions* to elaborate *precise analytical models* [R6]). For n Degree of Freedom systems $n \times n$ dimensional Orthogonal Matrices and main diagonal stretches/shrinks of tuneable parameters were introduced to diagonalize a rough initial model of the inertia matrix. I have shown that for this purpose the number of the freely tunable parameters is $n+(n^2-n)/2$.
- I have illustrated via simulation examples that this technique can improve the tracking properties of the control.
- To improve the capacities of the novel uniform modeling technique I analyzed the internal symmetry of the *Canonical Equations of Motion.* I also analyzed the phenomenological aspects related to the *Symplectic Group* that conserves this internal symmetry. I applied Lajos Jánossy's *Deformation Principle* for developing uniform

models and *restricted Symplectic Transformations* for tuning to be used in the adaptive control of *Classical Mechanical Devices*.

- I have pointed out that the number of the free parameters of the *Symplectic Transformations* not mixing the **q** and **p** components in the case of an n DOF system is altogether n^2 that normally is considerably more than the $n+(n^2-n)/2$ parameters of the transformations diagonalizing the inertia matrix.
- I have shown that for parameterizing purposes *symmetric* and *skew symmetric* generators can be introduced for describing the *Symplectic Group*, and elaborated a simple method to express the uniform structures in closed analytical form.
- I have shown that the above *Symplectic Transformations* can be used for realizing a *partial and situation-dependent identification of the uniform models of the classical mechanical systems.* These models need continuous maintenance via *observing the "realized behavior"* and mapping it to the "*expected behavior*" calculated on the basis of the actual model.
- *I have shown that this simple approach has the great advantage that no any effort it needs to distinguish between the effects of the improper system model and that of the external perturbations. Their overall effects are taken into account in the construction of a simple mapping. This is significant advantage in comparison with the Adaptive Inverse Dynamics Control, or with Slotine's and Li's methods in which either the lack, or at least temporal and insignificant nature, or exact knowledge of the external perturbations are assumed, and the adaptation is based on complicated calculations based on Lyapunov's 2^nd^ Method.*
- *I have shown that the modeling method proposed has the great advantage in comparison with the Classic Soft Computing approaches that the sizes of the applicable uniform structures and the numbers of their tunable parameters essentially are determined by the Degrees of Freedom of the systems under consideration. They are limited, and independent of the systems' analytical models. In contrast to that, for instance in a Multilayer Perceptron for modeling nonlinear mapping only the number of the necessary layers is bounded and pre-determined. The number of the free parameters to be tuned is determined by the required precision of the approximation, and strongly depends on the details of the particular problem under consideration.*
- Via discussing the results of simulation examples I illustrated the applicability of this proposed method in the case of a 3 Degree of Freedom *Classical Mechanical System* under external perturbation.

The most important publications related to the contents of Thesis 2 are as follows: [B1], [B2], [J1], [J2], [J3], [J4], [J5], [J6], [J7], [J13], [C1], [C2], [C3], [C4], [C5], [C6], [C7], [C8], [C9], [C10], [C11], [C12], [C13], [C14], [C15], [C16], [C17], [C18], [C19], [C20], [C21], [C22], [C23], [C24], [C25], [C26], [C27], [C28], [C29], [C30], [C31], [C40]. Furthermore, the parts of publications conveying some "criticism" are also relevant here as [J14], [C102], [C106].

Chapter 7: Adaptive Control of Particular Physical Systems by the Abstract Use of Special Elements of Various Lie Groups

As it was already briefly outlined in “Chapter 6: Introduction of Uniform Model Structures for Partial, Temporal, and Situation-Dependent Identification on Phenomenological Basis” It seemed to be reasonable to risk the assumption that “generality” and “uniformity” of the “traditional SC structures” exclude the application of plausible simplifications which may be characteristic to a whole set of typical tasks. This made the idea rise that several “simplified” branches of *SC* could be developed for narrower problem classes if more specific features could be identified and taken into account in the uniform structures.

The first steps in this direction were made in the field of *Classical Mechanical Systems (CMSs)* [J2], while further refinements were published in [J4], on the basis of principles detailed e.g. in [R25]. This approach used the internal symmetry of *CMS*s, the *Symplectic Group (SG)* of *Symplectic Geometry* in the tangent space of the physical states of the system. The result of the situation-dependent system identification was a symplectic matrix mirroring the effects of the inaccuracy of the rough dynamic model initially used as well as the external dynamic interactions not modeled by the controller.

By considering the problem from a purely mathematical point of view quite independently of the phenomenology of *CMS*s, it became clear that all the essential steps used in the control can be realized by other mathematical means than symplectic matrices. The *SG* can be replaced by other Lie groups defined in a similar manner by the use of some “basic quadratic expression” [J5]. In this approach the Lie group used in the control does not describe any internal physical symmetry of the system to be controlled. In the next paragraph the details of this idea are developed.

7.1. The Idea of Cumulative Control Using Minimum Operation Transformations

As is well known, a discrete time model of a causal physical system can be formulated in the form of a difference equation with an external input $\{u_k\}$ that is usually considered to be known (*Autoregressive Moving Average Model with external input - ARMAX*) (e.g. [R113]):

$$y_{k+1} \approx \sum_{s=0}^{N} a_s y_{k-s} + \sum_{w=0}^{M} b_w u_{k-w} \tag{7.1.1}$$

For instance, in the so-called Takagi-Sugeno fuzzy models the consequent parts are expressed by analytical expressions similar to (7.1.1) and they use some linear combinations of the (7.1.1)-type rules in which the coefficients depend on the antecedents. With the help of such Takagi-Sugeno fuzzy IF-THEN rules sufficient conditions to check the stability of fuzzy control systems are now available (e.g. [R114]).

From our particular point of view the most important feature of the model described by (7.1.1) is that *the fading consequences of the past are taken into account in it in an additive manner* with expectedly vanishing linear coefficients i.e. a_s, $b_s \to 0$ as $s \to \infty$. This additive structure entails significant consequences regarding the mathematical form of the various proofs elaborated for describing the behavior of such systems or dealing with the problem of the identification of these coefficients.

Quite different formal approach can be developed for transformations that can be described by the use of Lie groups. For instance, consider the parametric matrix transformation of some "initial" array $\mathbf{a}(t_0)$ in which the transformation $\mathbf{g}(t_n,t_0)$ is the element of certain Lie group:

$$\mathbf{a}(t_n)=\mathbf{g}(t_n,t_0)\mathbf{a}(t_0)=\mathbf{g}(t_n,t_{n-1})\mathbf{g}(t_{n-1},t_{n-2})\mathbf{g}(t_{n-2},t_{n-3})\cdots\mathbf{g}(t_1,t_0)\mathbf{a}(t_0) \quad (7.1.2)$$

Equation (7.1.2) is valid because of the group properties and it interprets the transformation $\mathbf{g}(t_n,t_0)$ as a sequence of consecutive transformations for an *arbitrary time-grid* $[t_n,t_{n-1},\ldots,t_1,t_0]$. If this grid is very fine, the consecutive transformations must be in the close vicinity of the identity operator so they must have the approximate form as

$$\mathbf{g}(t_n,t_{n-1})\approx\mathbf{I}+(t_n-t_{n-1})\mathbf{G}_n \quad (7.1.3)$$

in which $\mathbf{G}_n$ is a certain generator of this group. Perhaps this simple observation gave M.S. Lie an impetus that led him to invent the concept of *Group Algebra* and to elaborate the details of his theory [R38]. It is evident in (7.1.2) that the effects of the past $\mathbf{a}(t_i)$ "states" are accumulated in it as well as in (7.1.1), but instead of the form of some sum it appears in the form of a product. If we take into account the accumulated past in the matrix $\mathbf{g}(t_{n-1},t_0)$, for finding the next step we have to make only some estimation for $\mathbf{G}_n$ that seems to be far simpler task than identifying some hypothetical *ARMAX* coefficients. This simple idea may be applied for modeling and control purposes as it will be detailed in the sequel.

From purely mathematical point of view several control problems can be formulated as follows: there is given some *imperfect model of the system* on the basis of which some *excitation* is calculated for a *desired reaction* of the system used as the input of the controller $\mathbf{i}^d$ as $\mathbf{e}=\boldsymbol{\varphi}(\mathbf{i}^d)$. The system has its inverse dynamics described by the unknown function $\boldsymbol{\psi}$ resulting in a *realized response* $\mathbf{i}^r$ instead of the desired one, $\mathbf{i}^d$: $\mathbf{i}^r=\boldsymbol{\psi}(\boldsymbol{\varphi}(\mathbf{i}^d)):=\mathbf{f}(\mathbf{i}^d)$. (In certain *Classical Mechanical Systems* these values are the *desired* and the *realized* joint accelerations, while the external disturbance forces and the joint velocities serve as the parameters of this temporarily valid and changing function.)

Normally any information on the behavior of the system can be obtained only via observing the actual value of the function $\mathbf{f}()$. In general it can considerably vary in time. Furthermore, no any possibility exists to "directly manipulate" the nature of this function with the exception of the direct manipulation of its actual input from $\mathbf{i}^d$ to certain $\mathbf{i}^{d*}$ that is referred to as the "*deformed input*". The controller's aim is to achieve and maintain the $\mathbf{i}^d=\mathbf{f}(\mathbf{i}^{d*})$ state. [Only the nature of the model function $\boldsymbol{\varphi}(\mathbf{i}^d)$ can directly be determined or manipulated.] According to [C51] the following iteration was suggested to find a proper "deformation" of the input argument:

$$\mathbf{i_0};\mathbf{S}_1\mathbf{f}(\mathbf{i_0})=\mathbf{i_0};\mathbf{i}_1=\mathbf{S}_1\mathbf{i_0};\ldots;\mathbf{S}_\mathrm{n}\mathbf{f}(\mathbf{i}_{n-1})=\mathbf{i_0};\quad \mathbf{i}_{n+1}=\mathbf{S}_{n+1}\mathbf{i}_n;\mathbf{S}_\mathrm{n}\xrightarrow[n\to\infty]{}\mathbf{I} \quad (7.1.4)$$

in which $\mathbf{i}_0$ denotes the initial estimation that can be calculated on the basis of the rough model. *Really, if the series of these consecutive linear transformations converges to the identity operator, no further deformation is needed, and the matrix obtained as the matrix product of the consecutively obtained matrices yields the necessary solution of the problem. This can be so interpreted that the controller "learns" the behavior of the observed system via step-by-step amending and maintaining the initial model.*

Equation (7.1.4) evidently does not yield unambiguous proposition to the necessary $\mathbf{S}_n$ matrices. Infinitely many matrices can be constructed that map a given vector to another given vector. For making the problem mathematically unambiguous this task can be transformed into a matrix equation by putting the values of $\mathbf{f}$ and $\mathbf{i}_0$ into well-defined blocks of bigger quadratic matrices having linearly independent columns as e.g. in the form

$$\mathbf{S}_n\left[\begin{array}{c|c}\mathbf{f}_{n-1} & \dots \\ d & \dots \\ \hline \dots & \dots\end{array}\right]=\left[\begin{array}{c|c}\mathbf{i}^d & \dots \\ d & \dots \\ \hline \dots & \dots\end{array}\right]\Rightarrow \mathbf{S}_n=\left[\begin{array}{c|c}\mathbf{i}^d & \dots \\ d & \dots \\ \hline \dots & \dots\end{array}\right]\times\left[\begin{array}{c|c}\mathbf{f}_{n-1} & \dots \\ d & \dots \\ \hline \dots & \dots\end{array}\right]^{-1}. \qquad (7.1.5)$$

Via the application of a physically not interpreted constant "dummy parameter" the occurrence of the mathematically dubious 0→0, 0→finite, finite→0 transformations can be evaded. If the "columns of the arbitrary parameters" are well defined continuous functions of the first column and a set of linearly independent initial vectors, then the $\mathbf{f}=\mathbf{i}^d$ case evidently results in $\mathbf{S}_n=\mathbf{I}$ that cannot cause computational problems. Via computing the inverse of the matrix containing $\mathbf{f}$ the problem becomes mathematically well-defined. For this purpose it is expedient to choose special matrices of fast and easy invertibility. The accumulated product of the linear transformations in (7.1.4) and the expected matrices converging to the identity operator $\mathbf{I}$ naturally make the idea arise that this approach can be implemented by the use of *Lie Groups*. This expectation is reasonable since the criterion $\mathbf{i}_{n+1}=\mathbf{S}_{n+1}\mathbf{i}_n$ is ambiguous and does not determine any unique matrix $\mathbf{S}_{n+1}$. *Therefore it can be hoped that by imposing extra criteria the solutions can be so restricted that they really belong to certain Lie Group.* On this reason, before going into the details of convergence issues it is expedient to consider certain *algebraic details* concerning the construction of the appropriate linear transformations.

The idea of ***Minimum Operation Transformations*** *means that in each step we try to construct matrices that make considerable transformation only in the 2D subspace spanned by* $\mathbf{f}$ *and* $\mathbf{i}_0$ *(i.e. the directions for which we have fresh information on the behavior of the controlled system), and leave their orthogonal subspaces invariant since no fresh information we have on the temporal behavior of the system in these directions.*

7.1.1. Introduction of Particular Symplectic Matrices

There are various algebraic possibilities to meet this requirement. Let $\mathbf{G}$ be nonsingular, quadratic, otherwise arbitrary constant matrix. The set of the $\mathbf{V}$ matrices for which

$$\det\mathbf{V}=1,\ \mathbf{V}^T\mathbf{G}\mathbf{V}=\mathbf{G}\Rightarrow\mathbf{V}^{-1}=\mathbf{G}^{-1}\mathbf{V}^T\mathbf{G} \qquad (7.1.6)$$

trivially form Lie groups that contain elements in arbitrary close vicinity of the unit matrix. For instance the special cases in which $\mathbf{G}$ corresponds to $\mathbf{I}$, $\Im:=\left[\begin{array}{c|c}\mathbf{0} & \mathbf{I}\\ \hline -\mathbf{I} & \mathbf{0}\end{array}\right]$,

and $\mathbf{g}=\left\langle 1,...,1,-c^2\right\rangle$, result in the *Orthogonal*, the *Symplectic*, and the *Generalized Lorentz Group*, respectively ("*c*", that is the velocity of light in the Lorentz group being the internal symmetry of electromagnetic and relativistic phenomena here can be chosen to be equal to 1). The appropriate special sets are the *Orthonormal*, the *Symplectic*, and the *Generalized Lorentzian Matrices*. It is very easy to construct certain special versions of these matrices in which the "arbitrary blocks" are properly selected [J10], [B3].

The *Minimum Operation Symplectic Transformations* introduced for this purpose have relatively big size but can be constructed by simple rotations and stretches [J5]. They have the structure as follows:

$$\mathbf{S}=\left[\begin{array}{cccc|cccc}\mathbf{f}^{(1)} & \mathbf{u}^{(2)} & \mathbf{e}^{(3)} & \ldots & -\tilde{\mathbf{f}}^{(2)} & -\tilde{\mathbf{u}}^{(1)} & \mathbf{0} & \ldots \\ \hline \mathbf{f}^{(2)} & \mathbf{u}^{(1)} & \mathbf{0} & \ldots & \tilde{\mathbf{f}}^{(1)} & \tilde{\mathbf{u}}^{(2)} & \mathbf{e}^{(3)} & \ldots\end{array}\right] \tag{7.1.7}$$

with

$$\begin{aligned}
\mathbf{u}^{(1)} &= \frac{1}{\mathbf{f}^{(1)T}\mathbf{f}^{(1)}}\left[\mathbf{f}^{(1)}-\frac{\mathbf{f}^{(2)T}\mathbf{f}^{(1)}}{\mathbf{f}^{(2)T}\mathbf{f}^{(2)}}\mathbf{f}^{(2)}\right]\\
\mathbf{u}^{(2)} &= \frac{1}{\mathbf{f}^{(2)T}\mathbf{f}^{(2)}}\left[\mathbf{f}^{(2)}-\frac{\mathbf{f}^{(1)T}\mathbf{f}^{(2)}}{\mathbf{f}^{(1)T}\mathbf{f}^{(1)}}\mathbf{f}^{(1)}\right]\\
\tilde{\mathbf{f}}^{(1)} &= \frac{\mathbf{f}^{(1)}}{\mathbf{f}^{(1)T}\mathbf{f}^{(1)}+\mathbf{f}^{(2)T}\mathbf{f}^{(2)}},\ \tilde{\mathbf{f}}^{(2)}=\frac{\mathbf{f}^{(2)}}{\mathbf{f}^{(1)T}\mathbf{f}^{(1)}+\mathbf{f}^{(2)T}\mathbf{f}^{(2)}}\\
\tilde{\mathbf{u}}^{(1)} &= \frac{\mathbf{u}^{(1)}}{\mathbf{u}^{(1)T}\mathbf{u}^{(1)}+\mathbf{u}^{(2)T}\mathbf{u}^{(2)}},\ \tilde{\mathbf{u}}^{(2)}=\frac{\mathbf{u}^{(2)}}{\mathbf{u}^{(1)T}\mathbf{u}^{(1)}+\mathbf{u}^{(2)T}\mathbf{u}^{(2)}}
\end{aligned}\ . \tag{7.1.8}$$

In (7.1.7) the *degree of freedom of the problem considered* (*DOF*), influences the dimension of the appropriate submatrices. $\mathbf{f}^{(1)}$ has the dimension of *DOF*+1 in which the last component is a nonzero constant to evade the problems of mapping zeros, while the other components are physically interpreted. $\mathbf{f}^{(2)}$ can be obtained from $\mathbf{f}^{(1)}$ e.g. via permutation of its components. Generally $\mathbf{f}^{(1)}$ and $\mathbf{f}^{(2)}$ must be linearly independent non-zero vectors. The $\left\{\mathbf{e}^{(j)}\in\Re^{DOF}\mid j=2,3,\ldots,DOF\right\}$ orthonormal set can arbitrarily be chosen in the orthogonal subset of $\left\{\mathbf{f}^{(1)},\mathbf{f}^{(2)}\right\}$. For realizing this, a full orthonormal set of (*DOF*+1) vectors can be chosen. The 1st vector of this set can be rotated into the direction of $\mathbf{f}^{(1)}$ in a way that leaves the orthogonal subspace of these two vectors invariant while rigidly rotating the whole set. Then the component of $\mathbf{f}^{(2)}$ orthogonal to $\mathbf{f}^{(1)}$ can be determined, and a similar rigid rotation of the previously rotated set can be executed in a special manner that transforms the 2nd vector of this set into this component of $\mathbf{f}^{(2)}$ and leaves the orthogonal subspace of these vectors invariant. (This latter rotation evidently does not alter the direction of the previously set 1st vector.) If it is needed by the particular application under consideration, for finding two nonzero, linearly independent vectors, more than one physically not interpreted constant components can be introduced in $\mathbf{f}^{(1)}$. The "desired" and the "observed" values can be substituted into the 1st *DOF* components of $\mathbf{f}^{(1)}$. By using the simple rules for *real vectors (arrays)* that $\mathbf{a}^T\mathbf{b}=\mathbf{b}^T\mathbf{a}$, as well as the orthogonality of its special block components, the fact that the matrix defined in (7.1.7) trivially can be proved by substituting it into the definition $\mathbf{S}^T\mathfrak{I}\mathbf{S}=\mathfrak{I}$. Actually, due to the orthogonality relationships of certain components only a few terms of this product is not completely trivial. The structure of the terms in (7.1.8) is so constructed that the required zeros, ones, and –ones can easily be obtained in the not completely trivial terms.

Though the above construction worked well according to the simulations, the relatively complicate structure of the matrix in (7.1.7) gave me impetus to find an even simpler structure. The solution came from the simple and trivial observation that the matrix $\mathfrak{I}$ itself is Symplectic. On this basis I tried to modify a little bit the simple structure of this matrix as

$$\mathbf{S} = \left[\begin{array}{ccc|ccc} & \mathbf{0} & & \frac{-1}{s}\mathbf{m}^{(1)} & \frac{-1}{s}\mathbf{m}^{(2)} & -\mathbf{e}^{(3)}\ldots-\mathbf{e}^{(5)} \\ \hline \mathbf{m}^{(1)} & \mathbf{m}^{(2)} & \mathbf{e}^{(3)}\ldots\mathbf{e}^{(5)} & & \mathbf{0} & \end{array}\right]. \quad (7.1.9)$$

The structure of the matrix in (7.1.9) is really similar to that of $\mathfrak{I}$ but in its upper right block instead of the pairwisely orthogonal components of the unit matrix pairwisely orthogonal vectors of not necessarily unit norm are situated. For instance, in the case of a 3 *DOF* physical system (7.1.9) can contain the following components:

$$\mathbf{M} = \left[\mathbf{m}^{(1)},\mathbf{m}^{(2)},\mathbf{m}^{(3)},\mathbf{m}^{(4)},\mathbf{m}^{(5)}\right] = \begin{bmatrix} \ddot{q}_1 & -\ddot{q}_1 & e_1^{(3)} & e_1^{(4)} & e_1^{(5)} \\ \ddot{q}_2 & -\ddot{q}_2 & e_2^{(3)} & e_2^{(4)} & e_2^{(5)} \\ \ddot{q}_3 & -\ddot{q}_3 & e_3^{(3)} & e_3^{(4)} & e_3^{(5)} \\ d & -d & e_4^{(3)} & e_4^{(3)} & e_4^{(5)} \\ D & \frac{\ddot{\mathbf{q}}^2 + d^2}{D} & e_5^{(3)} & e_5^{(4)} & e_5^{(5)} \end{bmatrix} \quad (7.1.10)$$

that evidently can be generalized for higher dimensions as follows: the lower left block has *DOF*+2 components, $\mathbf{m}^{(1)}$ and $\mathbf{m}^{(2)}$ are trivially orthogonal to each other, and they are orthogonal to the pairwisely orthogonal unit vectors of *DOF*+2 dimensions $\{\mathbf{e}^{(3)},\ldots,\mathbf{e}^{DOF+2}\}$. It is not difficult to construct such vectors: one can start with the columns of the unit matrix that is an orthonormal set; its columns can rigidly be rotated in a special rotation modifying only the 2D subspace spanned by $\mathbf{e}^{(1)}$ and $\mathbf{m}^{(1)}$ so that $\mathbf{e}^{(1)'}$ will be parallel to $\mathbf{m}^{(1)}$. Since $\mathbf{m}^{(2)}$ and $\mathbf{e}^{(2)'}$ are in the orthogonal subspace of $\mathbf{m}^{(1)}$, a similar rigid rotation can be constructed that makes $\mathbf{e}^{(2)''}$ parallel with $\mathbf{m}^{(2)}$. This rotation can leave the orthogonal subspace of $\mathbf{e}^{(2)'}$ and $\mathbf{m}^{(2)}$ invariant, that means that in the result $\mathbf{e}^{(1)'}$=$\mathbf{e}^{(1)''}$. Again, it is very easy to see that the matrix defined by (7.1.9) is symplectic. If it is substituted into the definition equation of the Symplectic matrices, due to the lot of zeros in the matrices and the orthogonality relationships only two non-trivial restrictions remain:

$$D^2 \equiv \ddot{\mathbf{q}}^T\ddot{\mathbf{q}} + d^2,\, s = 2D^2 \quad (7.1.11)$$

that certainly can be satisfied. This idea was published in [C53], it was called as "*Special Symplectic Transformations*", and was applied in numerous numerical simulation tests. Before showing simulation example for its application, in the sequel the construction of other special transformations will be considered.

7.1.2. Introduction of Other Special Transformations

As an alternative possibility, the *Generalized Lorentz Group* can be considered. Really, it is very easy to construct special Lorentzian matrices for control purposes. For instance, it is easy to prove that the columns of the following matrix form a generalized Lorentzian set [C42]:

$$\left[\begin{array}{c|c|c|c|c} \mathbf{e}^{(f)}\sqrt{f^2/c^2+1} & \mathbf{e}^{(2)} & \ldots & \mathbf{e}^{(DOF)} & \mathbf{f} \\ \hline f/c^2 & \mathbf{0} & \ldots & \mathbf{0} & \sqrt{f^2/c^2+1} \end{array}\right]. \quad (7.1.12)$$

The size of this matrix is determined by the number of the modeled/observed/controlled degrees of freedom of the physical system to be controlled (*DOF*). The physically interpreted vector **f** is accomplished with a fictitious $(DOF+1)^{th}$ component, and it is placed into the last column of a generalized Lorentzian. The scalar f denotes the absolute value of **f** (Frobenius norm). [The $\mathbf{e}^{(2)}$, …, $\mathbf{e}^{(DOF)}$ set of pairwisely orthogonal unit vectors within the orthogonal subspace of **f**. They can be obtained by rigidly so rotating a whole set of orthonormal basis vectors, e.g. the columns of the unit matrix, that this operation alters only the 2D subspace spanned by **f** and $\mathbf{e}^{(1)}$ by rotating $\mathbf{e}^{(1)}$ into the direction of **f**. $\mathbf{e}^{(f)}$ denotes the unit vector of the direction of **f**.] It is again trivial to prove that (7.1.12) is Lorentzian: it has to be substituted into the quadratic part of the equation of the definition. This construction was also used in several simulation tests.

The "*Minimum Operation Symplectic Transformations*", the "*Special Symplectic Matrices*", and the "*Special Generalized Lorentzian Matrices*" share the common feature that the Frobenius norm of their appropriate block into which the physically interpreted arrays have to be placed is not bounded. The *Orthogonal Group* as a plausible and simple Lie group does not have such a property. On this reason some attempts were done to introduce the combination of rotations and isotropic stretch operation as follows:

$$\mathbf{T} = s\mathbf{W} \tag{7.1.13}$$

in which **W** is orthogonal matrix, and $s>0$ is a positive parameter. These **T** matrices trivially form a Lie group and they can easily be constructed for a pair of non-zero vectors **a** and **b** so that **a**=**Tb**. In this operation simply $s=|\mathbf{a}|/|\mathbf{b}|$, and **W** makes a rotation concerning only the two-dimensional sub-space stretched by **a** and **b**. It is clear that while the calculation of these "*Stretched Orthogonal Transformations*" is very simple, and they immediately yield the solution needed, their stretch factor s concerns any sub-spaces. This construction evidently does not correspond to the principle of "*Minimum Operation Transformations*" while "*Special Generalized Lorentzian Matrices*", the "*Minimum Operation Symplectic Matrices*", and the "*Special Symplectic Transformations*" correspond to it because they apply stretch/shrink only in well-defined sub-spaces, while the other sub-spaces are rotated only. This may explain that the application of the "*Stretched Orthogonal Group*" normally gave considerably weaker results than the other matrices.

The special transformations introduced above provide us with a convenient mathematical framework within which linear transformations converging to the identity matrix **I** can conveniently be constructed. The next question is whether what conditions are needed to guarantee this desired convergence. From this point of view the properties of the appropriate physical system under consideration as well as that of their approximate models used by the controller in the beginning of the control process are important factors. In the next subsection we show that the iterative learning expressed by (7.1.4) can be convergent for a wide class of physical systems.

7.2. Proof of Complete Stability for a Wide Class of Physical Systems

In the realm of *Cellular Neural Networks* the concept of *Complete Stability* is often used as a criterion [R19]. It means that for a constant input excitation the network asymptotically converges to a constant response. If the variation of the input is far slower than the network's dynamics, with a good accuracy, the net provides with a continuous response corresponding to some mapping of the time-varying input. The same idea is applied when we seek for series pertaining to a constant desired response.

The $\mathbf{f}(\mathbf{i}_n) \to \mathbf{i}_0$ requirement can be expressed in more or less restricted forms. For instance, assume, that there exists 0<*K*<1 for which

$$\left\|\mathbf{f}(\mathbf{i}_n)-\mathbf{i}_0\right\| \le K\left\|\mathbf{f}(\mathbf{i}_{n-1})-\mathbf{i}_0\right\| \le \ldots \le K^n\left\|\mathbf{f}(\mathbf{i}_0)-\mathbf{i}_0\right\| \qquad (7.2.14)$$

This requirement trivially guarantees the desired complete stability with a convergence to the desired value. So assume that there is given an unknown, differentiable, invertible function **f**(**x**) for which there exists an inverse of $\mathbf{x}^d$ as $\hat{\mathbf{x}} = \mathbf{f}^{-1}(\mathbf{x}^d) \ne 0$. Let the Jacobian of **f** that is $\mathbf{f}'(\hat{\mathbf{x}}) \equiv \frac{\partial \mathbf{f}}{\partial \mathbf{x}}$ be positive definite and of a norm considerably smaller than 1. Furthermore, let us assume that the actual estimation of the deformed input **x** is quite close to the proper inverse of $\mathbf{x}^d$. Consequently two near-identity linear transformations must exist in the chosen group for which $\mathbf{T}\hat{\mathbf{x}} = \mathbf{x}$, and $\mathbf{S}\mathbf{f}(\mathbf{x}) = \mathbf{x}^d$. Following the classical perturbation theory, if variable ξ is chosen to be the "small variable" the above operators can be written as $\mathbf{T} = \mathbf{I} + \xi\mathbf{G}, \mathbf{S} = \mathbf{I} + \xi\mathbf{H}$ in which **G** and **H** must be certain generators of the given group used for describing the transformations. Taking into account only the 0th and the 1st order terms in ξ we obtain the estimations as

$$\begin{gathered}\mathbf{f}(\mathbf{x}) \cong \mathbf{f}([\mathbf{I}+\xi\mathbf{G}]\hat{\mathbf{x}}) \cong \mathbf{x}^d + \xi\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{G}\hat{\mathbf{x}} \\ \mathbf{S}\mathbf{f}(\mathbf{x}) \cong (\mathbf{I}+\xi\mathbf{H})\left(\mathbf{x}^d + \xi\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{G}\hat{\mathbf{x}}\right) \cong \mathbf{x}^d + \xi\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{G}\hat{\mathbf{x}} + \mathbf{H}\mathbf{x}^d\right) + O(\xi^2) = \mathbf{x}^d\end{gathered} \qquad (7.2.15)$$

This implies that

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{G}\hat{\mathbf{x}} + \mathbf{H}\mathbf{x}^d = 0. \qquad (7.2.16)$$

The next approximation so is determined as

$$\begin{gathered}\mathbf{f}(\mathbf{S}\mathbf{x}) = \mathbf{f}(\mathbf{S}\mathbf{T}\hat{\mathbf{x}}) \cong \\ \cong \mathbf{f}([\mathbf{I}+\xi\mathbf{H}][\mathbf{I}+\xi\mathbf{G}]\hat{\mathbf{x}}) \cong \mathbf{x}^d + \xi\frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{G}+\mathbf{H})\hat{\mathbf{x}}\end{gathered} \qquad (7.2.17)$$

For the convergence we need decreasing error, that is $\left\|\mathbf{f}(\mathbf{S}\mathbf{x})-\mathbf{x}^d\right\| \le K\left\|\mathbf{f}(\mathbf{x})-\mathbf{x}^d\right\|$ which means that

$$\left\|\xi\frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{G}+\mathbf{H})\hat{\mathbf{x}}\right\| \le K\left\|\xi\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{G}\hat{\mathbf{x}}\right\| \text{ or } \left\|\mathbf{H}\mathbf{x}^d - \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{H}\hat{\mathbf{x}}\right\| \le K\left\|\mathbf{H}\mathbf{x}^d\right\| \qquad (7.2.18)$$

For a finite generator **H** $\mathbf{H}\hat{\mathbf{x}}$ and $\mathbf{H}\mathbf{x}^d$ must be approximately of the same norm and same direction that is the angle between them is acute because it was assumed that $\hat{\mathbf{x}} \cong \mathbf{x}^d$, and because the matrix multiplication is a continuous operation. Due to the positive definite nature of $\mathbf{f}' := \frac{\partial \mathbf{f}}{\partial \mathbf{x}}$ multiplication with it also can result in an acute angle between $\mathbf{H}\mathbf{x}^d$ and $-\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{H}\hat{\mathbf{x}}$. Therefore (7.2.17) has the following geometric interpretation [Fig. 7.2.1.]:

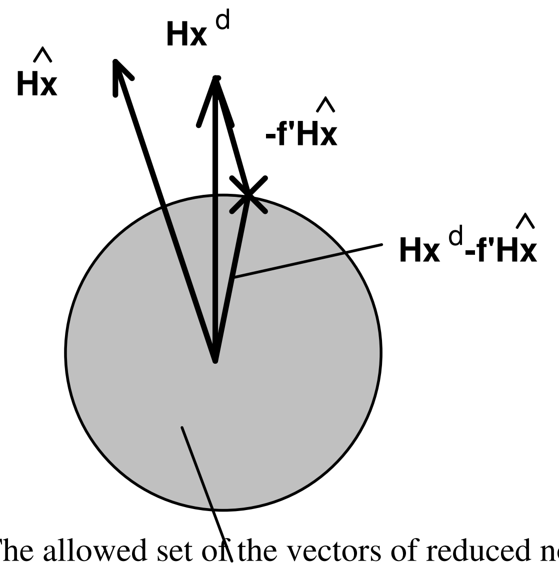


Figure 7.2.1. Geometric interpretation of the convergence criteria

On the basis of Fig. 7.2.1. it is evident that under quite general conditions the algorithm can converge to the desired solution. For instance, in the Euler-Lagrange equations of a *Classical Mechanical System* the inertia matrix can be roughly approximated by a *small scalar inertia plus a big additive „vector" term*. The positive definite nature of the inverse of the actual inertia matrix of the system can guarantee convergence for this problem class.

To keep the desired value in the vicinity of the actual one, which also is a necessary criterion of convergence, instead of the joint accelerations its "interpolated" version is taken into account, that is a "regulated correction" is applied as:

$$\xi = \frac{\left\|\mathbf{x}^d - \mathbf{f}\right\|}{1 + \max\left(\left\|\mathbf{x}^d\right\|, \left\|\mathbf{f}\right\|\right)},\ \lambda = 1 + \varepsilon_1 + \left(\varepsilon_2 - 1 - \varepsilon_1\right)\frac{\upsilon\xi}{1+\upsilon\xi},\ \hat{\mathbf{x}}^{\mathrm{d}} = \mathbf{f} + \lambda\left(\hat{\mathbf{x}}^{\mathrm{d}} - \mathbf{f}\right). \quad (7.2.19)$$

For large relative difference the "regulating factor" $\lambda \to \varepsilon_2$ (a small value moderating the norm of the transformation matrices to be applied), while for small difference it approaches $(1+\varepsilon_1)$, with a mall positive ε_1 meaning a kind of slight extrapolation of the tendency. The $\upsilon>0$ variable has the function of a shape-parameter.

7.3. Simulation Example for Potential Application of the Special Symplectic Matrices

In the forthcoming simulations the *Special Symplectic Matrices* defined in (7.1.10) are applied. An important aspect in connection with incomplete modeling is the existence of two possible alternative approaches: application of a single, complex rough initial model containing each modeled degree of freedom, or tackling the problem in a "distributed" manner in which certain subsystems are controlled by independent controllers modeling and controlling only certain degrees of freedom of the subsystem in their care. In this case, for the local, decentralized controllers, any dynamic coupling between the locally controlled subsystems appears as external perturbation influencing the behavior of the subsystem under their control. This problem was discussed in details e.g. in a plenary speech by D'Andrea in connection with the dynamic coupling of wings located in each other's vicinity in flowing air [R120]. Since the above discussed approach offers simple and convenient implementation for both "centralized" and "distributed" approach to control the incompletely modeled system, in Appendix A.5. its operation was investigated via simulation. It can be stated that the simulation results well illustrated the applicability of the proposed ideas.

7.4. Thesis 3: Adaptive Control of Particular Physical Systems by the Abstract Use of Special Elements of Various Lie Groups (Summary of the Results of Chapter 7)

This Thesis concentrates on the *potential application of the formal mathematical properties* of particular Lie groups. The elements of these groups occur in *multiplicative terms* realizing a cumulative control by multiplying near-identity matrices in the realization of the *situation-dependent, temporal, and partial system identification*. I have elaborated various particular matrices of very simple structure. Instead *parameter tuning* I introduced "*Minimum Operation Transformations*" that map the "*expected*" and "*observed*" behavior of the controlled systems to each other. These mappings are so constructed that essential modification happens only in the *freshest available information in the direction of the recently observed subspaces* of the systems' behavior. This means a restriction on the arbitrary parameters of the possible mappings. The controller utilizes the inverses of certain matrices. Since these matrices belong to special Lie Groups the computation of their inverse is very cost effective (the maximum effort is the calculation of two matrix products). More specifically:

- For application purposes I invented *two kinds of particular symplectic matrices*, invented *special generalized Lorentzian matrices*, and introduced the *stretched orthogonal transformations*.
- I have provided a proof that shows that the so constructed adaptive control can be convergent for a wide class of physical systems. The proof is based on considerations similar to those that are in use in "*perturbation approximations*" (*Perturbation Calculus*);
- I have invented *ancillary methods* as the application of the "*regulating*" and "*weighting*" *factors* that can keep the control convergent in the initial phase when the appropriate transformation matrices are very far from the identity transformation, so in this stage of the control the use of the *Perturbation Calculus* would not be justified;
- I have demonstrated the potential use of these approaches in various simulation tasks, among others in the control of partially and imprecisely modeled, dynamically coupled subsystems in a *centralized* and in a *distributed implementation*.

In general it can be stated that the present approach uses far simpler uniform structures and procedures than any conventional soft computing approach, does not apply any parameter tuning, and does not require the construction of a complicated Lyapunov function. Its deficiency is that it can guarantee complete stability on the basis of local (i.e. not global) basis. The convergence of the so constructed control can be lost outside of a local basin of attraction.

The most important publications strictly related to the contents of Thesis 3 are as follows: [B3], [B6], [J5], [J6], [J7], [J8], [J9], [J10], [C32], [C33], [C34],

[C35], [C36], [C37], [C38], [C39], [C40], [C41], [C42], [C43], [C44], [C45], [C46], [C47], [C48], [C49], [C50], [C51], [C52], [C53], [C54], [C56], [C57], [C58], [C59], [C60], [C61], [C62], [C63], [C64], [C65], [C67], [C68], [C74]. Other related publications are as follows: [C80], [C83], [C84], [C93], [C96].

Chapter 8: Introduction of Various Parametric Fixed Point Transformations for the Adaptive Control of Special SISO and MIMO Systems

In this chapter an even more simple possible control effort is considered than that detailed in the previous chapters. While the above approaches invested some energy into the task of "identifying" some temporal and situation-dependent "system model of uniform structure and limited size" via uniformized algorithms or procedures of strongly reduced number of operations, in the present chapter *we investigate the possibility of developing adaptive control without any particular identification effort. The convergence of this approach can be guaranteed only by certain simple, qualitative properties of the system to be controlled.* These "qualitative" features known in advance mean the "speciality" of these systems. As it will be shown in the sequel the proposed solutions can be embedded into the general mathematical framework of *convergent sequences generated by contractive mappings defined over Banach Spaces*. Their convergence can be proved by using simple geometric analogies. The first attempts were made for *Single Input – Single Output (SISO)* systems, and then later the idea was generalized to *Multiple Input – Multiple Output (MIMO)* systems by using various norms in the appropriate *Banach Spaces* and norms for the operators defined over these spaces. Another important fact is the introduction of the "robust variant" of these transformations applying strongly saturated sigmoids for the generation of a local basin of attraction for these sequences.

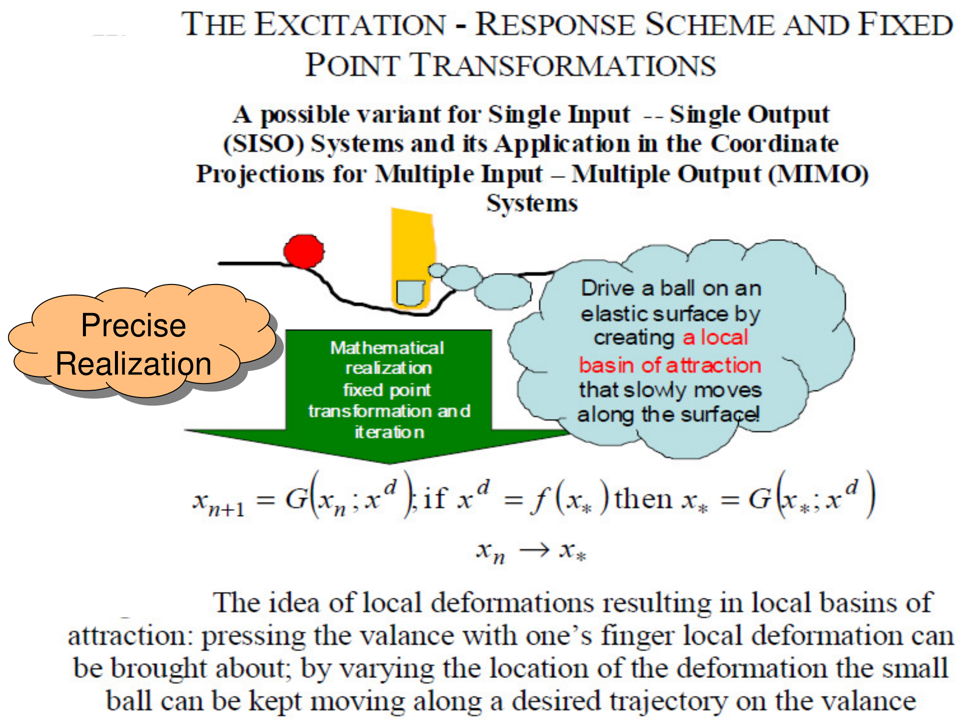
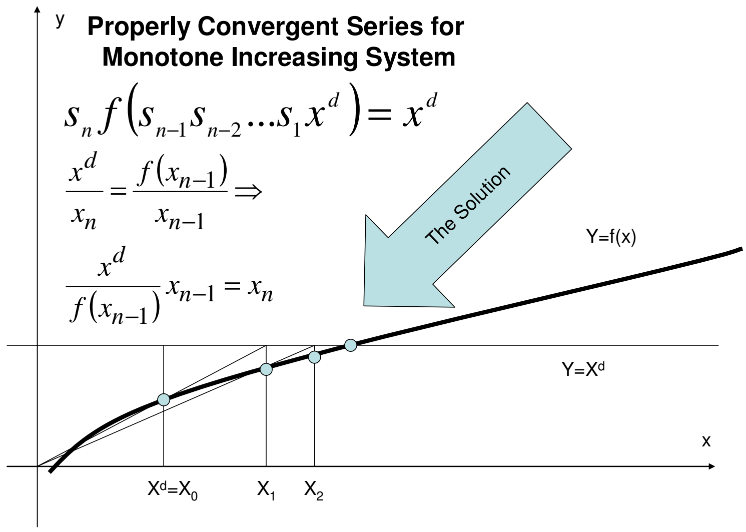


Figure 8. A rough sketch concerning the idea of applying local deformations for convergent adaptive control

In contrast to the Lyapunov function based techniques that normally try to guarantee "*Global Stability*" i.e. forming an unbounded basin of attraction for the initial errors, the present approach is satisfied by creating some local basin of attraction as it is intuitively outlined in Fig. 8.

8.1. Fixed Point Transformations with a Few Parameters for "Increasing" and "Decreasing" SISO Systems

As in the case of "*Chapter 7.1. The Idea of Cumulative Control Using Minimum Operation Transformations*" the (in this case SISO) system is considered according to the "*expected and realized response scheme*". As it is indicated by the forthcoming pictures it is possible to create properly and improperly convergent sequences by considering the geometric similarity of various simple triangles for learning sequences as



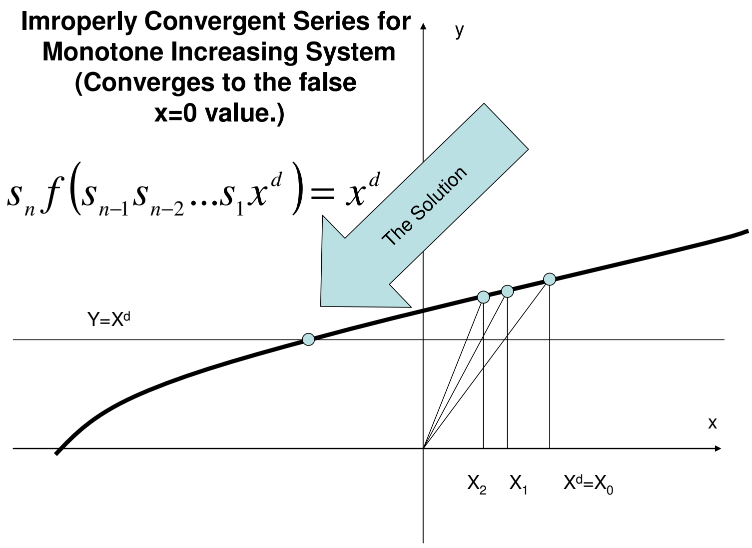


Figure 8.1.1. Properly and improperly convergent sequences for "increasing system"

As it is clear from Fig. 8.1.1. by simple manipulations involving the "origin", on the basis of some qualitative and certain approximate quantitative information concerning the behavior of the controlled system. In similar manner, by the application of some “mirroring” technique similar propositions can be done for “decreasing systems” (Fig. A.6.2. in the Appendix).

It is evident that the above ideas can be more systematically applied if the “special role” of the origin is evaded by translating the appropriate vertex of the geometrically similar triangles. In this manner a systematic set of transformations using only two parameters for the adaptive control of SISO systems was introduced as follows: a) for increasing systems [Fig. A.6.4.], and b) for decreasing systems [Fig. A.6.6.]. These figures indicate the combinations of two special functions in the role of function G of Fig. 8. with two parameters as follows:

$$\begin{gathered} h\left(x \mid x^d, D_-, \Delta_\mp\right) := \frac{x^d - \Delta_\mp}{f(x) - \Delta_\mp}(x - D_-) + D_- \\ h\left(x_* \mid x^d, D_-, \Delta_\mp\right) := x_* \\ h' = \frac{x^d - \Delta_\mp}{(f(x) - \Delta_\mp)^2}\left[-(x - D_-)f'(x) + f(x) - \Delta_\mp\right], \\ h'\left(x_* \mid x^d, D_-, \Delta_\mp\right) = 1 - \frac{x_* - D_-}{f(x_*) - \Delta_\mp} f'(x_*) \end{gathered} \tag{8.1.1}$$

and

$$\begin{gathered} g\left(x \mid x^d, D_-, \Delta_\pm\right) := \frac{f(x) - \Delta_\pm}{x^d - \Delta_\pm}(x - D_-) + D_- \\ g\left(x_* \mid x^d, D_-, \Delta_\pm\right) := x_* \\ g' = \frac{f'(x)}{x^d - \Delta_\pm}(x - D_-) + \frac{f(x) - \Delta_\pm}{x^d - \Delta_\pm} \\ g'\left(x_* \mid x^d, D_-, \Delta_\pm\right) = \frac{f'(x_*)}{x^d - \Delta_\pm}(x_* - D_-) + 1 \end{gathered} \tag{8.1.2}$$

in which $f(x_*) = x^d$. It is evident that by properly manipulating the parameters $\Delta_\mp$, $\Delta_\pm$, and D_- it is possible to obtain *contractive mapping in the vicinity of the solution*, x_*, therefore either for *“decreasing”* or *“increasing”* systems properly convergent sequences can be obtained.

Regarding the question of “*designing the parameters*” of the adaptive control at first the qualitative properties of the system to be controlled has to be studied. At first information has to be obtained on the expected sign of *f’* in the vicinity of the estimated solution. Following that, according to (8.1.1) and (8.1.2) a properly big absolute value has to be given to $\Delta_\mp$ or $\Delta_\pm$ to meet the conditions $\left|x^d\right| << \left|\Delta_\mp\right|$ or $\left|x^d\right| << \left|\Delta_\pm\right|$. In the final step proper sign and absolute value has to be given to D_- to guarantee the small absolute value of the factors $f'(x_* - D_-)/\left(x^d - \Delta_\pm\right)$ or $f'(x_* - D_-)/(f(x_*) - \Delta_\mp)$. In this manner small negative correction can be given to the value 1 in *h’* and *g’* to guarantee contractive mapping and convergence. For this purpose some qualitative and rough quantitative information in many cases is quite enough.

In the next section some application case will be studied for a 4^{th} order physical system in which the appropriate system response to be manipulated is the 4^{th} time-derivative of the coordinate values.

8.1.1. A Higher Order Application Example for Fixed Point Transformations of a Few Parameters

The physical paradigm considered for the investigations is outlined in Fig. A.6.1.1. of Appendix A.6. also containing the equations of motion of the system. The results are convincing and well illustrate the applicability of the proposed adaptive control method.

As it was indicated by the "ball-beam paradigm" the simple adaptive control based on the idea of geometrically similar triangles can work well. However, one of its deficiencies may be the fact that the size of the appropriate triangles influenced by the parameters $\Delta_{\mp}$, $\Delta_{\pm}$, and D_{-} may be varied depending on the "steepness" of the almost unknown response function *f*. With other words, this solution for creating local deformations and basin of attraction for the process of iterative learning is not very much robust as far as the variation of *f* is concerned. In the next section, in order to tackle this deficiency *"robust fixed point transformations"* will be introduced, at first for SISO systems. As it will be also shown, this latter variant will be easily extended to MIMO systems in two possible ways, too.

8.2. Robust Fixed Point Transformations for SISO Systems

In the above suggested fixed point transformations in (8.1.1) and (8.1.2) the response error occurs either in the numerator or the denominator of a fractional term. That means that the "wideness" of the basin of attraction cannot very efficiently be manipulated in this solution. With other words, this solution is not very "robust". To address the problem of "robustness" of the similar triangles based fixed points transformations better ones were proposed in [C105] in the form as

$$
\begin{gathered}
G\left(x;x^d\right)=(x+K)\left[1+B\tanh\left(A\left[f(x)-x^d\right]\right)\right]-K \\
G\left(x_*;x^d\right)=x_*,\quad G\left(-K;x^d\right)=-K \\
G'=(x+K)\frac{ABf'(x)}{\cosh^2\left(A\left[f(x)-x^d\right]\right)}+\left[1+B\tanh\left(A\left[f(x)-x^d\right]\right)\right] \\
G'\left(x_*;x^d\right)=(x_*+K)ABf'(x_*)+1
\end{gathered}
\tag{8.2.1}
$$

It is evident that the transformation defined in (8.2.1) has a *"proper"* and a *"false"* fixed point, but by properly manipulating the *A*, *B* and *K* control parameters the good fixed point can be located within its basin of attraction, and the requirement of $|G'(x_*)|<1$ can be guaranteed, too. This means that the iteration can have considerable speed of convergence even nearby x_*, and the strongly saturated *tanh* function can make it more robust in its vicinity, that is the properties of *f*(*x*) have less influence on the behavior of *G*.

Regarding the convergence issues the following simple formal consideration can be applied. Assume that for fixed x^d the restriction $|G'|:=\left|\partial G\left(x;x^d\right)/\partial x\right|\le H<1$ is valid for a region. This naturally means that for arbitrary *a* and *b* values within this region $\left|G\left(a;x^d\right)-G\left(b;x^d\right)\right|=\left|\int_a^b G'\left(\xi;x^d\right)d\xi\right|\le\int_a^b\left|G'\left(\xi;x^d\right)\right|d\xi\le H|a-b|$, that is we have a *contractive mapping*. This entails the consequence for the sequence obtained as $x_{n+1}:=G(x_n;x^d)$ that

$$\begin{aligned}&\left|x_{n+L}-x_n\right|=\left|G\left(x_{n-1+L};x^d\right)-G\left(x_{n-1};x^d\right)\right|\le H\left|x_{n-1+L}-x_{n-1}\right|\le\ldots\\&\le H^n\left|x_L-x_0\right|\to 0\text{ as }n\to\infty\end{aligned}\quad. \tag{8.2.2}$$

This means that we have a *Cauchy Sequence* in a *complete, linear, normed metric space*, therefore it must have a limit value x_*. It is easy to show that this limit value converges to the fixed point of G:

$$\begin{aligned}&\left|G\left(x_*;x^d\right)-x_*\right|=\left|G\left(x_*;x^d\right)-x_n+x_n-x_*\right|\le\left|G\left(x_*;x^d\right)-x_n\right|+\left|x_n-x_*\right|\le\\&\le H\left|x_*-x_{n-1}\right|+\left|x_n-x_*\right|\to 0\text{ as }n\to\infty\end{aligned}. \tag{8.2.3}$$

Regarding the issue of the "design of the control parameters" the following simple practice can be carried out: parameter B can be made equal to 1; according to the saturated nature of the *tanh* function, parameter A determines the "*sampling width*" within which the variation of the "*response error*" $f\text{-}x^d$ can be "monitored": very small A means wide, very big A means very narrow range of monitoring. Via making simulations by the use of a simple non-adaptive, e.g. PID-like controller the numerical range (order of magnitude) of the occurring responses can be determine/estimated. The proper absolute value of parameter K can be a few times of the occurring maximum. It is not very difficult to satisfy the condition of $|G'(x_*)|<1$ in the possession of the above estimations. Furthermore, by properly manipulating the signs of A and K, both "*increasing*" and "*decreasing*" systems can be tackled in this manner, according to the last equation of the group (8.2.1). Before showing any application example possible ways of generalizing the transformation (8.2.1) for MIMO system will be considered in the next section.

8.2.1. Possible Generalizations for MIMO Systems

The generalization of (8.2.1) for *Multiple Input – Multiple Output* (*MIMO*) systems may be done in different manners. A possibility is the use of the norm for the system-response $\|\mathbf{f}\|=\sum_i|f_i|$, and a multiple dimensional *sigmoid function* in the role of the *tanh* function as $\boldsymbol{\sigma}(\mathbf{f}):\Re^n\to\Re^n$ as $y_i=\sigma^{(i)}(f_i)$ in which each function $\sigma^{(i)}(\ )$ is a single-dimensional sigmoid. If each of them is contractive, i.e. $\forall i\ \exists 0\le M_i<1$ so that $\left|\sigma^{(i)}(a)-\sigma^{(i)}(b)\right|\le M_i|a-b|$ then it can be stated that

$$\begin{aligned}&\|\boldsymbol{\sigma}(\mathbf{a})-\boldsymbol{\sigma}(\mathbf{b})\|:=\sum_i\left|\sigma^{(i)}(a_i)-\sigma^{(i)}(b_i)\right|\le\sum_i M_i\left|a_i-b_i\right|\le\\&\le\underset{i}{Max}\{M_i\}\sum_i\left|a_i-b_i\right|=M\|\mathbf{a}-\mathbf{b}\|,0\le M<1\end{aligned} \tag{8.2.4}$$

that means that this multiple dimensional sigmoid function is contractive in a Banach space. In this case it is possible to find the A_i, B_i, and K_i control parameters for each component i.

An alternative possibility is to define the *response error* and its *direction* in the n^{th} control step as $\mathbf{h}_n := \mathbf{f}(\mathbf{r}_n) - \mathbf{r}^d, \mathbf{e}_n := \mathbf{h}_n / \|\mathbf{h}_n\|$ (here the Euclidean or Frobenius norm is in use), and apply the following transformation:

$$\text{if } \|\mathbf{h}_n\| > \varepsilon \text{ then } \mathbf{x}_{n+1} = \left(1 + \tilde{B}\right)\mathbf{x}_n + \tilde{B}K\mathbf{e} \text{ else } \mathbf{x}_{n+1} = \mathbf{x}_n,\ \tilde{B} := B\sigma\left(A\|\mathbf{h}_n\|\right) \quad (8.2.5)$$

in which ε is a small positive threshold value for the response error. If the response error is quite small, the system already attained the fixed point and no any manipulation is needed with the unit vector the computation of which would be near singular. In the case of this implementation we have four control parameters, ε, A, B, and K, and a single sigmoid function $\sigma()$. This realization applies correction in the direction of the response error only, and normally leads to more precise tracking than the more complicated one using separate control parameters for various directions. In the next section a possible application will be discussed. In the sequel typical application examples will be discussed.

8.2.2. Application Example a): Precise Control of an AGV Equipped with Omnidirectional Wheels

As is well known, in contrast to the so called *Ackerman Devices*, *Automatic Guided Vehicles (AGVs)* using omnidirectional wheels (e.g. [R121]) can precisely track arbitrary trajectories at least from kinematic point of view. On this reason for our purposes a triangular structure similar to that in [R121] was chosen as a paradigm detailed in Appendix A.6.2.

8.2.3. Application Example b): Precise Control of the Cart-Beam-Hamper System

Since the more traditional "*Adaptive Inverse Dynamics*" and "*Slotine-Li Adaptive Controller*" and their modifications as inclusion of integrated feedback terms and dropping the use of Lyapunov function for tuning were quantitatively analyzed in "*4.4. Adaptive Inverse Dynamics Control of Robots*" and in "*4.5. Adaptive Slotine-Li Controller for Robots*", for the purposes of comparison it is expedient to give here some simulation results for the "alternative generalization" of the "*Robust Fixed Point Transformations*" as defined in (8.2.5) applied for the same physical system as paradigm (Fig. A.1.1.). In the comparison the same nominal trajectory was applied with a trajectory tracking prescription $\ddot{\mathbf{q}}^{Des} = \ddot{\mathbf{q}}^N + 3\mathbf{\Lambda}\ddot{\xi}(t) + 3\mathbf{\Lambda}^2\dot{\xi}(t) + \mathbf{\Lambda}^3\xi(t)$ with $\xi(t) := \int_0^t \mathbf{e}(\tau)d\tau$ with $\mathbf{\Lambda}$=10×$\mathbf{I}$ [1/s] and besides the friction hectic disturbance force was applied in the linear direction (it was generated by 3rd order periodic spline functions). Detailed results are given in Appendix A.6.3.

As a summation of the here presented results and that of many others published in other papers it can be stated that on the basis of simple qualitative information and some rough quantitative knowledge in many cases quite satisfactory adaptive control can be constructed by the use of local deformations guaranteeing stability within a reasonably wide region though no "global stability" can be guaranteed in this manner.

8.3. Convergence Stabilization by Tuning only one Adaptive Control Parameter

For guaranteeing the stability of the control the motion must be kept within the basin of attraction of the fixed point of the iterative sequence. A plausible possibility is tuning only A if K and B were already properly estimated. For this purpose we have to note that in the vicinity of the fixed point in (8.2.5) $\tilde{B} << 1$ and $\tilde{B}\mathbf{e} = B\sigma(A\|\mathbf{h}\|)\mathbf{h}/\|\mathbf{h}\| \approx BA\mathbf{h}$, so $\|\tilde{B}\mathbf{r}\| << K\tilde{B}\mathbf{e}$. Therefore, instead of the formerly successfully used version e.g. in the examples of Appendix A.6.2. and A.6.3. we proposed the following iterative transformations for MIMO Systems, for the $(n+1)^{\text{th}}$ control cycle:

$$\mathbf{r}_{n+1} := \mathbf{r}_n + KB\,\frac{\sigma(A\|\mathbf{h}_n\|)}{\|\mathbf{h}_n\|}\mathbf{h}_n,\ \mathbf{h}_n := \mathbf{f}(\mathbf{r}_n) - \mathbf{r}^d_{n+1} \tag{8.3.1}$$

In the vicinity of the fixed point (that is when the difference $\|\mathbf{r}_{n+1}-\mathbf{r}_n\|$ is small, the modification of the response error h can be estimated by the use of the 1st order term of the Taylor series of $\mathbf{f}(\mathbf{r})$ as follows:

$$\begin{gathered}\mathbf{f}(\mathbf{r}_{n+1}) \approx \mathbf{f}(\mathbf{r}_n) + \frac{\partial \mathbf{f}(\mathbf{r}_n)}{\partial \mathbf{r}} KBA\ \mathbf{h}_n, \\ \mathbf{f}(\mathbf{r}_{n+1}) - \mathbf{r}^d_{n+2} \approx \mathbf{f}(\mathbf{r}_n) - \mathbf{r}^d_{n+1} + \mathbf{r}^d_{n+1} - \mathbf{r}^d_{n+2} + \frac{\partial \mathbf{f}(\mathbf{r}_n)}{\partial \mathbf{r}} KBA\ \mathbf{h}_n\end{gathered}. \tag{8.3.2}$$

This means that

$$\mathbf{h}_{n+1} = \left[\mathbf{I} + KBA\frac{\partial \mathbf{f}(\mathbf{r}_n)}{\partial \mathbf{r}}\right]\mathbf{h}_n - \left(\mathbf{r}^d_{n+2} - \mathbf{r}^d_{n+1}\right). \tag{8.3.3}$$

For normal cases the desired control signal varies only slowly, therefore this second term is not significant, and it is expected that for the reduction of the response error the first term in the brackets must be properly set. For instance, if it can be known in advance that $\partial\mathbf{f}/\partial\mathbf{r}$ is positive definite (this situation can be met in the great majority of the fully actuated Classical Mechanical Systems, for small $A>0$ and $B=1$ $KBA(\partial\mathbf{f}/\partial\mathbf{r})\mathbf{h}_n$ is a small vector of direction approximately opposite to that of $\mathbf{h}_n$ that corresponds to step by step decreasing response error. The same holds if $\partial\mathbf{f}/\partial\mathbf{r}$ is not completely symmetric, but its symmetric part is positive definite (for negative definite systems the setting $B=-1$ can be used in similar manner): its skew–symmetric part yields some contribution that is orthogonal to $\mathbf{h}_n$. If the controller stores certain past data the necessary modification of A can be estimated if the controller computes the quantity in the $(n+2)^{\text{th}}$ control cycle:

$$\varepsilon_{est} := \frac{\mathbf{h}_n^T\left[\mathbf{h}_{n+1} - \mathbf{h}_n + \left(\mathbf{r}^d_{n+2} - \mathbf{r}^d_{n+1}\right)\right]}{\mathbf{h}_n^T\mathbf{h}_n}. \tag{8.3.4}$$

By assuming positive definite $\partial\mathbf{f}/\partial\mathbf{r}$ the following tuning rule can be suggested:

$$\dot{A} = \alpha\sigma\left(\varepsilon_{est} - \varepsilon_{goal}\right)A \tag{8.3.5}$$

that tries to stabilize ε_{est} about $\varepsilon_{goal}\approx-0.5$ to keep the control within the center of the basin of attraction of the iteration ($\alpha>0$). (For avoiding extreme tuning the same saturated sigmoid function that was used in the fixed point transformations (8.2.5) is applied here, too.)

8.3.1. Possible Application:Control of the Cart and Double Pendulum System

In the sequel a possible application will be shown for a Classical Mechanical paradigm, viz. the cart + double pendulum system already depicted in details in Fig. A.5.1. with The Euler-Lagrange equations of motion given in (A.5.4). However, in these examples it was considered as an *underactuated system* that means that the linear degree of freedom (q_3) was left without own drives, i.e. $Q_3 \equiv 0$ was assumed. The motion in the linear direction was controlled through the dynamic coupling between the linear axis and the two rotary ones. With other words it means that the reaction forces needed for moving the two "counterweights" m_1 and m_2 were used for generating acceleration along q_3.

Detailed simulation results are given in Appendix A.6.4. obtained by simple SCILAB programs with Euler integration and the more sophisticated integrator of SCICOS co-simulator. The results well exemplify the consistent behavior of tuning and the comparable output of the simple and sophisticated simulators. The superiority of the SCICOS-based computation reveals itself in the values of the ε_{est} estimated values according to (8.3.4).

8.4. Thesis 4: Introduction of Various Parametric Fixed Point Transformations for the Adaptive Control of Special SISO and MIMO Systems (Summary of the Results of Chapter 8)

On the basis of simple geometrical observation I developed a special class of *situation-dependent, temporal adaptive control* for special Single Input – a Single Output nonlinear system that practically does *not need any partial system identification.* Instead of that it uses certain "*qualitative information*" that corresponds to the "*specialty*" of the system to be controlled. The system's response function must be either clearly "*increasing*" or clearly "*decreasing*". The method is based on local deformations creating a local basin of attraction for the result of the iterative learning process it applies. It can compensate the effects of unknown external disturbances that partly may origin from dynamic coupling with unmodeled subsystems. For this purpose

- I introduced four variants of fixed point transformations to be used in adaptive control. These transformations have two adaptive parameters only. These parameters have simple geometric interpretation related to geometrically similar triangles defined in the "*input – response space*" of the system to be controlled;
- The control produces iterative learning resulting in convergent Cauchy sequences in the input space;
- I have shown that the convergence of the method can simply be proved by using the concept of "*linear, normed, complete metric spaces*" (i.e. *Banach Spaces*) with contractive mappings;
- I have shown that since this concept allows the use of various norms to be applied over the same set, the method is very versatile and may have numerous particular variants;
- I have proposed a very simple design method setting the much reduced number of the constant adaptive parameters of this method. This is a considerable advantage in comparison with the traditional adaptive control methods like "*Adaptive Inverse Dynamics*" or "*Adaptive Slotine – Li Control*" that have to use very detailed and complicated system models, have to tune a lot of model parameters,

and have to apply a lot of "adaptive control parameters"; Furthermore, numerical details can be obtained only by numerical simulations for these classic approximations, too;

- I have shown that – in contrast to the very sophisticated "*Adaptive Inverse Dynamics*" and "*Adaptive Slotine – Li Control*" methods – the here proposed one separates from each other the phases of controller design and prescription of the trajectory tracking. The desired trajectory tracking can arbitrarily be formed using only purely kinematic terms. In this manner the here proposed method is very flexible.
- I have shown that in contrast to the "*Adaptive Inverse Dynamics*" and "*Adaptive Slotine – Li Control*" the here proposed method is able to simultaneously compensate the effects of unknown external perturbations and that of the existence of unknown and not controlled subsystems in dynamic coupling with the controlled one;
- I have elaborated a "*more robust version*" of this control applying saturated sigmoidal functions. This form has three parameters of which practically two parameters must be set, the third one normally may be taken equal to ±1; I have interpreted its operation on geometrical basis;
- I have elaborated two kinds of generalization of this method for special Multiple Input – Multiple Output (MIMO) systems in the case of which the "*increasing*" or "*decreasing*" nature can be generalized by using the concepts of *"vectors approximately of the same direction"* in real Hilbert spaces;
- In order to test the potential applicability of the proposed method I made investigations for 2nd, 3rd, 4th, and fractional order systems (in this latter case the system's response function may be a fractional order derivative of the state variable) including holonomic and non-holonomic mechanical and electromechanical systems, underactuated classical mechanical systems, electrostatic microactuator, and the model of a chemical reaction describing a polymerization process. I also made comparisons regarding the results of the present approach and that of "ad hoc" solutions using similar qualitative information.
- To extend the applicability of the method based on iterations of local basin of attraction I introduced a tuning procedure for one of the three adaptive control parameters of the Robust Fixed Point Transformations; I have shown mathematically and illustrated via SILAB and SCICOS based simulations that this complementary tuning stabilizes the controller near the fixed point that is the solution of the controller's task. In this manner the proposed method can be competitive with the traditional ones offering global stability.

The most important publications strictly related to the contents of Thesis 4 are as follows: [B7], [B8], [J5], [J15], [J16], [J18], [J19], [J20], [C77], [C85], [C86], [C87], [C97], [C100], [C101], [C103], [C105], [C107], [C108], [C109], [C110], [C111], [C113], [C114], [C116], [C117], [C122]. Other related publications are as follows: [C89], [C91], [C96].

9. Novel Approach in Model Reference Adaptive Control: Replacement of Lyapunov's Direct Method with Robust Fixed Point Transformations

Adaptive control of physical systems having uncertain, time-varying, directly neither observable nor controllable, dynamically coupled subsystems still is an interesting challenge. The classical "*parameter adaptive*" approaches as "*Adaptive Inverse Dynamics*" or "*Adaptive Slotine-Li Controllers*" e.g. in Robotics try to exactly learn the dynamic parameters of the systems under control. They commence their operation with initial approximate model parameters that they tune until reaching their exact values. These approaches naturally have to cope with either the lack or infinite complexity of the appropriate analytical system models in a wider scope. A typical example is a not completely full tank containing some wobbling fluid that dynamically interacts with the tank's wall. Development and real-time identification of any fluid model would evidently be a hopeless for precisely controlling the motion of the tank.

The so called "*signal adaptive*" controllers have far simpler construction than the above mentioned ones. *They do not wish to compensate the observed discrepancies in the system's behavior by tuning the parameters of any analytical model.* Instead of that they quickly manipulate certain additive and/or gain parameters for error compensation. To this class belongs the idea of the "*Model Reference Adaptive Control (MRAC)*". The MRAC technique is a popular and efficient approach in the adaptive control of nonlinear systems e.g. in robotics. A great manifold of appropriate papers can be found for the application of MRAC from the early nineties (e.g. [R135]) to our days (e.g.[R138]). One of its early applications was a breakthrough in adaptive control. In [R136] C. Nguyen presented the implementation of a joint-space adaptive control scheme that was used for the control of a non-compliant motion of a Stewart platform-based manipulator that was used in the Hardware Real-Time Emulator developed at Goddard Space Flight Center to emulate space operations.

The essence of the idea of MRAC is the transformation of the *actual system under control* into a well behaving *reference system* (reference model) for which simple controllers can be designed. In the practice the *reference model* used to be stable linear system of constant coefficients. *In particular cases the reference models can also be the nonlinear analytical models of the systems built up of their nominal parameters.* The controllers normally are constructed by the use of the Lyapunov function technique, too. Recently it became clear that the "Robust Fixed Point Transformations" [C105] can be also used for developing novel simple versions of MRAC controllers without using Lyapunov's complicated technique (e.g. in [C118], [C119]). In these early papers simultaneous compensation of the effects of modeling imprecision and external disturbances were considered.

Assume that on purely kinematical basis we prescribe a trajectory tracking policy that needs a *desired acceleration* of the mechanical system as $(d^2q/dt^2)^D$. From the behavior of the reference model for that acceleration we can calculate the physical agent that could result in the response $(d^2q/dt^2)^D$ for the reference model (in our case the generalized force components are denoted by U^D). The direct application of this U^D for the actual system could result in different response since its physical behavior differs from that of the reference model. Therefore it can be "*deformed*" into a "*required*" U^{Req} value that directly can be applied to the *actual system*. Via substituting the realized response of the actual system d^2q/dt^2 into the reference model the "*realized control action*" U^R can be obtained instead of the "desired one" U^D. Our aim is to find the proper deformation by the application of which U^R well approaches U^D, that is *at which the controlled system seems to behave as the reference system*. The proper deformation may be found by the application of an iteration as follows. Consider the iteration generated by some function G as $U^{Req}_{n+1}=G(U^{Req}_n,U^R_n,U^D_{n+1})$ in which n is the index of the control cycle. For slowly varying desired value U^D can be considered to be constant. In this case the iteration is reduced to $U^{Req}_{n+1}=G(U^{Req}_n,U^R_n,U^D)$ that must be made convergent to U^{Req}_*.

It is evident that the same function G and the same considerations can be applied in this case as that detailed in Section 8.2. with the same extension to MIMO systems as in Subsection 8.2.1. Furthermore, the same convergence stabilization by tuning as applied is Section 8.3. can be in this case, too. The comparison of the "traditional" and the "novel" schemes is given in Fig. 9.1.

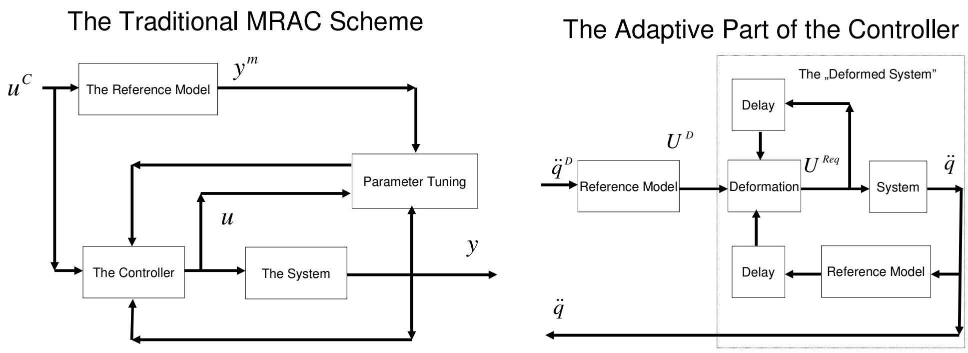


Figure 9.1. The "traditional" MRAC scheme operated by some Lyapunov function based parameter tuning, and the novel one based on "Robust Fixed Point Transformations"

The scheme in Fig. 9.1. does not need any further sophisticated mathematical analysis. If it works it evidently has to result in precise trajectory, velocity, and acceleration tracking, and also determines the appropriate deformation of the force / torque signal calculated to the *reference model* to achieve appropriate acceleration of the actual system under control. Therefore in the sequel potential application examples are given.

9.1. Application Examples

In this section the results of a comparative analysis with that of the more "*traditional*" MRAC strategies will be given. The "traditional MRAC philosophy" is a wide framework that can be filled in with various particular solutions. For comparison we choose a relatively simple implementation containing integrated feedback in the tracking error. Let the tracking error be denoted as $\mathbf{e} := \mathbf{q}^N - \mathbf{q}$ and let $\xi(t) := \int_0^t \mathbf{e}(\tau)d\tau$ ($\mathbf{q}^N$ denotes the *nominal*, $\mathbf{q}$ is the *actual* trajectory). The kinematically prescribed trajectory tracking can be defined by the positive definite matrix $\mathbf{\Lambda}$ and the „error metrics" of the VS/SM controllers as $\mathbf{S} := \left(\frac{d}{dt} + \mathbf{\Lambda}\right)^3 \xi(t) = 0$

leading to $\ddot{\mathbf{q}}^D = \ddot{\mathbf{q}}^N + \mathbf{\Lambda}^3\xi + 3\mathbf{\Lambda}^2\mathbf{e} + 3\mathbf{\Lambda}\dot{\mathbf{e}}$ as the *desired joint acceleration*. Let the *reference model* consist of two symmetric positive definite constant matrices as $\mathbf{M}^{\mathrm{Ref}}\ddot{\mathbf{q}} + \mathbf{B}^{\mathrm{Ref}}\dot{\mathbf{q}} = \mathbf{Q}^{\mathrm{Ref}}$ where $\mathbf{Q}^{\mathrm{Ref}}$ corresponds to its force/torque need for the actual $\ddot{\mathbf{q}}, \dot{\mathbf{q}}$ values. Let $\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{Q}$ be the actual system's equation of motion. By „copying" the idea of the *Adaptive Inverse Dynamics Controller* let the exerted force/torque be $\mathbf{M}^{\mathrm{Ref}}\ddot{\mathbf{q}}^D + \mathbf{B}^{\mathrm{Ref}}\dot{\mathbf{q}} + \mathbf{D} = \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{Q}$ in which $\mathbf{D}$ corresponds to an *additive force* to be determined by the MRAC controller. Via subtracting $\mathbf{M}^{\mathrm{Ref}}\ddot{\mathbf{q}}$ from both sides we can express the known difference of the *desired* and *actual* joint accelerations as $\ddot{\mathbf{q}}^D - \ddot{\mathbf{q}} = \mathbf{M}^{\mathrm{Ref}-1}\left[\left(\mathbf{H}(\mathbf{q}) - \mathbf{M}^{\mathrm{Ref}}\right)\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{B}^{\mathrm{Ref}}\dot{\mathbf{q}} - \mathbf{D}\right]$. By the introduction of the arrays $\mathbf{x} := \left[\xi^T, \mathbf{e}^T, \dot{\mathbf{e}}^T\right]^T$ and $\dot{\mathbf{x}} := \left[\mathbf{e}^T, \dot{\mathbf{e}}^T, \ddot{\mathbf{e}}^T\right]^T$ the following equation of motion holds:

$$\begin{aligned}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{\Phi} = &\begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ -\mathbf{\Lambda}^3 & -3\mathbf{\Lambda}^2 & -3\mathbf{\Lambda} \end{bmatrix} \begin{bmatrix} \xi \\ \mathbf{e} \\ \dot{\mathbf{e}} \end{bmatrix} + \\ &+ \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}^{\mathrm{Ref}-1}\left[\left(\mathbf{H}(\mathbf{q}) - \mathbf{M}^{\mathrm{Ref}}\right)\ddot{\mathbf{q}} + \mathbf{h} - \mathbf{B}^{\mathrm{Ref}}\dot{\mathbf{q}} - \mathbf{D}\right] \end{bmatrix}.\end{aligned} \tag{9.1.1}$$

With a positive definite matrix $\mathbf{P}$ the Lyapunov function $V=\mathbf{x}^T\mathbf{P}\mathbf{x}$ can be introduced with the desired negative time-derivative $\dot{V} = \mathbf{x}^T\left(\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A}\right)\mathbf{x} + 2\mathbf{x}^T\mathbf{P}\mathbf{\Phi} < 0$. By solving the Lyapunov equation the term quadratic in $\mathbf{x}$ can be made negative, therefore it is sufficient to guarantee the non-positive nature of the remaining one. Since $\ddot{\mathbf{q}}^D - \ddot{\mathbf{q}}$ is known the remaining part consists of the sum of known and unknown terms as

$$\begin{aligned}\mathbf{x}^T\mathbf{P}\mathbf{\Phi} &= \overbrace{\mathbf{x}^T\mathbf{P}[\mathbf{0}^T;\mathbf{0}^T;\left(\ddot{\mathbf{q}}^D - \ddot{\mathbf{q}}\right)^T]}^{z_{meas}} = \\ &= \overbrace{\mathbf{x}^T\mathbf{P}[\mathbf{0}^T;\mathbf{0}^T;\mathbf{M}^{\mathrm{Ref}-1}]\left\{\left(\mathbf{H} - \mathbf{M}^{\mathrm{Ref}}\right)\ddot{\mathbf{q}} + \mathbf{h} - \mathbf{B}\dot{\mathbf{q}}\right\}}^{u} - \overbrace{\mathbf{x}^T\mathbf{P}[\mathbf{0}^T;\mathbf{0}^T;\mathbf{M}^{\mathrm{Ref}-1}]}^{\mathbf{w}^T}\mathbf{D} < 0\end{aligned} \tag{9.1.2}$$

in which z_{meas} and $\mathbf{w}$ are *known quantities*, and u is not known. Let us seek $\mathbf{D}$ in the form of $\alpha(t)\mathbf{w}$! Then the condition $z_{meas}=u - \alpha(t)w^Tw<0$ should be achieved. Since $w^Tw \geq 0$, in the possession of z_{meas} we have idea if $\alpha(t)$ must be increased or decreased. Let us apply a *tuning rule* with $\kappa>0$ as follows: $\dot{\alpha} = \kappa\left[1 + \mathrm{sgn}\left(z_{meas}\right)\right]z_{meas}$. With properly great κ and $\mathbf{P}$ this tuning can soon lead to decreasing Lyapunov function, i.e. to stable control. This tuning leaves the negative terms unchanged but decreases the positive ones.

The novel MRAC approach was simulated according to Fig. 9.1.

Two interesting application examples are presented in this Thesis. Comparison of the operation of the "traditional" and "novel" is given for the Cart + Beam + Hamper system as depicted in Fig. A.1.1. using simple SCILAB programs.

The other application is a pendulum of uncertain mass center point for which only the novel approach was investigated by the professional numerical integrator of SCILAB-SCICOS co-simulator.

9.1.1. Possible Applications: a) MRAC Control of the Cart + Beam + Hamper System

The comparative simulations were made for the paradigm depicted in Fig. A.1.1. The system's parameters were M=30 kg, m=10 kg, L=2 m, Θ=20 kg×m^2, and g=10 m/s^2 (gravitational acceleration). The „*reference model*" was defined by $\mathbf{M}^{Ref}=20\times\mathbf{I}$, $\mathbf{B}^{Ref}=3\times\mathbf{I}$. In both cases a strong feedback gain was chosen for trajectory tracking as $\mathbf{\Lambda}=10\times\mathbf{I}$. In the „traditional case" κ=15; and $\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}=-600\times\mathbf{I}$ was chosen.

The adaptive control parameters of the "*adaptive control parameters*" (without tuning parameter A) were: K=7000, B=-1, A=10^{-5}. Detailed results are given in Appendix A.7.1. Fig. A.7.1.3. well illustrate that the torque need of the reference model is in close vicinity of the "recalculated" values, that is the MRAC idea is well realized by the novel approach. The appropriate results of Figs. A.7.1.4. and Fig. A.7.1.5. that were made for the lack of external disturbances are even more convincing: the "non-adaptive" part of the controller has the "illusion" that it calculates the torque / force needs of the *reference model* and obtains appropriate 2nd time-derivatives of the generalized coordinates accordingly.

9.1.2. Possible Applications: b) Novel MRAC Control of a Pendulum of Uncertain Mass Center Point

In this example the controller calculates with a rigid pendulum of 1 DOF while the actual system has 2 DOFs: inside the jig of the pendulum a ball of significant mass positioned by a spring of limited stiffness is located. As the pendulum rotates the ball can considerably be translated in the radial direction. In this case we have dynamic coupling with a not modeled subsystem. The simulation investigations revealed that in this case very aggressive tuning was necessary to the adaptive control parameter A (it is detailed in Appendix A.7.2.). Since the controller assumed an 1 DOF system the here applied estimation a little bit differed from that we used for the MIMO systems in Section 8.3. In the earlier investigations it was found that the use of constant adaptive control parameters once estimated were satisfactory during the whole control session. However, for r^d *considerably varying in time* the following estimation can be done in the vicinity of the fixed point when $|r_n-r_{n-1}|$ is small: $r_{n+1}-r_n=G(r_n,r^d_n)-G(r_{n-1},r^d_{n-1})\approx[\partial G(r_{n-1},r^d_{n-1})/\partial r](r_n-r_{n-1})+[\partial G(r_{n-1},r^d_{n-1})/\partial r^d](r^d_n-r^d_{n-1})$. Since from the analytical form of $\sigma(x)$ $[\partial G(r_{n-1}.r^d_{n-1})/\partial r^d]$ is known, and the past "desired" inputs as well as the arguments of function G are also known, this equation can be used for real-time estimation of $[\partial G(r_{n-1},r^d_{n-1})/\partial r]$ therefore for calculating the *estimated actual value* in $\partial G(r,r^d)/\partial r=1-\varepsilon_{est}$:

$$\varepsilon_{est}\approx\frac{r_{n+1}-r_n+(r_n+K)BA_0\sigma'\left(A\left[f_n-r_n^d\right]\right)\left(r_{n+1}^d-r_n^d\right)}{r_n-r_{n-1}}-1 \qquad (9.1.2.1)$$

[$\sigma'(x)$ denotes the derivative of $\sigma(x)$].

For the novel MRAC control of this pendulum detailed simulation results are given in Appendix A.7.2. These results are quite convincing, too.

9.2. Thesis 5: Replacement of Lyapunov's Direct Method with Robust Fixed Point Transformations in Model Reference Adaptive Control (Summary of the Results of Chapter 9)

I have realized that the "*Robust Fixed Point Transformations*" with the *convergence stabilizer parameter tuning* and the "*Expected – Realized Response*

Scheme" can replace Lyapunov's direct method in "*Model Reference Adaptive Control*".

- I have shown that for this purpose in the role of the "*expected response*" the generalized force components calculated from the reference model (nominal system) and a kinematically prescribed joint coordinate acceleration, and in the place of the "*realized response*" the reference model's torque needs calculated from the observed accelerations of the controlled system have to stand;
- By the use of simulation investigations I have shown that the "novel" approach precisely can realize the main idea of MRAC: it provides the model based controller with the illusion that the controlled system behaves like the reference model, i.e. it responses to the same generalized force with the same joint accelerations like the reference model;
- The design of the novel controller is far simpler task than the application of the technique that uses Lyapunov functions.

Publications related this Thesis are as follows: [C118], [C119], and [C121].

10. Adaptive Control for MIMO Systems by the Use of Approximate SVD of the Available Approximate Model

In this section a further step ahead is discussed concerning the novel approach to develop adaptive nonlinear control for non-special *MIMO* systems. This means a kind of generalization of the solutions elaborated for *SISO* systems in section 8.1. The previously applied parametric transformations were tailored to systems in which such concepts as "*increasing*", "*decreasing*", "*greater/smaller than* ..." had definite meaning. However, in the state space of multiple dimensional systems uncountable manifold of the possible directions exists, so the above concepts cannot directly be applied for them. The idea behind the here proposed generalization is very simple. By utilizing the fact that in any *Real Hilbert Space* the *scalar product of the elements, therefore the angles between two vectors* can be defined, it can be said that if two vectors define an *acute angle* this means that these vectors have "*approximately the same direction*", while if they define an *obtuse angle* the situation can be interpreted that these vectors have "*approximately opposite directions*". Therefore instead of "*quantities that increase in a given step*", we can speak of "*vectors that vary approximately in the direction of the original vector*". In similar manner, instead of "*quantities that decrease in a given step*", we can speak of "*vectors that vary approximately in the direction opposite to that of the original one*".

The only problem is that in the case of such systems no any a priori information we have on the relationship connecting the directions of the modification of the control agent and the modification in the response caused by it. The nature of this connection can vary in time during the motion (state propagation) of the controlled system. Fortunately enough, if some *approximate analytical model* of the system under control is available together with the very rough model used for control purposes, by the use of the method of *SVD* this relationship can roughly be estimated, and on this basis some adaptive controller can be designed or at least outlined. (We have similar situation in the case of the sophisticated traditional adaptive controllers, i.e. in the case of the "*Adaptive Inverse Dynamics*" and the "*Adaptive Slotine - Li Controllers*" that assume that the exact *analytical form of the model of the system to be controlled is available, and we have imprecision only in the values of the dynamical parameters.*)

In the sequel it will be shown in details that this idea can be utilized by using the means of *Singular Value Decomposition (SVD)* of real matrices for the purposes of developing adaptive control for *MIMO* dynamical systems. At first we consider the geometric interpretation of *SVD* as is given in Appendix A.11. by Eq. (A.11.7) as

$$\begin{aligned}
\mathbf{b} = \mathbf{V}\mathbf{D}\mathbf{U}^T\mathbf{a} &= \left[\mathbf{v}^{(1)} \mid \cdots \mid \mathbf{v}^{(n)}\right]\begin{bmatrix} D_{11} & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & D_{nm}\end{bmatrix}\begin{bmatrix} \mathbf{u}^{(1)T} \\ \hline \cdots \\ \hline \mathbf{u}^{(m)T}\end{bmatrix}\mathbf{a} = \\
&= \left[D_{11}\,\mathbf{v}^{(1)} \mid \cdots \mid D_{kk}\mathbf{v}^{(k)} \mid \cdots\right]\begin{bmatrix} \left(\mathbf{u}^{(1)T},\mathbf{a}\right) \\ \hline \cdots \\ \hline \left(\mathbf{u}^{(m)T},\mathbf{a}\right)\end{bmatrix} = \\
&= \left(\mathbf{u}^{(1)T},\mathbf{a}\right)D_{11}\mathbf{v}^{(1)} + \ldots + \left(\mathbf{u}^{(k)T},\mathbf{a}\right)D_{kk}\mathbf{v}^{(k)}
\end{aligned} \qquad \text{(A.11.7)}$$

in which k=min(n,m), and in the central line following the matrix element D_{kk} in "[…|D_{kk}|…]" either nothing stands or zeros are located. The geometric interpretation of (A.11.7) is straightforward: *characteristic pairs of orthogonal directions* are found in the *input* and the *output spaces* to which *characteristic stretch/shrink* denoted by the *singular values* $D_{ii}\geq 0$ belong. *To zero singular values special directions pertain that do not take part in the mapping realized by the linear operator under consideration.* By using this geometric interpretation we can create appropriate, convergent *Cauchy Sequences* as solutions of the control problem in the case of *MIMO* systems almost exactly in the same manner as it was done by the parametric fixed point transformations we applied for the *SISO* systems.

10.1. Mathematical Formulation

Consider the following task: it is given an initial $\mathbf{x}_0$ value, a smooth $\mathbf{f}:\Re^n\rightarrow\Re^n$ function, an $\mathbf{x}^d$ "*desired value*", and the appropriate solution $\mathbf{x}_*$ is sought for which $\mathbf{x}^d=\mathbf{f}(\mathbf{x}_*)$. We should like to achieve a first order correction in the value of $\mathbf{f}(\mathbf{x})$ that moves $\mathbf{f}$ *in the direction of* $\mathbf{x}^d$, that is a positive number $\alpha>0$ can be introduced as

$$\Delta\mathbf{f}=\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\Delta\mathbf{x}=\alpha\left[\mathbf{x}^d-\mathbf{f}(\mathbf{x})\right] \qquad (10.1.1)$$

If the Jacobian of $\mathbf{f}$ can be inverted the following sequence of points can be defined by (10.1.1):

$$\mathbf{x}_{n+1}=\mathbf{x}_n+\alpha\left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right]^{-1}\left[\mathbf{x}^d-\mathbf{f}(\mathbf{x}_n)\right] \qquad (10.1.2)$$

To estimate the approximation error belonging to $\mathbf{x}_{n+1}$ the first order Taylor series expansion of $\mathbf{f}$ can be used as

$$\begin{aligned}\mathbf{x}^d-\mathbf{f}(\mathbf{x}_{n+1})&\approx\mathbf{x}^d-\mathbf{f}\left(\mathbf{x}_n+\alpha\left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right]^{-1}\left[\mathbf{x}^d-\mathbf{f}(\mathbf{x}_n)\right]\right)\approx\\ &\approx\mathbf{x}^d-\mathbf{f}(\mathbf{x}_n)-\alpha\left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right]\left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right]^{-1}\left[\mathbf{x}^d-\mathbf{f}(\mathbf{x}_n)\right]\approx\\ &\approx(1-\alpha)\left[\mathbf{x}^d-\mathbf{f}(\mathbf{x}_n)\right]\end{aligned} \qquad (10.1.3)$$

This error in absolute value evidently can be decreased if approximately $0<\alpha<2$. Normally (10.1.3) cannot exactly be realized since $\partial\mathbf{f}/\partial\mathbf{x}$ is not exactly known. To obtain better idea on the possibilities for reducing the approximation error, imagine the application of the *SVD* for $\partial\mathbf{f}/\partial\mathbf{x}$ as $\left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right]=\mathbf{UDV}^T$, $\left[\frac{\partial\mathbf{f}}{\partial\mathbf{x}}\right]^{-1}=\mathbf{VD}^{-1}\mathbf{U}^T$ that leads to the step $\Delta\mathbf{x}$ in the above outlined iteration as $\Delta\mathbf{x}=\alpha\mathbf{VD}^{-1}\mathbf{U}^T\left[\mathbf{x}^d-\mathbf{f}(\mathbf{x})\right]$. Now apply the form (A.11.7) by expressing the actual error used for calculating the next step with the components of the orthogonal matrix $\mathbf{U}$. In the ideal case

$$\begin{aligned}&\mathbf{x}^d-\mathbf{f}(\mathbf{x})=\sum_l c_l\mathbf{u}^{(l)},\ c_l=\mathbf{u}^{(l)T}\left[\mathbf{x}^d-\mathbf{f}(\mathbf{x})\right]\\ &\Delta\mathbf{x}\approx\alpha\sum_l D_{ll}^{-1}c_l\mathbf{v}^{(l)}\end{aligned} \qquad (10.1.4)$$

For guaranteeing convergence small $\mathbf{\Delta x}$ is needed. Since the *SVD* for an invertible quadratic matrix yields $D_{11}\geq D_{22}\geq...\geq D_{nn}$ it can be said that $0<D^{-1}_{11}\leq...\leq D^{-1}_{nn}$ so $\|\Delta\mathbf{x}\|\leq(\sqrt{n})\ \alpha\ \max^{n}_{l=1}\{|c_l|\}D^{-1}_{nn}$. For introducing the quantity K expressing the *maximum allowable step length in the* **x** *space* the proper value of the proposed maximum of α can be estimated as

$$\alpha_{\max}\approx\frac{KD_{nn}}{\max\left(\max^{n}_{l=1}\{|c_l|\},C_{cut}\right)\sqrt{n}} \tag{10.1.5}$$

in which the parameter C_{cut} has the function of limiting α in the case of small c_l coefficients in (10.1.4).

The geometric way of thinking here can be utilized as follows: it is not necessary to exactly move in the **x** space as it is defined in (10.1.1): it is just enough to make a small step "*approximately in the same direction*". Therefore, if we have some *approximate model of the Jacobian of* **f** *of our system*, only one times executing *SVD* on this approximation may be satisfactory to approximate the **U**, **D**, and **V** matrices that can be used for estimating the factor α, and the system can be directed to the direction of the decreasing error even if not exactly the direction of the "*steepest descent*" according to (10.1.4) is achieved. In the sequel this idea will be applied in the adaptive control of the *cart plus double pendulum system*.

10.2. Application Example: Adaptive Control of the Cart plus Double Pendulum System

Certain excerpts of the consideration obtained here have been published in [C102]. The structure of the paradigm for the control of which the proposed method was proved is described in Fig. A.5.1. and with the equations of motion (A.5.4) of the Appendix and also discussed in details in [C63]. However, the dynamic parameters of the system were different in this case. The cart the mass of M=5 [*kg*], the pendulums assembled on the cart by parallel shafts and arms having negligible masses and lengths L_1=2 and L_2=3 [*m*], respectively. At the end of each arm balls of negligible sizes and considerable masses of m_1=6 and m_2=4 [*kg*] are attached, respectively, and the gravitational acceleration was g=9.81 [m/s^2]. Detailed simulation results are provided in Appendix A.8. These results well exemplify the applicability of the proposed method.

10.3. Thesis 6: Adaptive Control for MIMO Systems by the Use of Approximate SVD of the Available Approximate Model (Summary of the Results of Chapter 10)

Based on the simple geometric interpretation of the *Singular Value Decomposition (SVD)* for real matrices I proposed a novel adaptive controller for non-special, nonlinear dynamical systems *the approximate analytical model of which is available*. This control has the following main features:

- It is a kind of generalization of the fixed point transformations using only two parameters, but in contrast to them, it is not restricted to either "*increasing*" or "*decreasing*" systems;
- The controlled system may have varying "*increasing*" and "*decreasing*" nature, the role of the rough analytical model and the SVD is to approximately "track" the variation of this nature in the state space of the controlled system;

- The proposed method does not require the real time application of SVD within the control cycles. Proper information on this nature of the system can be observed in the grid points of a mesh before exerting any control effort, and the result of these observations can be applied in the control process by the use of some interpolation technique, e.g. by applying Support Vector Machine applying radial basis functions.
- The operation of the proposed control was demonstrated via numerical simulations for an appropriate nonlinear paradigm.

The publications strictly related to this Thesis are as follows: [J14], [C102], [C104], [C106].

11. Approximation and Application of Fractional Order Derivatives in the Time Domain

In various physical, chemical, economic, etc. processes typical momentum is the existence of a great number of dynamically coupled degrees of freedom. Normally we can concentrate on modeling and control only of a few of them while no detailed information is available on the state of the other, not modeled and directly not controlled variables. Within the framework of Classical Physics, if $\mathbf{x}$ denotes those variables of the coupled system that are in the centre of our attention, the formal state propagation equation of type $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u})$ containing the controller's action in variable $\mathbf{u}$ will not be valid due to the presence of the not modeled coupled subsystems. A plausible way of modeling approximation is to assume that the coupled subsystems also are excited by the control action $\mathbf{u}$, and the state propagation of these subsystems may reveal itself in a kind of memory of the system that depends on the relaxation properties of the excited subsystems. A formal possibility to introduce "memory" into the model is the replacement of the *d/dt* differential operator with another operator that conveys some memory. For this purport the concept of the so-called "fractional derivatives" were found to be excellent tools.

Though the formal mathematical idea of introducing non-integer order derivatives can be traced from the 17th century in a letter by L'Hospital in which he asked Leibniz what would be the meaning of $D^n y$ if n = ½ in 1695 [R122], it was better outlined in the 19th century [R123]-[R125]. Due to the lack of their physical interpretation their first applications in Physics appeared only later, in the 20th century, in connection with visco-elastic phenomena [R126]-[R127]. The topic later obtained quite general interests [R128]-[R130], and also obtained new applications in material science [R131], analysis of earth-quake signals [R132], control of robots [R133], and in the description of diffusion [R134].

The concept of fractional derivatives has many, only more or less equivalent definitions, e.g. by Riemann-Liouville, Caputo, Grünwald-Letnikov, Hadamard, Marchaud, Riesz, etc. For our purposes, for its lucidity and simplicity, we use the discrete time resolution approximation of the form invented by Caputo that also was used in a fractional order controller developed for integer order system e.g. in [C94]. It will be shown that this approximation is applicable for modeling the dynamic behavior stable dissipative and unstable physical systems damped/excited by dynamical coupling to unmodeled internal degrees of freedom. Furthermore, it will be shown that an adaptive control method originally elaborated for integer order systems can be extended to the control of systems of fractional order dynamics.

11.1. Numerical Approximation of Caputo's Fractional Order Derivatives

The definition given by Caputo for the $\beta \in (0,1)$ order derivative of a function $u(t)$ for $a \le t$ is given as

$$ {}_a^C u_t^\beta := u^{(\beta)}(t) := \int_a^t \frac{\dot{u}(\tau)(t-\tau)^{-\beta}}{\Gamma(1-\beta)} d\tau \qquad (11.1.1) $$

in which the parameter a is in the role of some initial condition. If we wish to apply this concept for describing physical systems no any special time-instant can be in some "distinguished" position. Instead of that it is more reasonable to assume that this operator has to describe the "*memory properties*" of the physical system that normally can be modeled by some "*finite length of memory*" *L.* According to that (11.1.1) can be applied in the form as

$$ {}_{a}^{C}u_{t}^{\beta} := u^{(\beta)}(t) := \int_{t-L}^{t} \frac{\dot{u}(\tau)(t-\tau)^{-\beta}}{\Gamma(1-\beta)} d\tau \tag{11.1.2} $$

that has the interesting property that for *du/dt*≡*const.* it also yields $u^{(\beta,T)}$=*const.* since by substitution $\xi := t-\tau$, $d\xi = -d\tau$, $\xi \in [L,0]$, it is obtained that

$$ u^{(\beta,L)}(t) := -\dot{u}\int_{L}^{0} \frac{\xi^{-\beta}}{\Gamma(1-\beta)} d\xi = \frac{-\dot{u}\left[0^{1-\beta} - L^{1-\beta}\right]}{\Gamma(1-\beta)(1-\beta)}. \tag{11.1.3} $$

The expression in (11.1.3) immediately suggest the following approximation for non constant classical first derivative: let us divide the [*t-L*,*t*] interval into small sub-intervals as [$t-T\delta t$, $t-(T-1)\delta t$], [$t-(T-1)\delta t$, $t-(T-2)\delta t$], [$t-(T-2)\delta t$, $t-(T-3)\delta t$], …, [$t-\delta t$, t] ($L=T\delta t$), and let us suppose that *du/dt*≈*const.* within these small intervals. In this manner (11.1.2) can be approximated as a sequence of discrete $u(s)$ values as

$$ \begin{gathered} u^{(\beta,T,\delta t)}(t) := \sum_{s=0}^{T} \frac{\delta t^{1-\beta}\dot{u}(t-s\delta t)\left[(s+1)^{1-\beta} - s^{1-\beta}\right]}{\Gamma(2-\beta)} \equiv \\ = \sum_{s=0}^{T} G_s \dot{u}(t-s\delta t),\ G_s := \frac{\delta t^{1-\beta}\left[(s+1)^{1-\beta} - s^{1-\beta}\right]}{\Gamma(2-\beta)} \end{gathered}. \tag{11.1.4} $$

If the interval δt is small enough, in (11.1.4) finite element approximation can be applied for the estimation of the first order derivative as

$$ \begin{gathered} u^{(\beta)}(t) = \sum_{s=0}^{T} G_s \dot{u}(t-s\delta t) \approx \\ \approx \sum_{s=0}^{T} G_s \frac{u(t-s\delta t) - u(t-[s+1]\delta t)}{\delta t} = \\ = \sum_{s=0}^{T} H_s u(t-s\delta t),\ H_0 = \frac{G_0}{\delta t},\ H_{i>0} = \frac{G_i - G_{i-1}}{\delta t} \end{gathered}. \tag{11.1.5} $$

In similar manner, for higher order fractional derivatives (11.1.1) can be generalized for $\beta \in (0,1)$ as

$$ {}_{a}^{C}u_{t}^{n-1+\beta} := u^{(n-1+\beta)}(t) := \int_{a}^{t} \frac{u^{(n)}(\tau)(t-\tau)^{-\beta}}{\Gamma(1-\beta)} d\tau . \tag{11.1.6} $$

that in strict analogy with the above considerations yields the following approximation

$$ u^{(n-1+\beta)}(t) \approx \sum_{s=0}^{T} G_s u^{(n)}(t-s\delta t) \tag{11.1.7} $$

in which

$$ G_s = \frac{\delta t^{1-\beta}\left[(s+1)^{1-\beta} - s^{1-\beta}\right]}{\Gamma(2-\beta)},\ s = 0,1,\ldots,T . \tag{11.1.8} $$

It is worth noting that for $\beta \in (0,1)$ $G_s>0$, and $G_{s+1}<G_s$. By applying one of the usual finite order approximation of the higher integer order derivatives similar time-

sequences can be obtained. In the sequel we restrict ourselves for n=2 for which the following sequences can be obtained:

$$
\begin{aligned}
& u^{(1+\beta)}(t) \approx \sum_{s=0}^{T} J_s u(t - s\delta t) \\
& J_0 = \frac{G_0}{\delta t^2}, J_1 = \frac{G_1 - 2G_0}{\delta t^2}, \\
& J_i = \frac{G_{i-1} - 2G_i + G_{i+1}}{\delta t^2}, i = 2,...,T \\
& J_{T+1} = \frac{G_{T-1} - 2G_T}{\delta t^2}, J_{T+2} = \frac{G_T}{\delta t^2},
\end{aligned}
. \tag{11.1.9}
$$

In the next step the behavior and modeling capabilities of this approximation is investigated.

11.2. The Behavior of the Proposed Numerical Approximation of Caputo's Fractional Order Derivatives

Consider the behavior of a hypothetical physical system that satisfies the "fractional order differential equation" in the form of the above finite approximation as

$$
u^{(\beta,T,\delta t)}(t) = -\alpha u(t) + g(t). \tag{11.2.1}
$$

that in the case of $\beta\approx 1$ must be similar to an exponentially damped system driven by the "driving force" $g(t)$. Applying the approximation (11.1.5), and considering the time t as a discrete variable from this point on for the simplicity, the following sequence is obtained for the $u(t)$ signals:

$$
u(t) = \frac{g(t) - \sum_{s=1}^{T+1} H_s u(t-s)}{H_0 + \alpha}. \tag{11.2.2}
$$

It is evident, that in this case the "initial values" $\{u(t-i)|\ i=1,2,\ldots,T+1\}$ that altogether can also be referred to as the "*preceding history*" uniquely determine the $u(t)$, $u(t+1)$, etc. future values so we obtained a causal system. In contrast to the integer order systems the number of the possible independent values defining the "preceding history" is independent from the order of differentiation. For studying the behavior of this system various estimations can be applied. The most efficient estimation can be based on the matrix form of (11.2.2) for the non-excited case of $g\equiv 0$:

$$\begin{bmatrix} u_t \\ u_{t-1} \\ \vdots \\ u_{t-T} \end{bmatrix} = \mathbf{M}_H \begin{bmatrix} u_{t-1} \\ u_{t-2} \\ \vdots \\ u_{t-(T+1)} \end{bmatrix} =$$

$$= \underbrace{\begin{bmatrix} \frac{-H_1}{H_0+\alpha} & \frac{-H_2}{H_0+\alpha} & \cdots & \frac{-H_{T+1}}{H_0+\alpha} \\ 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\mathbf{M}_H} \begin{bmatrix} u_{t-1} \\ u_{t-2} \\ \vdots \\ u_{t-(T+1)} \end{bmatrix}. \tag{11.2.3}$$

The propagation of the system's state is evidently described the increasing powers of the $\mathbf{M}_H$ matrix of very special structure. Therefore it is interesting to see the spectrum and the eigenvalues of these matrices. In the vicinity of the "classical limit", i.e. short memory (T=2 is the smallest available number) and almost integer order derivative ($\beta \to 1$) with some small $\delta t \to 0$. For calculating the eigenvalues of the $\mathbf{M}_H$ matrix the following secular equation has to be solved [due to the simple structure of the matrix in (11.2.4) it is not difficult to calculate it in closed analytical form]:

$$\det\begin{bmatrix} \frac{-H_1}{H_0+\alpha} - \lambda & \frac{-H_2}{H_0+\alpha} & \frac{-H_3}{H_0+\alpha} \\ 1 & -\lambda & 0 \\ 0 & 1 & -\lambda \end{bmatrix} = \tag{11.2.4}$$

$$= \left(\frac{-H_1}{H_0+\alpha} - \lambda \right) \lambda^2 - \frac{H_2}{H_0+\alpha} \lambda - \frac{-H_3}{H_0+\alpha} = 0$$

According to (11.1.4) if $\beta \to 1$ $G_0 = \delta t^{(1-\beta)}/\Gamma(2-\beta) \to 1$, and if $i>0$ $G_i \to 0$, so $H_0 \to 1/\delta t$, $H_1 \to -1/\delta t$, $H_2 \to 0$, $H_3 \to 0$ that in the limit case reduces the secular equation to

$$= \left(\frac{1/\delta t}{1/\delta t + \alpha} - \lambda \right) \lambda^2 = 0 . \tag{11.2.5}$$

The nonzero eigenvalue works as follows: during time δt the appropriate eigenvector is multiplied by $\lambda = 1/(1+\alpha\delta t)$, so the variation of a quantity x during one cycle is as follows:

$$\frac{\delta x}{\delta t} = \frac{(\lambda - 1)}{\delta t} x = \frac{1}{\delta t} \frac{1-(1+\alpha\delta t)}{(1+\alpha\delta t)^2} x \approx -\alpha x \qquad \text{if } |\alpha\delta t| << 1 \tag{11.2.6}$$

That evidently corresponds to an exponential damping with the exponent $-\alpha$ as it is expected in the case of an integer order system.

For the near-limit case numerical calculations have to be done because in general the secular equations do not have solutions of closed analytical form. For instance, in the vicinity of the "classical limit", i.e. short memory (T=2) and almost integer order derivative (β=0.99) with δt=0.01 the following matrix is obtained for the case α=1:

$$\mathbf{M}_H = \begin{bmatrix} 0.9828115 & 0.0028349 & 0.0040490 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{11.2.7}$$

The eigenvalues of this matrix easily can be calculated by some popular software as e.g. SCILAB. They are {0.9898084, - 0.0034984 + 0.0638626*i*, - 0.0034984 - 0.0638626*i*} (each of them is smaller than 1 in its absolute value, while the determinant of the matrix **V** the columns of which are made of the appropriate eigenvectors is - 3.903D-17 - 0.0735043$i \neq 0$, that is any complex (therefore as special case real) initial vector can be calculated as a linear combination of the columns of **V**. The appropriate components of this sum are multiplied by the appropriate eigenvalues of the matrix $\mathbf{M}_H$ in each time step. Since the absolute values of these eigenvalues are smaller than 1, the system is stable and its “initial perturbation” relaxes to zero. It is worth noting that the real eigenvalue has the biggest absolute value, so it corresponds to the slowest relaxation. The absolute values of the two complex eigenvalues that describe some damped oscillation are very small, so these oscillations are relaxed very quickly. Therefore the components of the “preceding history” belonging to the eigenvectors of very fast relaxation die out very quickly, and the effect of a single “initial condition” seems to be more or less lasting [Fig. 11.2.1.].

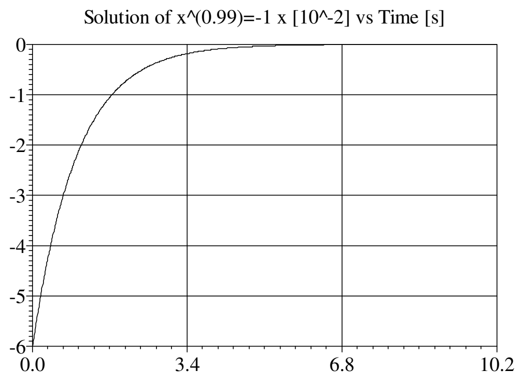


Figure 11.2.1. The approximately exponential relaxation of the “preceding history” nearby the classical limit

According to the terminology we already used, the “very fractional limit” means long memory (e.g. T=100), and small order of differentiation (e.g. β=0.01). In this numerical example the eigenvectors form a complete system (the appropriate determinant is equal to 0.0129997 - 1.388D-17$i \neq 0$), and the absolute value of the biggest eigenvector is 0.9931726. However, there are eigenvalues of considerable real parts and very small imaginary ones, therefore this system relaxes slowly and the structural richness of the “preceding history” does not die out quickly. This situation is well demonstrated by Fig. 11.2.2.

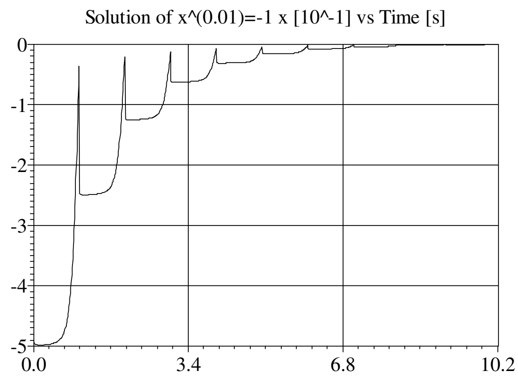


Figure 11.2.2. The relaxation of the "preceding history" in the "very fractional limit"

Quite similar considerations can be done for the fractional order derivatives higher than one. According to (11.1.9) the appropriate structure is as follows.

$$\begin{bmatrix} u_t \\ u_{t-1} \\ \vdots \\ u_{t-(T+1)} \end{bmatrix} = \mathbf{M}_J \begin{bmatrix} u_{t-1} \\ u_{t-2} \\ \vdots \\ u_{t-(T+2)} \end{bmatrix} =$$

$$= \underbrace{\begin{bmatrix} \frac{-J_1}{J_0+\alpha} & \frac{-J_2}{J_0+\alpha} & \cdots & \frac{-J_{T+2}}{J_0+\alpha} \\ 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\mathbf{M}_J} \begin{bmatrix} u_{t-1} \\ u_{t-2} \\ \vdots \\ u_{t-(T+2)} \end{bmatrix}. \tag{11.2.8}$$

In this case the "classical limit" corresponds to T=3 and $\beta\to 1$. The secular equation belonging to ((11.2.8)) has quite similar structure as (11.2.4) and the appropriate determinant can easily be computed:

$$-\lambda^5 - \frac{J_1}{J_0+\alpha}\lambda^4 - \frac{J_2}{J_0+\alpha}\lambda^3 - \frac{J_3}{J_0+\alpha}\lambda^2 + \frac{J_4}{J_0+\alpha}\lambda - \frac{J_5}{J_0+\alpha} = 0. \tag{11.2.9}$$

In the classical limit according to (11.1.9) $J_0\to 1/\delta t^2$, $J_1\to -2/\delta t^2$, $J_2\to 1/\delta t^2$, $J_3\to 0$, $J_4\to 0$, $J_5\to 0$. This reduces (11.2.9) to

$$\lambda^3\left(-\lambda^2 + \frac{2}{1+\alpha\delta t^2}\lambda - \frac{1}{1+\alpha\delta t^2}\right) = 0. \tag{11.2.10}$$

The nontrivial solution can exactly be obtained by making the 2nd order term in the parentheses equal to zero yielding

$$\lambda_{1,2} = \frac{1 \mp i\delta t\sqrt{\alpha}}{1+\alpha\delta t^2}. \tag{11.2.11}$$

Assuming that $|1+\alpha\delta t^2| << 1$ the polar form of these eigenvalues approximately is $\exp(\pm i\delta t\sqrt{\alpha})$. Since during the time-slot of length δt one matrix multiplication happens this corresponds to the circular frequency $\omega \approx \delta\varphi / \delta t = \sqrt{\alpha}$. This evidently corresponds to the non-damped harmonic oscillation of the classical limit. As a numerical example, T=3, δt=0.01, and β=0.99 can be considered. The appropriate eigenvalues of $\mathbf{M}_J$ are as follows: {0.9998961 + 0.0101323i, 0.9998961 - 0.0101323i, 0.0642578 + 0.1318036i, 0.0642578 - 0.1318036i, - 0.1354708} that corresponds to a complex conjugate pair of slowly relaxing eigenvalues, while the others relax relatively quickly [Fig. 11.2.3.]. The determinant of the matrix of the eigenvectors is 0.0000610 - 4.382D-19i, that is this system is nonsingular again.

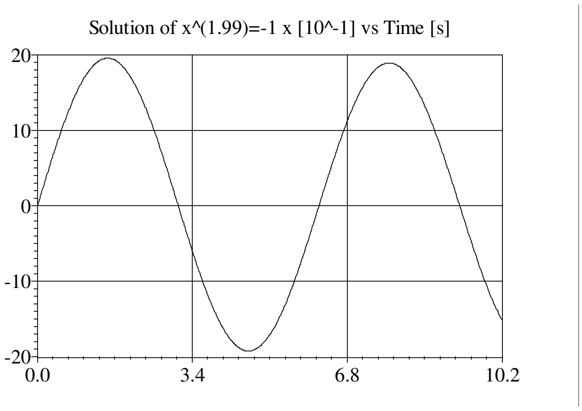


Figure 11.2.3. The approximately exponential relaxation of the "preceding history" nearby the classical limit

Since during δt exactly one multiplication happens with the matrix, that corresponds to a circular frequency ω = 0.0101334 /δt=1.01334 *rad/s* and a period $2\pi/\omega$≈6.2 *s*. This well agrees with the period of the signal in the upper chart of Fig. 11.2.3.. (In the case of the integer 2nd order derivatives ω would be √1=1.) Similar calculation for α=100 yields a period of 0.6222019 *s* for the slowly damped pair of eigenvalues that also is in good agreement with the charts, and with the period of the second order system. (For the integer order system $2\pi/10$≈0.6283185 *s*.)

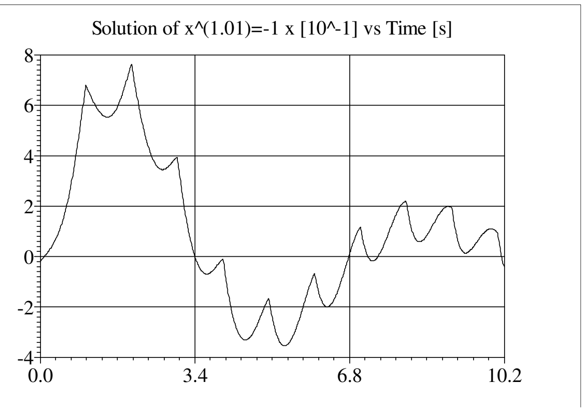


Figure 11.2.4. The relaxation of the "preceding history" in the "very fractional limit"

The case "far from the classical limit" is defined by T=100, and β=0.01. (The biggest eigenvalue then was 0.9996263.) The results of certain numerical calculations are given in Fig. 11.2.4. The determinant of the matrix of the eigenvectors was - 8.327D-17 - 0.1067265i, that is this system was far from being singular again. The slowly relaxing pair of eigenvalues of the near classical limit have the exponential form 0.9999475 × exp(±i0.0101334). Simulation results for a "medium value" are given in Fig. 11.2.5.

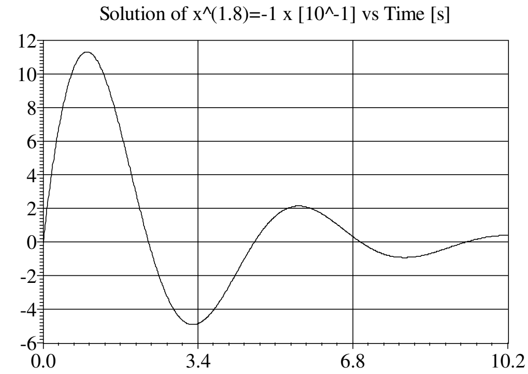


Figure 11.2.5. The relaxation of the "preceding history" in a "medium" case

It is interesting to see what happens if in the closed analytical formulae obtained as the numerical approximation of the Caputo derivatives $2>\beta>1$. Though the original integrals cannot be calculated for such cases, the approximations yield physically interpretable behavior [Fig. 11.2.6.].

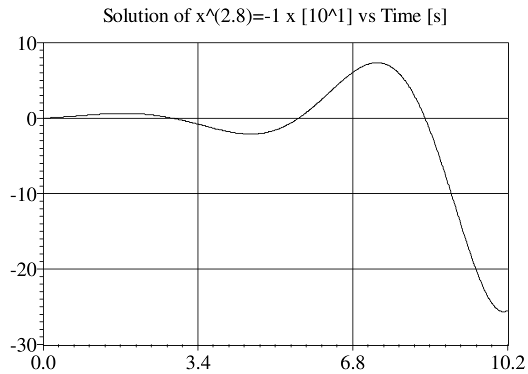


Figure 11.2.6. $2>\beta>1$ case far from the classical limit: $T=100$, $\beta=1.8$

The approximation from the "lower order side" yields relaxing oscillations, while from the higher order side we obtain unstable system with the eigenvalue of maximal absolute value 1.004497.

To sum up it can be stated that this simple discrete approximation of the Caputo derivatives seems to be a useful means of modeling fractional order systems in the time domain.

In the sequel potential applications of the proposed approximation are discussed.

11.3. Application Example: the Use of Fractional Order Terms in the Control of Integer Order Systems

For application example the control of the same physical system already considered in section "7.3. Simulation Example for Potential Application of the Special Symplectic Matrices", i.e. the control of two coupled cart+double pendulum system [depicted in Fig. A.5.1. and described by (A.5.4) and (A.5.5)] was chosen. The same kinematically prescribed tracking strategy was considered as in (A.5.6) with the same adaptive control for the "centralized" approach using the special symplectic matrices of size 12×12 defined in (7.1.10) but the computation of the torque to be exerted happened in different manner. Observing that according to (11.1.1), (11.1.2), (11.1.4), (11.1.5) a constant integer order derivative of a function d$u(t)$/dt=const. yields constant fractional order derivative $u^{(\beta)}(t)$, and that the ratio of the two constants is very close to 1 if β is close to 1, the rough initial system model was utilized to exert the generalized forces. In the simulation the *rough initial system model* for both carts was $\mathbf{Q}=10\mathbf{q}^{(1+\beta)Des}+10\times[1,1,1]^T$ instead of $\mathbf{Q}=10\ddot{\mathbf{q}}^{Des}+10\times[1,1,1]^T$. It is reasonable to expect for $0<\beta<1$ this degrades the tracking accuracy, however, due to the "internal memory" of the fractional order derivators it can smooth the fast fluctuation in the torque typically appearing in Fig. A.5.2. Detailed computational results are given in Appendix A.9.

11.4. Thesis 7: Numerical Approximation of Fractional Order Derivatives and Their Potential Applications (Summary of the Results of Chapter 11)

I have introduced a discrete approximation of the Fractional Order derivatives defined by Caputo.

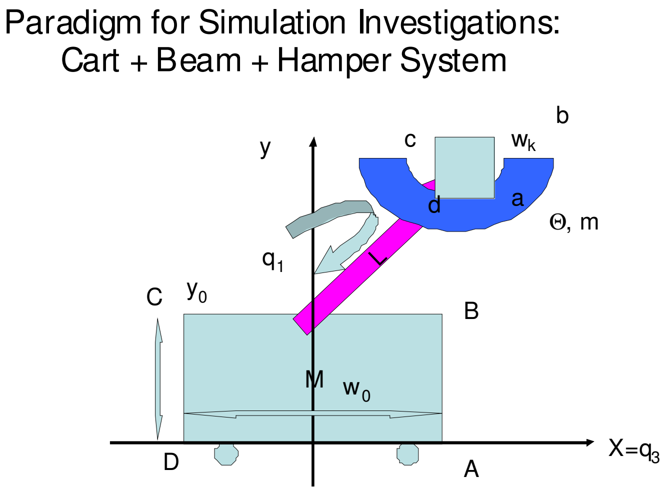
- I have shown that the proposed approximation has three parameters as follows: the time-step of discretization, the memory length of the approximation, and the order of differentiation;
- Instead of the concept of "initial condition" usually used in the literature I proposed to apply the concept of "preceding history" that naturally takes into account the memory length of the approximation of the *operator* and makes the distinguished position of the "initial time instant" cease;
- Via considering the numerical solutions of homogeneous, linear, fractional order differential equations with constant coefficients I have shown that in the limit case the proposed approximation yields the common integer order derivatives; For this purpose the eigenvalues of special matrices were calculated in analytical form;
- I have shown that by extending the "order" parameter of the derivation to a higher possible set of values than that allowed by the original definition given by Caputo both stable dissipative and unstable systems can simply be modeled;
- I have shown that the numerical approximation can well be combined with the novel adaptive control approaches I formerly introduced;
- I have shown that the frequency filtering property of the proposed approximation can well be used for smoothing the operation of the adaptive controller.

To the subject area of Thesis 7 the following publications are strictly related: [C66], [C69], [C70], [C71], [C72], [C73], [C75], [C76], [C78], [C79], [C81], [C82], [C92], [C93], [J13].

Appendix

A.1. Simulation Results for Section "4.4.1. Modification of the Tuning Rule of the Adaptive Inverse Dynamics Controller"

The appropriate tuning strategies were numerically studied by the use of a paradigm sketched in Fig. A.1.1. The exact parameters of the system were M=30 *kg*, m=10 *kg*, L=2 *m*, Θ=20 $kg\times m^2$, g=10 m/s^2. The approximate model parameters in the dynamic model were $\hat{M}$ =60 *kg*, $\hat{m}$ =20 *kg*, $\hat{L}$ =2.5 *m*, $\hat{\Theta}$ =50 $kg\times m^2$, and $\hat{g}$ =8 m/s^2.



$$
\begin{bmatrix} \left(mL^2+\Theta\right) & \Theta & mL\cos q_1 \\ \Theta & \Theta & 0 \\ mL\cos q_1 & 0 & \left(m+M\right) \end{bmatrix}
\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix}
+
\begin{bmatrix} -mgL\sin q_1 \\ 0 \\ -mL\sin q_1\dot{q}_1^2 \end{bmatrix}
=
\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}
$$

Figure A.1.1. The schematic picture and the equations of motion of the cart+beam+hamper system

In Fig. A.1.2. the operation of the "conventional" *Adaptive Inverse Dynamics Controller* is shown for the parameter setting $K_0 = K_0\times\mathbf{I}$, $\mathbf{K}_1=2\sqrt{K_0}\times\mathbf{I}$, $\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}=10^6\mathbf{I}$, $\mathbf{R}=5\mathbf{I}$.

We note that for smaller **R** value (i.e. faster tuning) the tuning process caused numerical overflow. Figure A.1.4. is the counterpart of Fig. A.1.2. with modified tuning based on the use of the directly available information and *SVD*. The improvement in the tracking accuracy is quite impressive. It is worthy of note that while the original, Lyapunov function based technique, due to the use of matrix **R**, almost unnecessarily modifies each parameters and cannot reach any settling point during the simulations, the *SVD*-based method reveals that only two of the three singular values differ significantly from zero, therefore the proper tuning concerns only two-dimensional subspaces of the space of the parameter errors.

Furthermore, in the cases when simultaneously each singular value is small but remains within the range for being kept, by the use of a few "brave" steps the tuning quickly is settled at good approximation of the dynamical parameters and the trajectory tracking becomes precise.

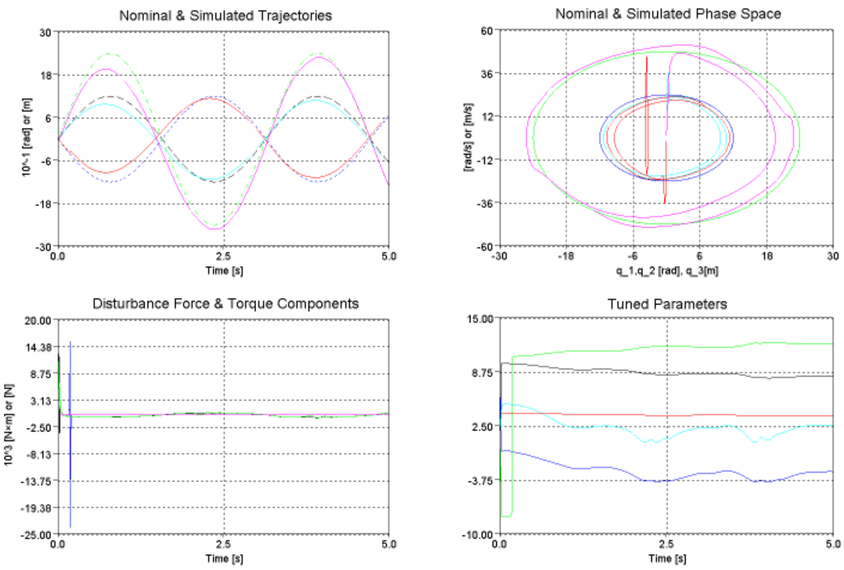


Figure A.1.2. The operation of the original "Adaptive Inverse Dynamics Controller" for $\mathbf{K}_0$=10 s^{-2}**I**, $\mathbf{K}_1$=2√*10*×**I**, $\mathbf{A}^T\mathbf{P}+\mathbf{PA}$=$10^6$**I**, **R**=5**I**

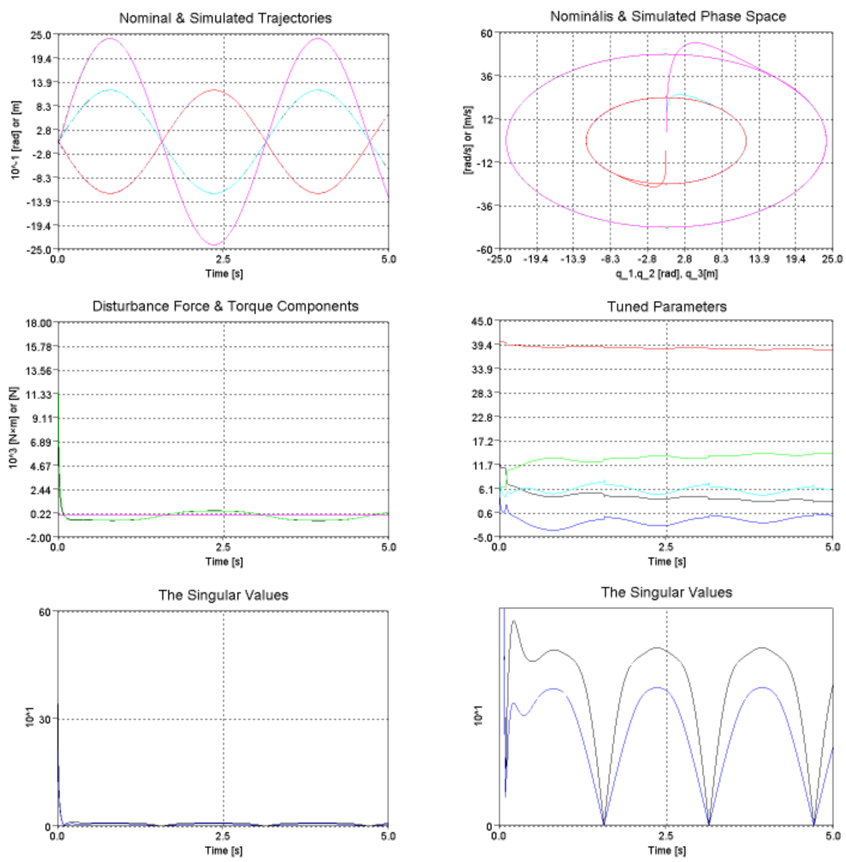


Figure A.1.3. The operation of the "Adaptive Inverse Dynamics Controller" with modified tuning and integrated feedback for $\mathbf{\Lambda}$=10×**I** s^{-2}, γ=50, *and the minimal singular value kept in the generalized inverse* ε_{SVD}=10^{-2};

The improvement in tuning is even more illustrative for the variant of *Adaptive Inverse Dynamics* also using integrated feedback and modified tuning (Fig. A.1.3.).

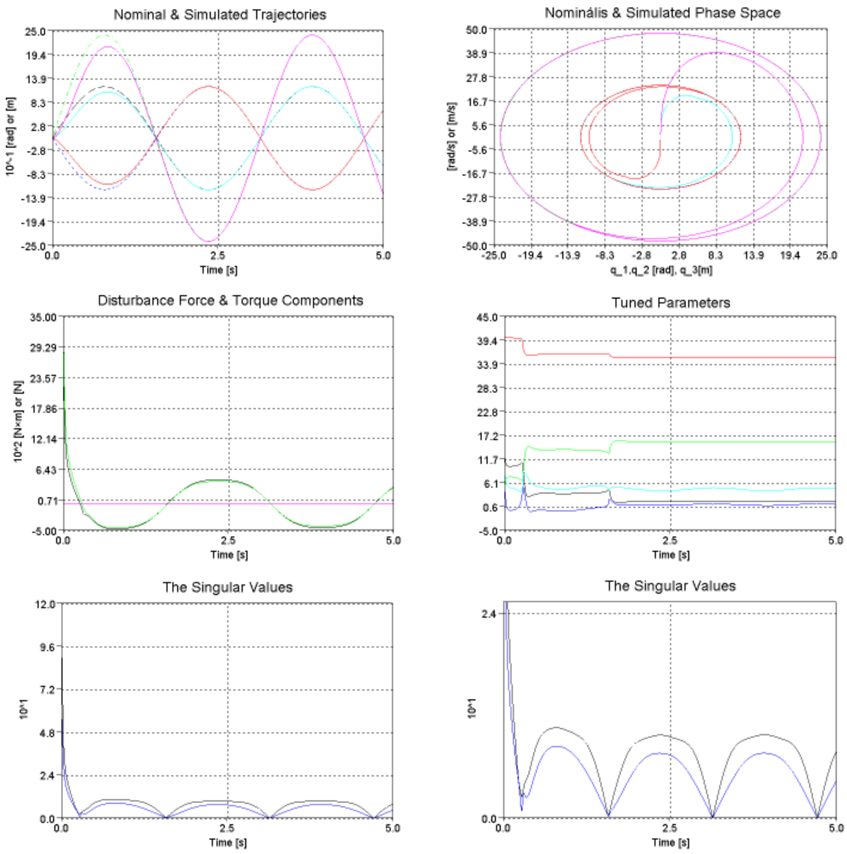


Figure A.1.4. The operation of the "Adaptive Inverse Dynamics Controller" with modified tuning for $\mathbf{K}_0$=10 $s^{-2}\mathbf{I}$, $\mathbf{K}_1=2\sqrt{10}\times\mathbf{I}$, γ=50, *and the minimal singular value kept in the generalized inverse* $\varepsilon_{SVD}=10^{-2}$;

It is worth noting that the parameter tuning again happens in two-dimensional subspaces of the space of the parameter estimation errors, furthermore, the presence of the integrated error-feedback results in more precise tracking from the onslaught.

In the sequel simulation investigations will show that ***even relatively insignificant external perturbations can considerably degrade the operation of these sophisticated controllers trough fobbing/misleading/foolishing their sensitive parameter tuning processes***. For this purpose it is satisfactory to show the phase trajectories of the controlled motion and certain excerpts revealing details of the tuning process.

Figure A.1.5. pertains to the original "*Adaptive Inverse Dynamics Controller*". The external disturbance was created by fitting a 3rd order periodic spline function to 11 randomly selected points within a narrow interval. It was applied as addition only to Q_3, its "insignificance in comparison with the exerted control forces" is well exemplified by the chart in the lower left corner of the figure that displays a zoomed excerpt of the chart in the upper right corner (the curve in magenta).

Similar effect can be observed in Fig. A.1.6. for the variant applying modified parameter tuning. The "hectic behavior" of the controlled system in the last seconds considered well exemplifies what may happen to a nonlinear system if it is kicked out of one its "normal regimes of operation". It cannot be taken granted that the controller can pull its state back to the normal regime. The same conclusion can be drawn from Fig. A.1.7. that belongs to the more efficient parameter tuning and integrated feedback.

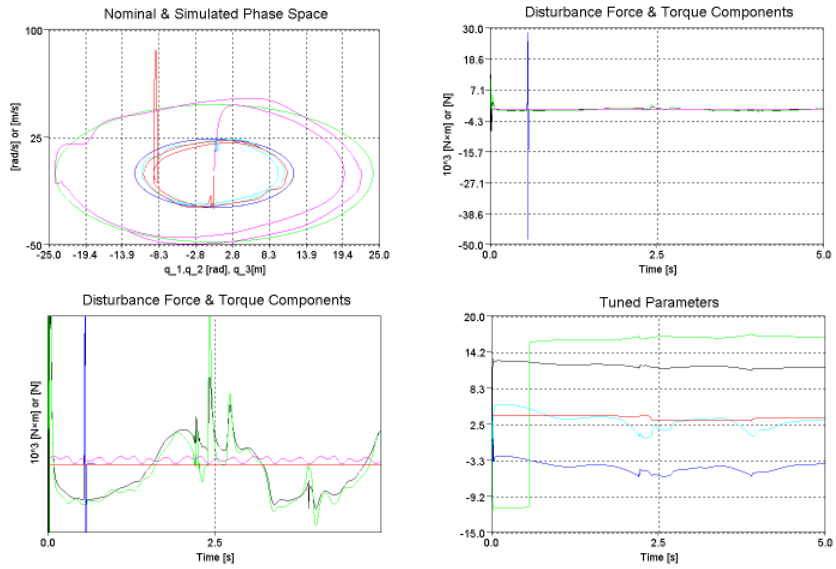


Figure A.1.5. The operation of the original "Adaptive Inverse Dynamics Controller" for $\mathbf{K}_0$=10 $s^{-2}\mathbf{I}$, $\mathbf{K}_1$=2√*10*×**I**, $\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}$=$10^6$**I**, **R**=5**I** *under external noises acting as addition to* Q_3 *only (the line in magenta in the force/torque diagram)*

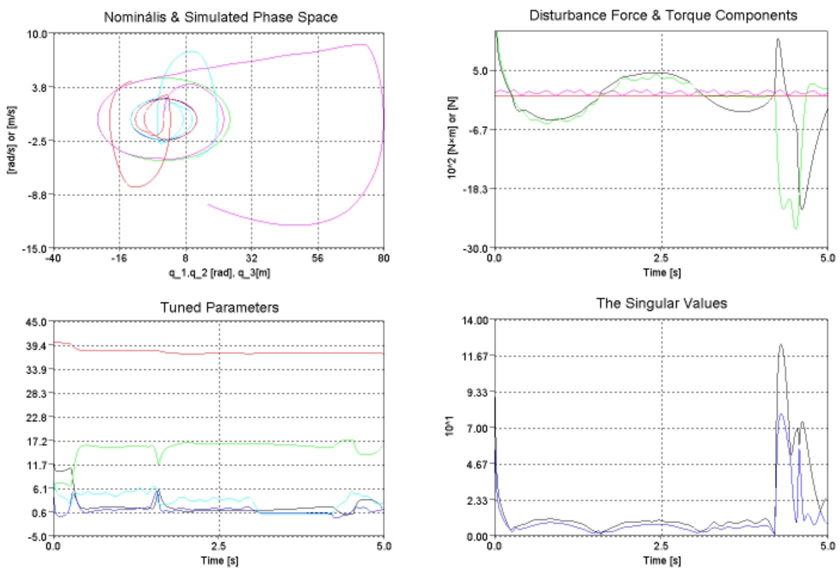


Figure A.1.6. The operation of the "Adaptive Inverse Dynamics Controller" with modified tuning for $\mathbf{K}_0$=10 $s^{-2}\mathbf{I}$, $\mathbf{K}_1$=2√*10*×**I**, γ=50, *and the minimal singular value kept in the generalized inverse* $\varepsilon_{SVD}=10^{-2}$ *under external noises acting as addition to* Q_3 *only (the line in magenta in the force/torque diagram)*

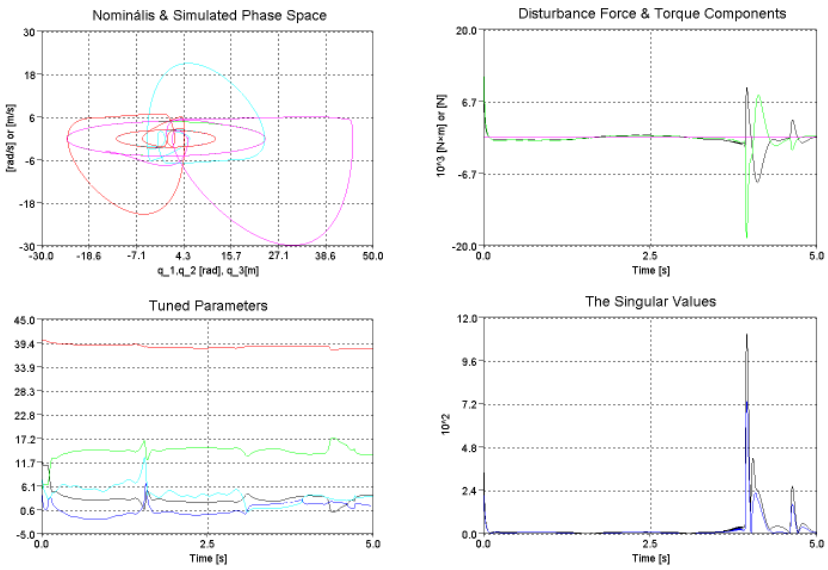


Figure A.1.7. The operation of the "Adaptive Inverse Dynamics Controller" with modified tuning and integrated feedback for $\mathbf{\Lambda}$=10×**I** s^{-2}, γ=50, *and the minimal singular value kept in the generalized inverse* ε_{SVD}=10^{-2} *under external noises acting as addition to* Q_3 *only (the line in magenta in the force/torque diagram)*

A.2. Simulation Results for Section "4.5.1. Modification of the Parameter Tuning Process in the Adaptive Slotine-Li Controller"

To illustrate the operation of the original and modified Slotine-Li controllers further simulations were made for the same physical system and nominal trajectory. Their results are discussed in the sequel. Figure A.2.1. belongs to the original version with the control parameters given in the caption of the figure. It can well be seen that due to the not very efficient tuning that is the consequence of insisting on the use of Lyapunov function, though in the beginning significant variation of the tuned parameters happens, within the duration of the simulations the parameters were not fairly tuned and the tracking errors remained significant. The introduction of integrated feedback [Fig. A.2.2.] allowed faster tuning and resulted in more precise tracking in general.

In Fig. A.2.3. the results describing the operation of "*Adaptive Slotine-Li Controller*" *modified by the introduction of integrated feedback term and SVD-based tuning* are given. The benefits of the modifications are quite similar to that obtained in the case of the modification of the "*Adaptive Inverse Dynamics Controller*". Though for a while a three-dimensional subspace of the five-dimensional parameter-error space was tuned in the beginning, later on the tuning process was restricted to two-dimensional subspaces. (Of course the direction of these subspaces can vary in time, therefore it is possible to study the whole five-dimensional space in this manner.)

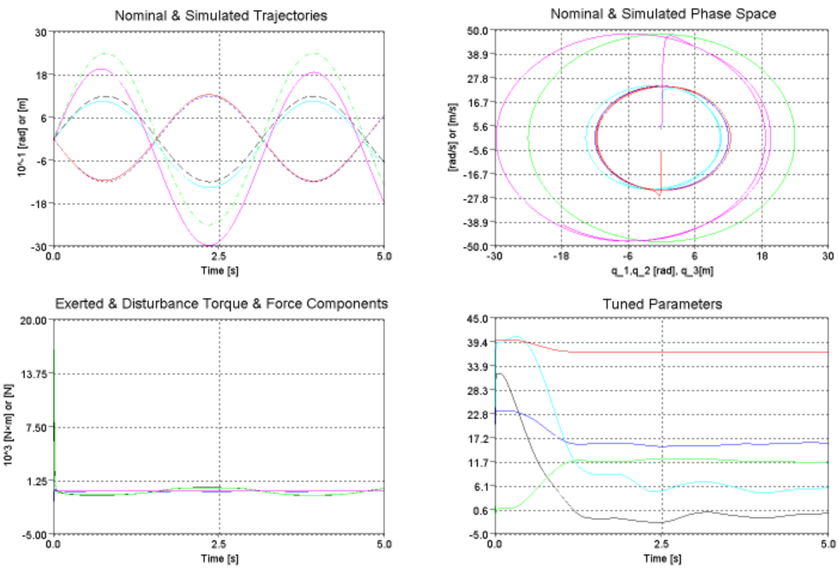


Figure A.2.1. The operation of the original "Adaptive Slotine-Li Controller" for $\mathbf{\Lambda}$=10×**I** *s*$^{-1}$, $\mathbf{K}_D$=100, $\mathbf{\Gamma}$=0.05×**I** *(for smaller* $\mathbf{\Gamma}$ *i.e. for faster tuning numerical overflow happened)*

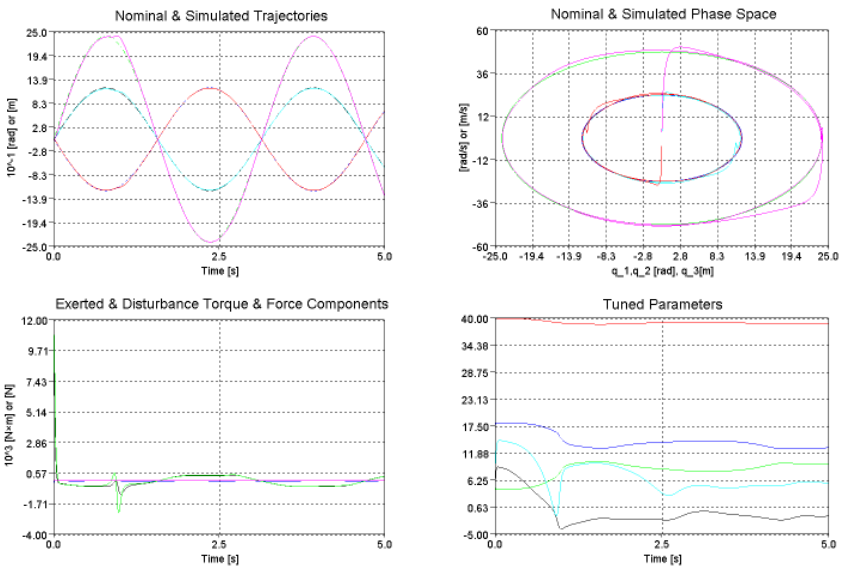
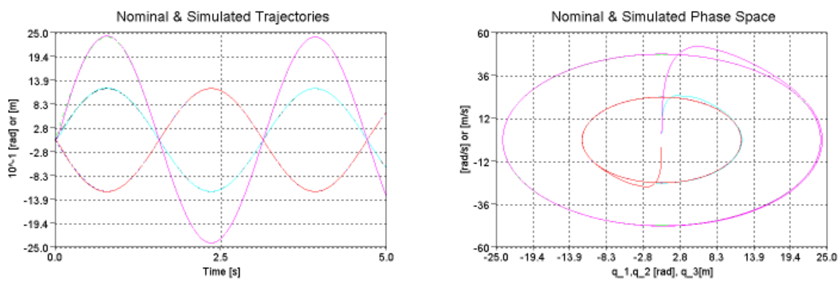


Figure A.2.2. The operation of the "Adaptive Slotine-Li Controller" modified by the introduction of integrated feedback for $\mathbf{\Lambda}$=10×**I** *s*$^{-1}$, $\mathbf{K}_D$=100, $\mathbf{\Gamma}$=0.01×**I**



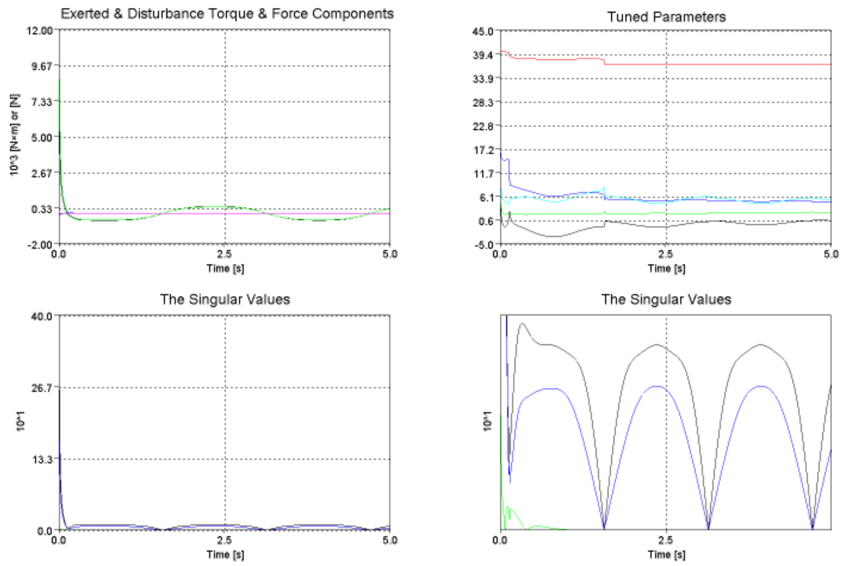


Figure A.2.3. The operation of the "Adaptive Slotine-Li Controller" modified by the introduction of integrated feedback and SVD-based tuning for $\mathbf{\Lambda}$=10×**I** s^{-1}, $\mathbf{K}_D$=100, γ=50×**I**

The next simulations belong to the presence of external noises. The original version of the "*Adaptive Slotine-Li Controller*" suffered the smallest catastrophe though its precision was very bad, too [Fig. A.2.4.].

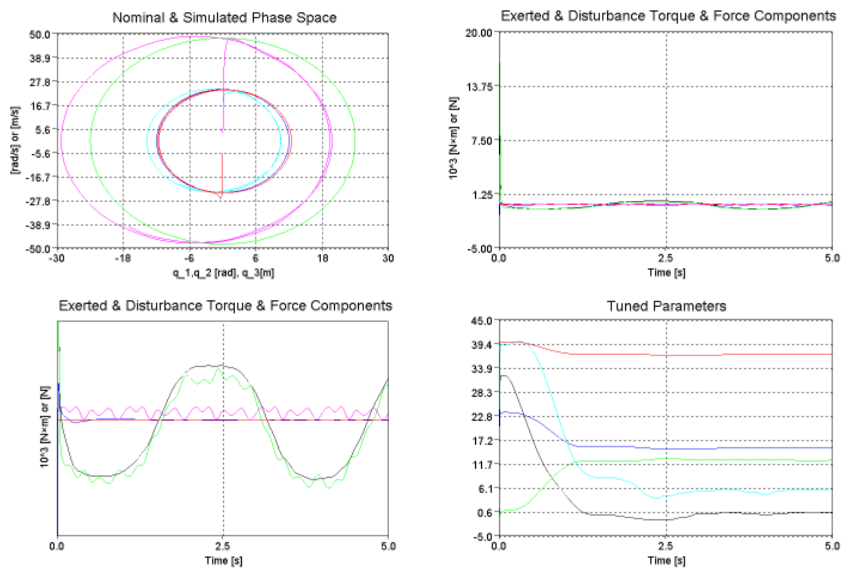


Figure A.2.4. The operation of the original "Adaptive Slotine-Li Controller" for $\mathbf{\Lambda}$=10×**I** s^{-1}, $\mathbf{K}_D$=100, $\mathbf{\Gamma}$=0.05×**I** *under external noises acting as addition to* Q_3 *only (the line in magenta in the force/torque diagram)*

Its completion with integrated feedback without speeded up parameter tuning remained relatively robust [Fig. A.2.5.]. The more drastic tuning in Fig. A.2.6. destroyed the stable behavior of this controller.

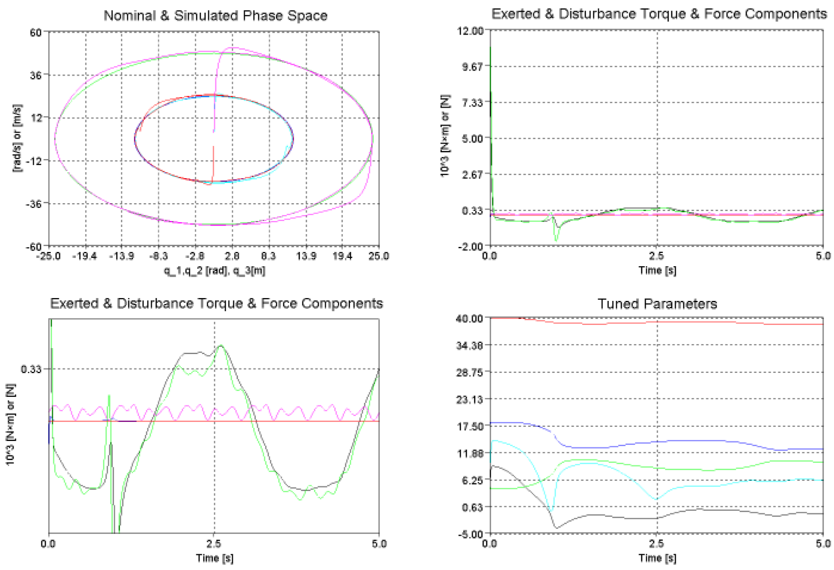


Figure A.2.5. The operation of the "Adaptive Slotine-Li Controller" modified by the introduction of integrated feedback for $\mathbf{\Lambda}$=10×$\mathbf{I}$ s^{-1}, $\mathbf{K}_D$=100, $\mathbf{\Gamma}$=0.01×$\mathbf{I}$ *under external noises acting as addition to* Q_3 *only (the line in magenta in the force/torque diagram)*

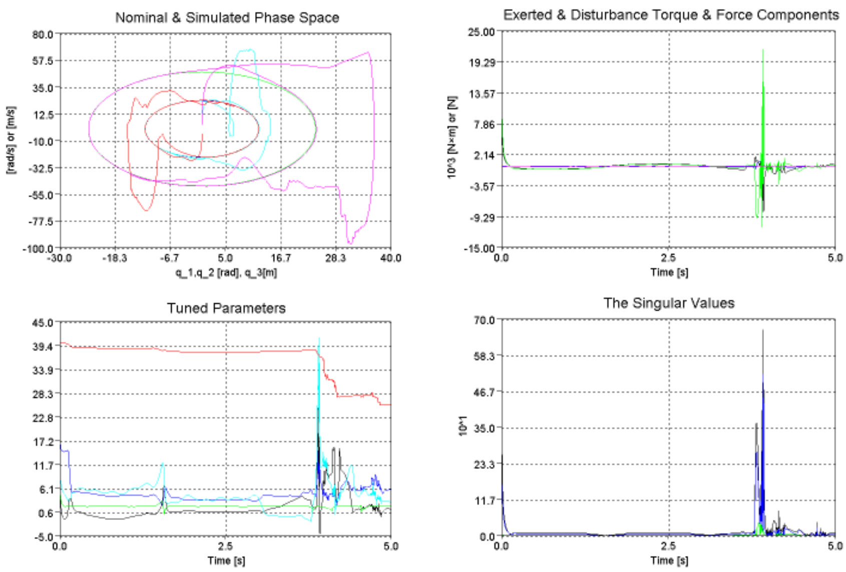


Figure A.2.6. The operation of the "Adaptive Slotine-Li Controller" modified by the introduction of integrated feedback and SVD-based tuning for $\mathbf{\Lambda}$=10×$\mathbf{I}$ s^{-1}, $\mathbf{K}_D$=100, γ=50×$\mathbf{I}$ *under external noises acting as addition to* Q_3 *only (the line in magenta in the force/torque diagram)*

A.3. Simulation Results for Section "6.1.2. Simulation Results for the Use of Diagonalization of the Inertia Matrix "

In Fig. A.3.1. the kinematic data of the motion are described, while Fig. A.3.2. reveals the error values regarding the trajectory and the contact force versus time.

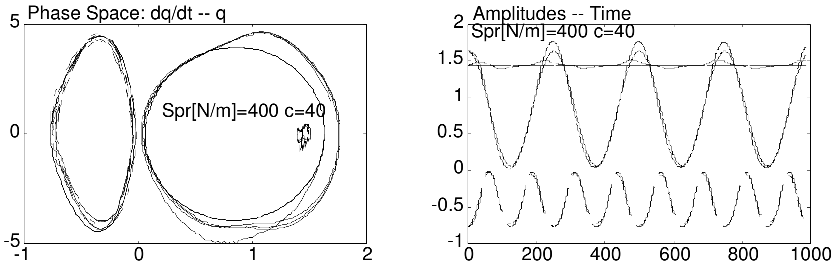


Figure A.3.1. The phase trajectory and the trajectory for the required (nominal) and the simulated motion ([m/s] vs. [m] and [rad/s] vs. [rad], time: 5 [ms] units).

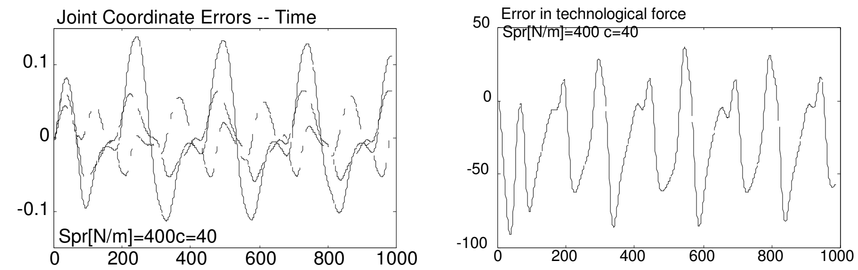


Figure A.3.2. The error in the trajectory and the contact force versus time (in [m] and [rad] and [N] and the time in 5 [ms] units, respectively).

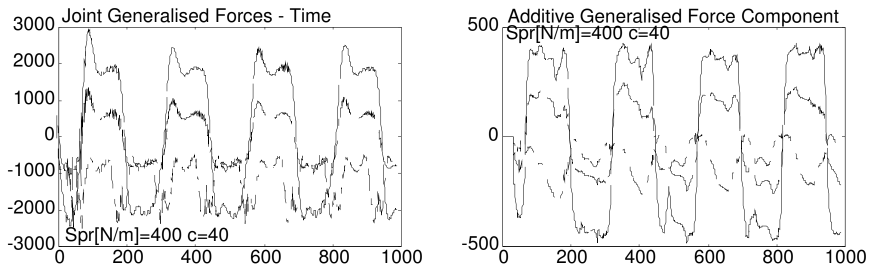


Figure A.3.3. The full amount of the generalized forces and the regression-based addition versus time (in [Nm] [N] and 5 [ms] units, respectively).

The maximum error in the contact force is about 80 [*N*] which is small enough if compared to the requested 1200 [*N*]. The error of the first rotational link is about 0.15 [*rad]*, the second one keeps its required constant value with the error of about 0.05 [*rad*], while the error of the telescopic axis is about 0.05 [*m*].

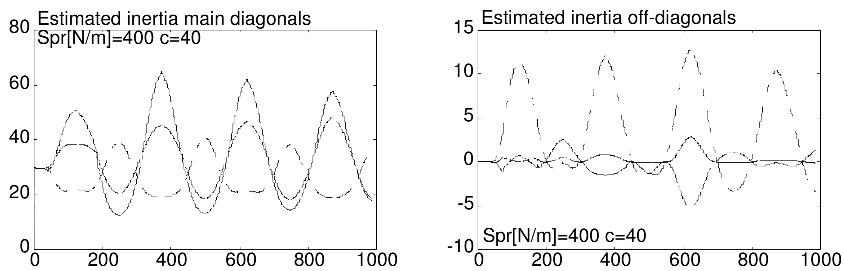


Figure A.3.4. The estimated "inertia data" in SI units (the various components of matrix H *have different physical dimensions).*

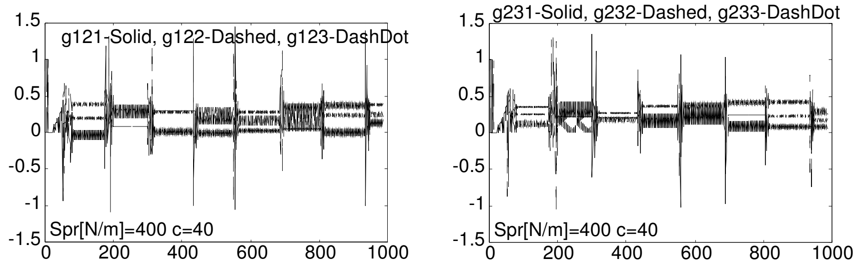


Figure A.3.5. The variation of six directly tuned parameters vs time (in 5 [ms] units).

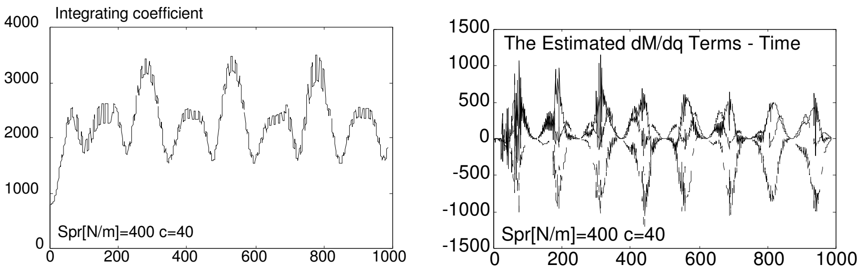


Figure A.3.6. The variation of the integrating term in the PID/ST part and that of one of the quadratic terms in the Euler-Lagrange equations vs. time (in 5 [ms] units).

Fig. A.3.3. reveals that the regression-based “addition” is quite significant that is this ancillary solution is quite useful in the control. To disclose the other “background processes” in Fig. A.3.4. the six independent elements of the estimated “inertia matrix” are plotted. The change in these matrix elements is quite considerable that means that this part of the control well cooperates with the other parts, too. The same can be told of the directly tuned parameters some of which described in Fig. A.3.5., and of the integrating term of the *PID/ST* part also displayed in Fig. A.3.6. It is worth noting that the quadratic terms in the Euler-Lagrange equations play a quite important role in the control according to Fig. A.3.6., too. Regarding the operation of the slower external loop the simulation results did not show considerable drift though considerably different initial values were investigated. It seems that the other parts of the control almost form a local "optimum" for these parameters at least in the cases investigated.

A.4. Simulation Results for Section “6.2.3. Application Example for the Use of Symplectic Transformations as the Sources of Uniform Structures in Classical Mechanics”

In the forthcoming simulation examples the same robot arm structure was used as that of Fig. A.4.1. [its Euler-Lagrange equations of motion are given in (A.4.1) and (A.4.2)].

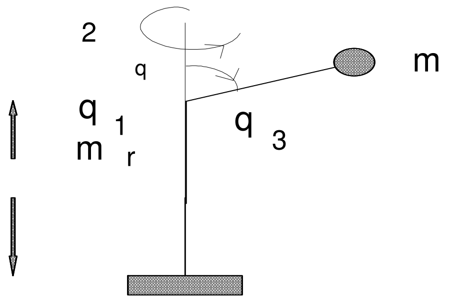


Figure A.4.1. The particular paradigm considered

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{k}(\mathbf{q},\dot{\mathbf{q}})+\mathbf{g}(\mathbf{q})=\mathbf{Q}\,. \tag{A.4.1}$$

$$\mathbf{M}=\begin{bmatrix} m_a+m_r & 0 & -m_aR_0Sinq_3 \\ 0 & m_aR_0^2Sin^2q_3 & 0 \\ -m_aR_0Sinq_3 & 0 & m_aR_0^2 \end{bmatrix},$$
$$\mathbf{k}=\begin{bmatrix} -m_aR_0Cosq_3\dot{q}_3^2 \\ 2m_aR_0^2Sinq_3Cosq_3\dot{q}_3\dot{q}_2 \\ -m_aR_0Cosq_3\dot{q}_1\dot{q}_3 \end{bmatrix},\ \mathbf{g}=\begin{bmatrix} m_ag+m_rg \\ 0 \\ -m_agR_0Sinq_3 \end{bmatrix} \tag{A.4.2}$$

The endpoint of the arm was connected to a dashpot producing elastic spring forces (with stiffness of 600 [*N/m*]) and viscous damping (100 [*Ns/m*]) as external perturbations. This manipulator arm consists of a vertical rod of 5 *kg* moving up and down (q_1 in *m*), rotating around itself as a vertical axis (q_2 in *rad*), and a second rod joined to it by a wrist tilting around a horizontal axis (q_3 in *rad*). This latter joint also was translated by q_1 and rotated by q_2. The second rod had negligible mass but carried a point-like small body of variable mass. It also had constant length (R_0=3 [*m*]). The three axes were controlled by drives exerting force for q_1 and torque for q_2 and q_3 prescribed by the control strategy. In each case considered the end-point of the robot arm was desired to be moved with circular frequency Ω [*rad/s*] along a circle of 0.5 *m* radius lying in a vertical plane at a distance of 2 *m* from the vertical axis. In each case the "*initial rough estimation*" of the dynamic model consisted of a non-singular, constant inertia matrix and a constant gravitational term. No quadratic velocity coupling was taken into account. Making all the further corrections was the task of the *Symplectizing Algorithm*.

The "canonical coordinates" without system identification were $[\mathbf{q}^T,(\mathbf{M}d\mathbf{q}/\mathrm{d}t)^T]^T$ with the estimated model inertia **M**. Simulations were run for pure application of the *Symplectizing Algorithm* and with complementary tuning only one of the Symplectic matrices in (6.2.5), namely matrix **B** by matrix **P**.

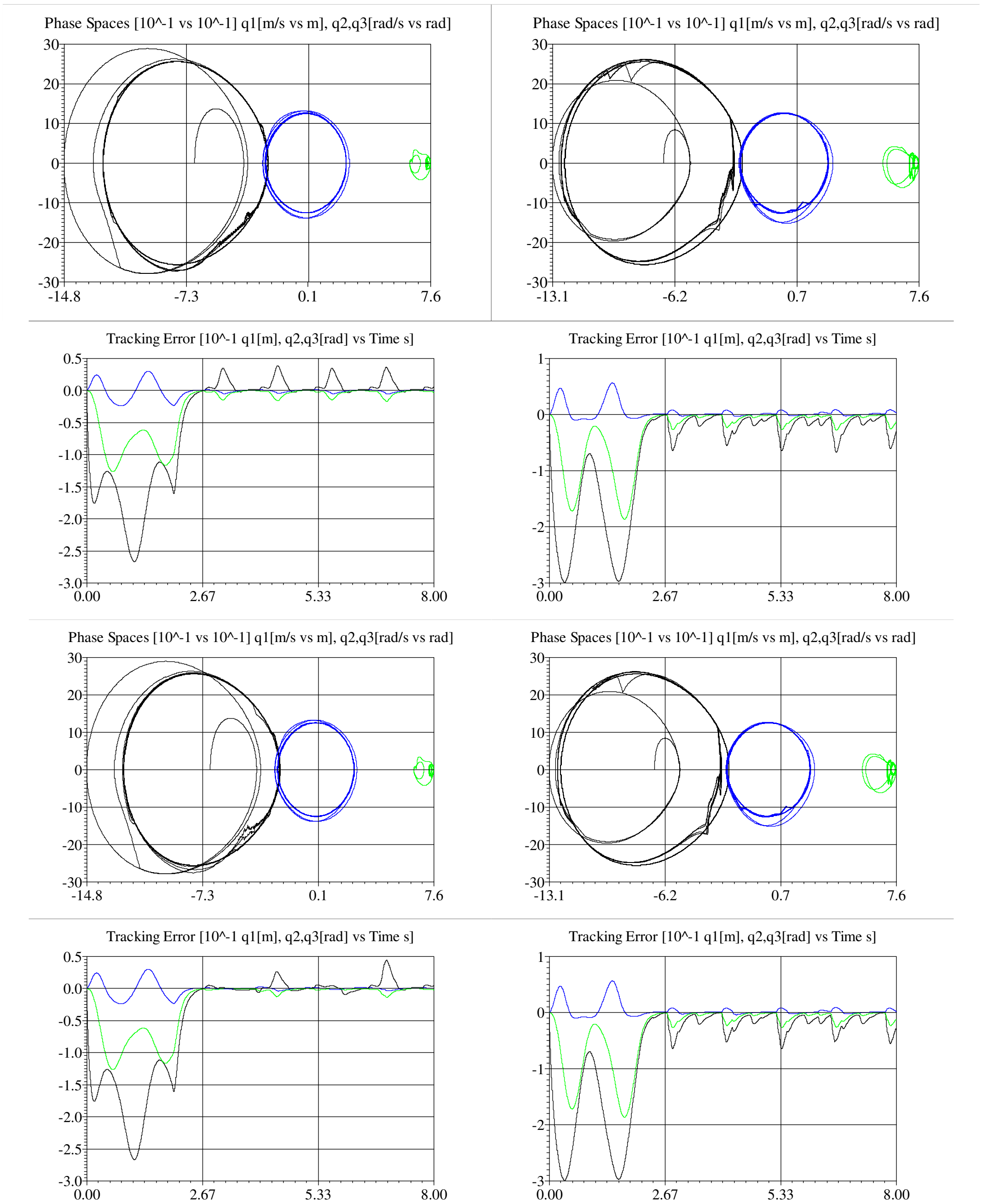


Figure A.4.2. Simulation results for "slow" motion Ω=5 [rad/s] without (LHS) and with (RHS) external perturbation without complementary tuning (the first two rows) and with complementary tuning of step length 10^{-8} [dimensionless] (3rd and 4th rows) [q_1:black, q_2:blue, q_3:green lines]

(For the sake of simplicity $\mathbf{P}^*\equiv\mathbf{I}$ was investigated only with independent tuned variables φ in (6.2.21) and ψ in (6.2.24).) In the simulations in the first half of the time considered no any Symplectic identification was applied, only the rough initial model was in use. In the second half of the time of the investigations the $[\mathbf{q}^T,(\mathbf{M}d\mathbf{q}/dt)^T]^T$ "canonical coordinates" were transformed by Symplectic matrices obtained by the Symplectizing Algorithm and the additional tuning if it was applied. For trajectory tracking the purely kinematically formulated

$$\ddot{\mathbf{q}}^D = \ddot{\mathbf{q}}^N - b\left(\dot{\mathbf{q}}^R - \dot{\mathbf{q}}^N\right) - c\left(\mathbf{q}^R - \mathbf{q}^N\right) \tag{A.4.3}$$

error relaxation was prescribed with b=30 [1/s], and c=0.8×(b^2)/4 [1/s^2] that guarantees oscillation-free desired tracking (the superscripts "R", "N" and "D" corresponds to the realized, the nominal, and the desired quantities).

In the calculations Ω=5 [*rad*/*s*], Ω=10 [*rad*/*s*], Ω=20 [*rad*/*s*] and Ω=25 [*rad*/*s*] nominal motions were considered referred to as "*slow*", "*normal*", "*fast*", and "*very fast*" nominal motions. The controller's cycle time was supposed to be *1* [*ms*].

The phase trajectories and the tracking errors with and without external perturbation (i.e. the dashpot) and with and without complementary tuning for "*slow*" motion are given in Fig. A.4.2. It is clear that in each case turning on the symplectic identification considerably improves the tracking accuracy and the phase trajectory, too. Due to the essentially exponential nature of the generators of the fine tuning in (6.2.21) and (6.2.24) small step length in fine tuning (10^{-8} [*dimensionless*]) was found to be reasonable. At slow motion no essential improvement by fine tuning was achieved.

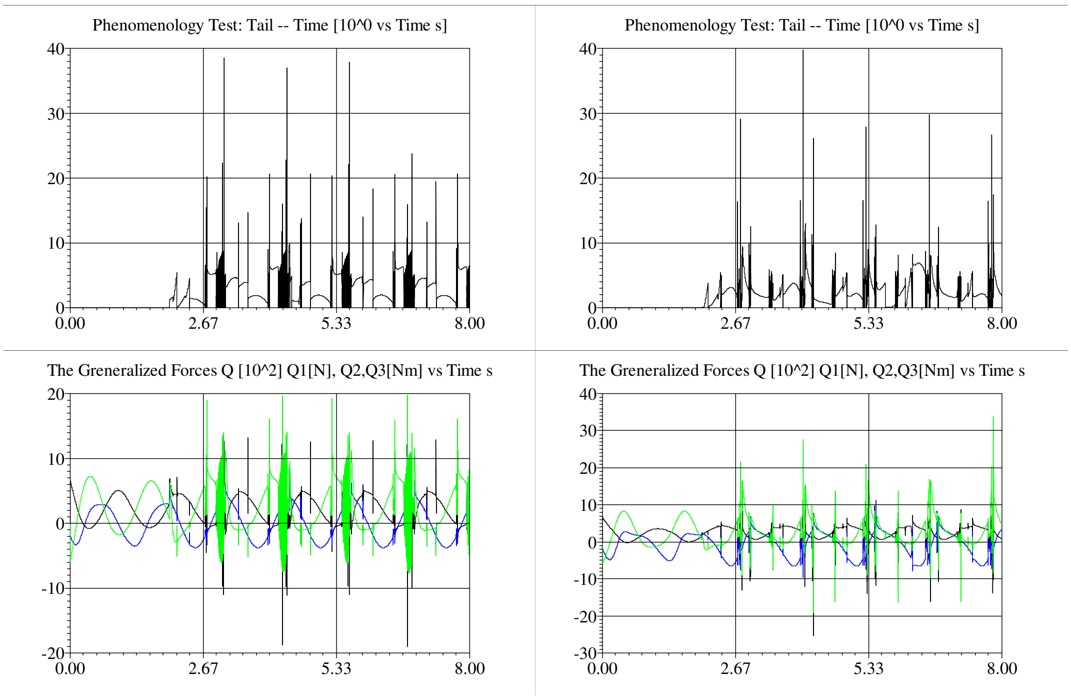


Figure A.4.3. Simulation results for "slow" motion Ω=5 [rad/s] without (LHS) and with (RHS) external perturbation using only the Symplectizing Algorithm: the norm of the "truncated generalized force components" (1^{st} row) and the generalized forces (2^{nd} row) [q_1:black, q_2:blue, q_3:green lines]

Since the main "*tool of system identification*" is the *Symplectizing Algorithm* in these simulations obtaining precisely and exceptionally "phenomenologically correct" block diagonal transformations was not guaranteed. On this reason the illegally nonzero components of the transformed generalized forces have been simply truncated from the resulting force after executing the multiplication by the symplectic matrix. In the sequel this "truncated" part (more precisely its norm according to Frobenius) is referred to as "tail" and is described in the charts called "*Phenomenology Test*".

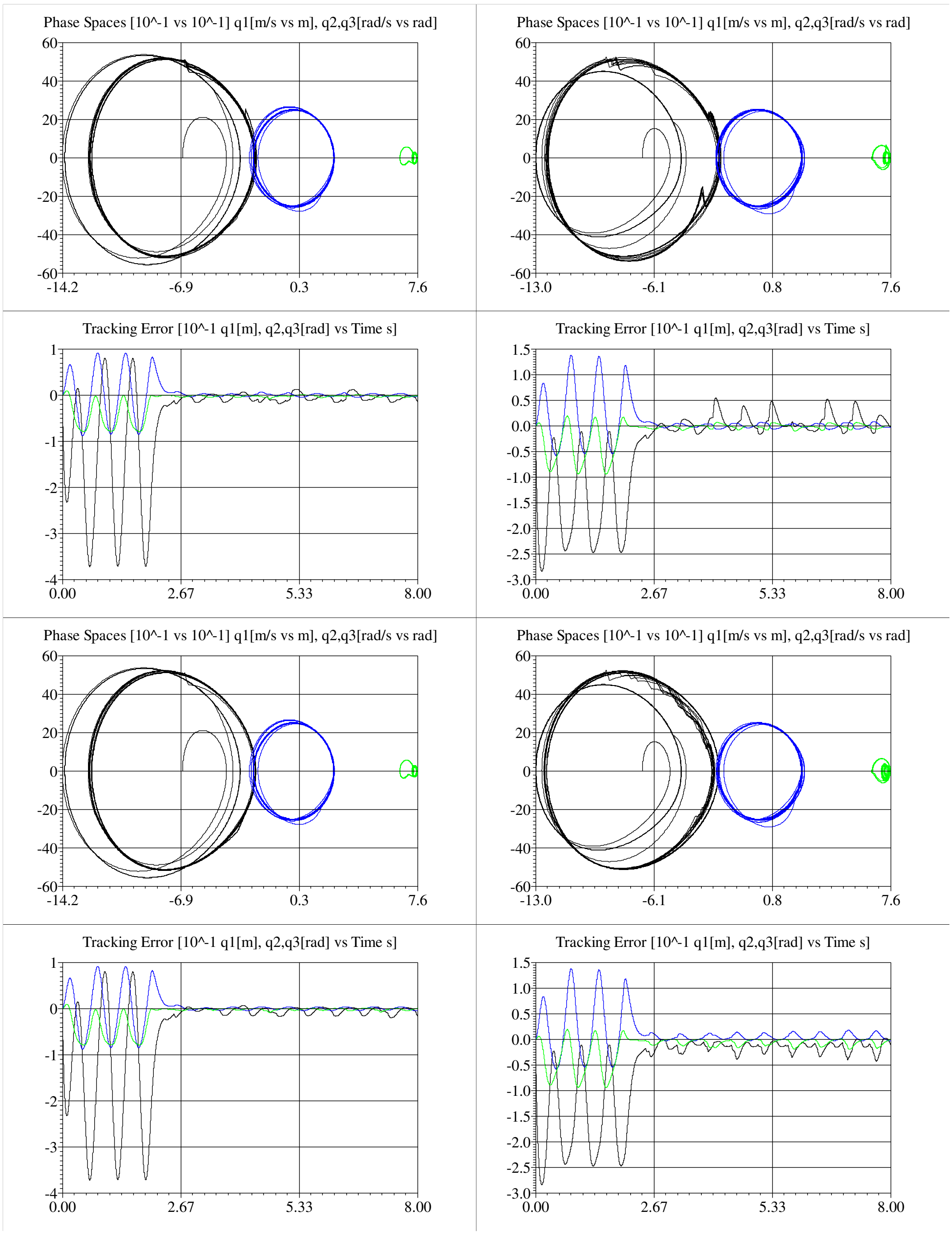


Figure A.4.4. Simulation results for "normal" motion Ω=10 [rad/s], without (LHS) and with (RHS) external perturbation without complementary tuning (the first two rows) and with complementary tuning of step length 10^{-6} [dimensionless] (3^{rd} and 4^{th} rows) [q_1:black, q_2:blue, q_3:green lines]

In Fig. A.4.3. the norm of the truncated components and the generalized forces are described for "*slow*" motion and the use of the "pure *Symplectizing Algorithm*". It can be seen that while the generalized forces are in the ≈500 or ≈1000 [*N*] or [*Nm*], the norm of the truncated components is small, about ≈5 ≈8 [*N*] or [*Nm*] only (with the exception of certain "extreme points" in which they achieve ≈40 [*N*] or [*Nm*] that is also small in comparison with the full force components). (On this reason was allowed in the fine tuning the use of non-block-diagonal generators, too.) The generally increased force component at the right hand side of the figures in Q_2 reveals the significance of the external perturbations. It is worth noting that the fine tuning did not essentially influence these charts in Fig. A.4.3., therefore, for saving room, these charts are not described here.

The appropriate counterpart of Fig. A.4.2. for "*normal*" speed of Ω=10 [*rad/s*] is given in Fig. A.4.4. It can well be seen that in this case the fine tuning definitely improved the tracking accuracy. Figure A.4.5. reveals some details on the variation of the generalized forces and the tuned parameters versus time. It has to be noted that for 10 consecutive steps only parameter φ, and following that, for the next 10 steps only parameter ψ was tuned by the *Simplex Algorithm*.

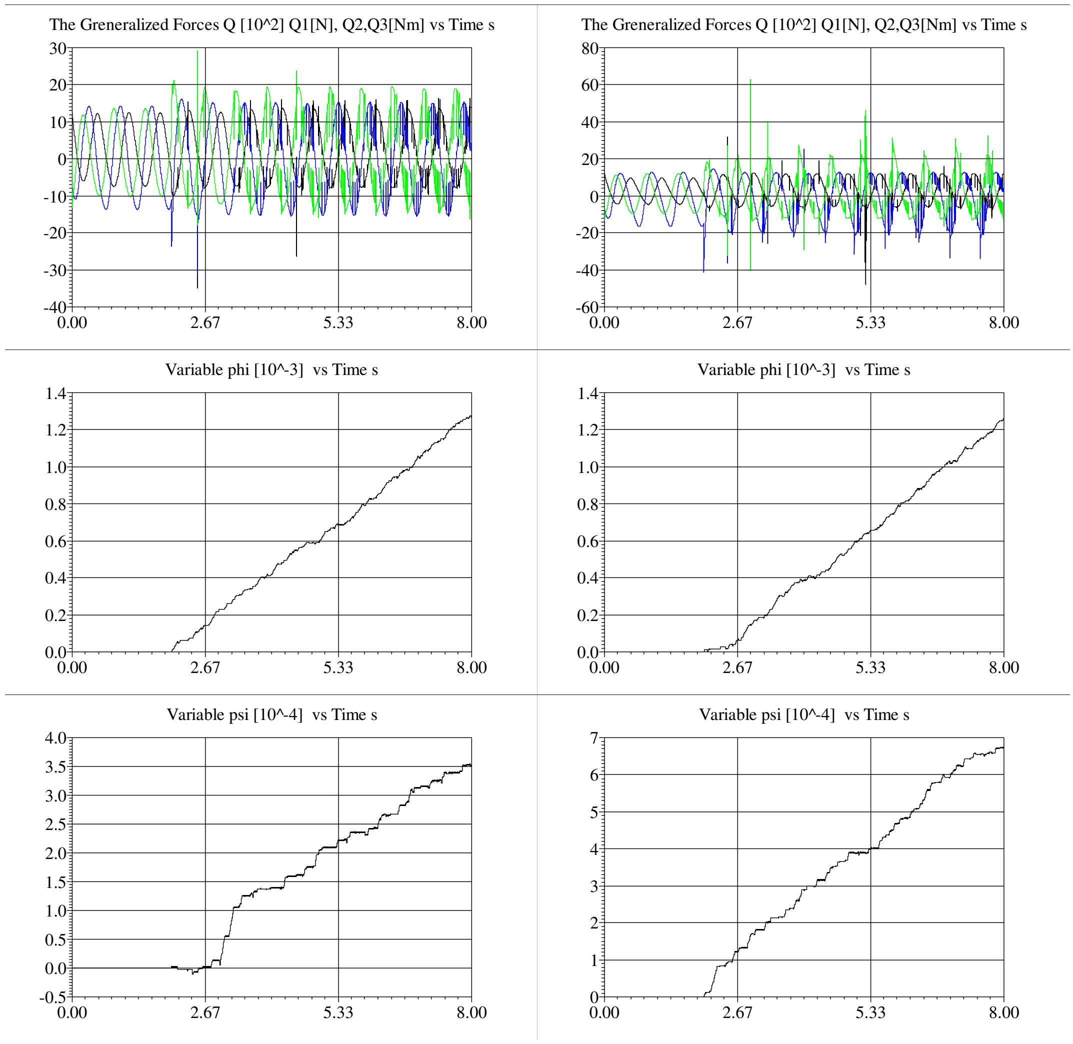


Figure A.4.5. Simulation results for "normal" motion Ω=10 [rad/s] without (LHS) and with (RHS) external perturbation (1st row) and the variation of the tuned parameters (2nd row and 3rd row) [q_1:black, q_2:blue, q_3:green lines]

Similar observations can be done in the case of "*fast*" motion of Ω=20 [*rad/s*] when the fine tuning rather "smoothes" the phase trajectories and has less influence on the tracking errors (Fig. A.4.6.).

In the case of the “*very fast*” motion (Fig. A.4.7.) it can well be observed that the fine tuning keeps the tracking errors “at bay”, i.e. makes their variation less chaotic than without fine tuning.

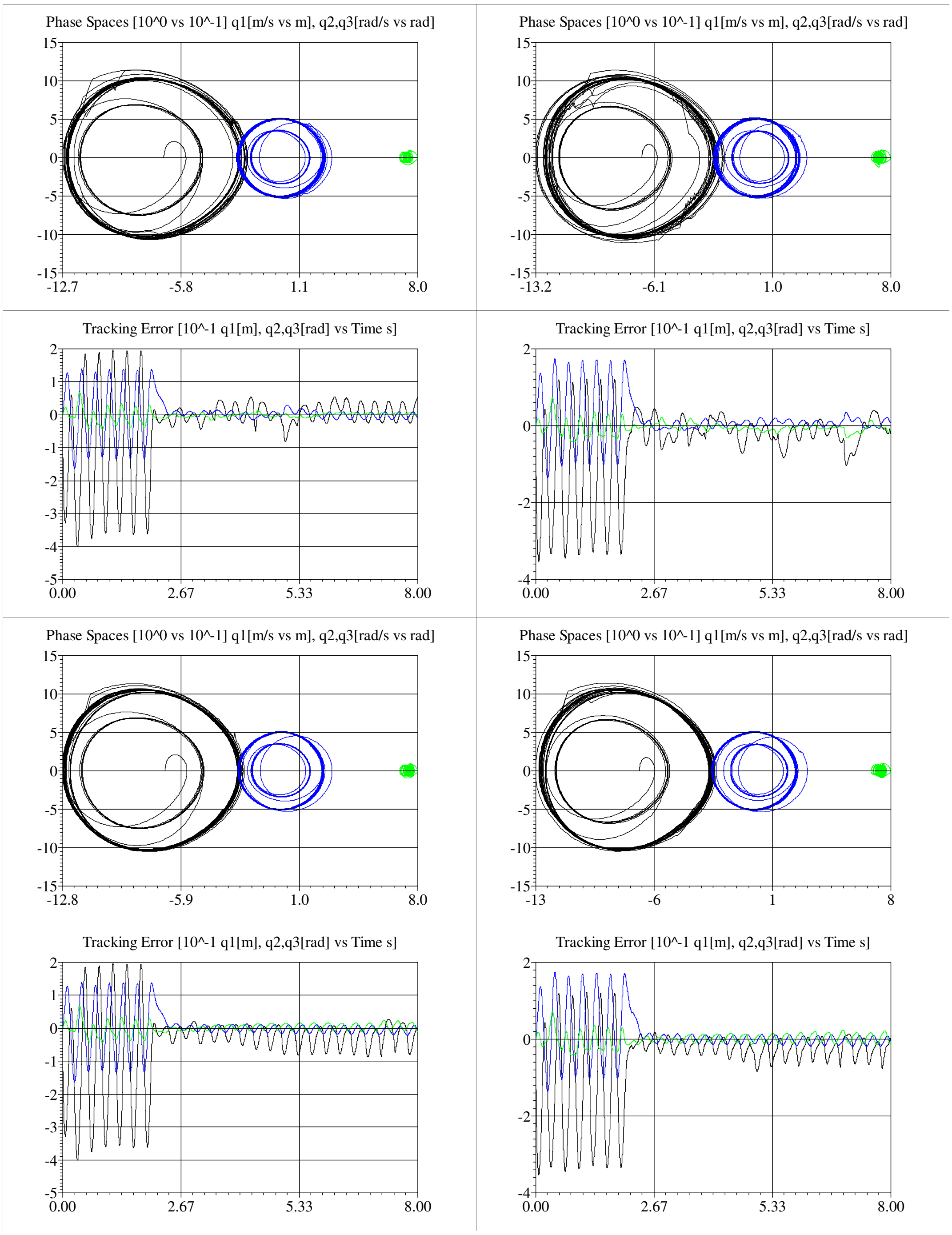


Figure A.4.6. Simulation results for “fast” motion Ω=20 [rad/s], without (LHS) and with (RHS) external perturbation without complementary tuning (the first two rows) and with complementary tuning of step length 5×10^{-6} [dimensionless] (3^{rd} and 4^{th} rows) [q_1:black, q_2:blue, q_3:green lines]

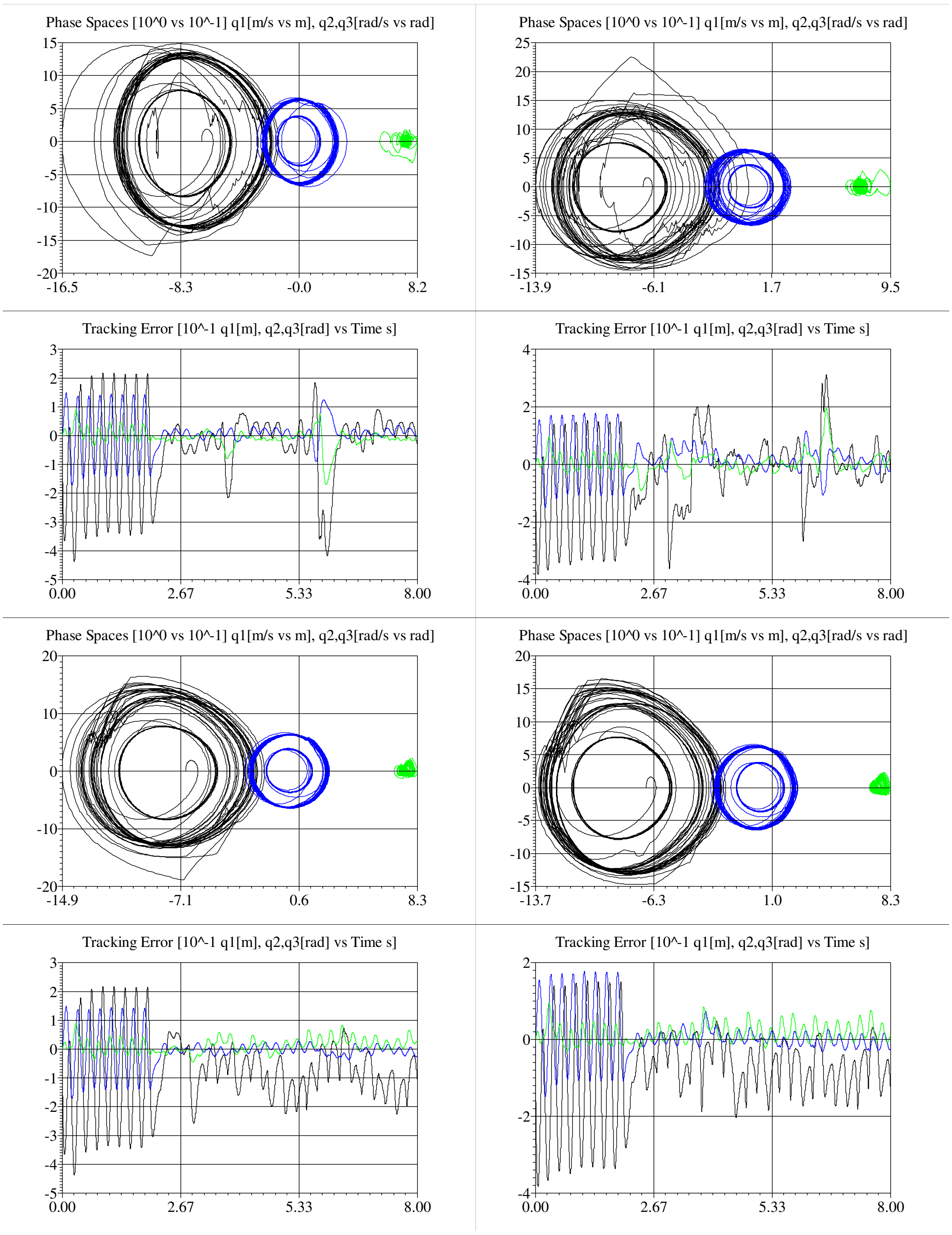


Figure A.4.7. Simulation results for "very fast" motion Ω=25 [rad/s], without (LHS) and with (RHS) external perturbation without complementary tuning (the first two rows) and with complementary tuning of step length 10^{-5} [dimensionless] (3^{rd} and 4^{th} rows) [q_1:black, q_2:blue, q_3:green lines]

A.5. Simulation Results for Section "7.3. Simulation Example for Potential Application of the Special Symplectic Matrices"

At first the paradigm used for the investigations is described mathematically, i.e. two coupled cart plus double pendulum systems.

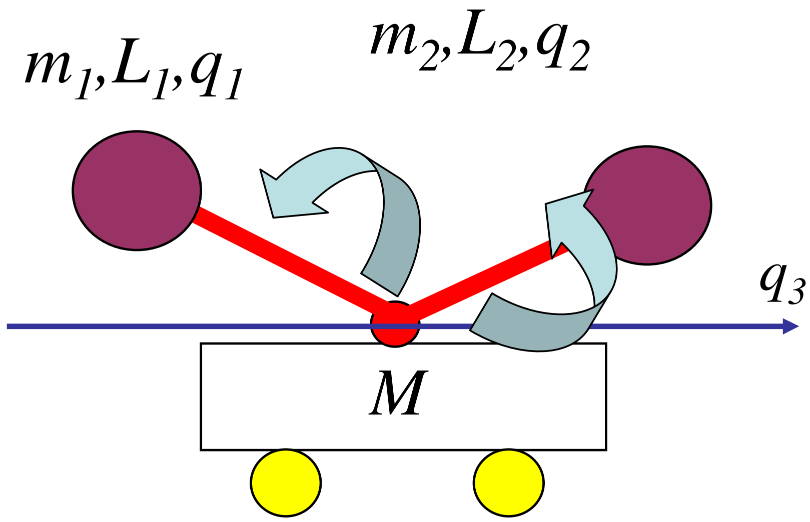


Figure A.5.1. The cart plus double pendulum system

Each cart under consideration consists of a body of considerable mass and wheels of negligible masses and momentums. The overall cart-masses are M^A=4 [*kg*], and M^B=4 [*kg*]. The pendulums are assembled on the cart by parallel shafts and arms of negligible masses and lengths L_1^A=2 and L_2^A=2 [*m*], L_1^B=1.5 and L_2^B=1.5 [*m*], respectively. At the end of each arm a ball of negligible size and considerable mass (m_1^A=10 and m_2^A=10, m_1^B=8 and m_2^B=7) [*kg*] are attached, respectively [Fig. A.5.1.]. The Euler-Lagrange equations of motion of a single cart are given as follows (A.5.4):

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} m_1 L_1^2 & 0 & -m_1 L_1 \sin q_1 \\ 0 & m_2 L_2^2 & -m_2 L_2 \sin q_2 \\ -m_1 L_1 \sin q_1 & -m_2 L_2 \sin q_2 & (M + m_1 + m_2) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \\ + \begin{bmatrix} m_1 g L_1 \cos q_1 \\ m_2 g L_2 \cos q_2 \\ -m_1 L_1 \cos q_1 \dot{q}_1^2 - m_2 L_2 \cos q_2 \dot{q}_2^2 \end{bmatrix} . \quad \text{(A.5.4)}$$

In the above formulae g denotes the gravitational acceleration [m/s^2], Q_1 and Q_2 [$N \times m$] denote the driving torque at shaft 1 and 2, respectively, and Q_3 [N] stands for the force moving the cart in the horizontal direction. The appropriate rotational angles are q_1 and q_2 [*rad*], and the linear degree of freedom belongs to q_3 [*m*]. *The 1st rotational and the linear degrees of freedom were the controlled and actuated ones, while the second rotary axis is without observation, control, and actuation that means that Q_2 takes the constant value zero.* Furthermore, two pieces of the above described subsystems are coupled along their linear direction of motion by the forces $Q_3^A = -Q_3^B$ given in [N] as

$$Q_3^A = k\left(q_3^B - q_3^A - L_0\right) + \frac{A}{\left(\varepsilon_{bump} + q_3^B - q_3^A - 1.5 \times L_0\right)^2} - \frac{A}{\left(\varepsilon_{bump} + q_3^B - q_3^A - 0.5 \times L_0\right)^2} \quad \text{(A.5.5)}$$

in which $k=10^4$ [N/m] describes a spring stiffness, and L_0=3 [*m*] belongs to the zero spring force length.

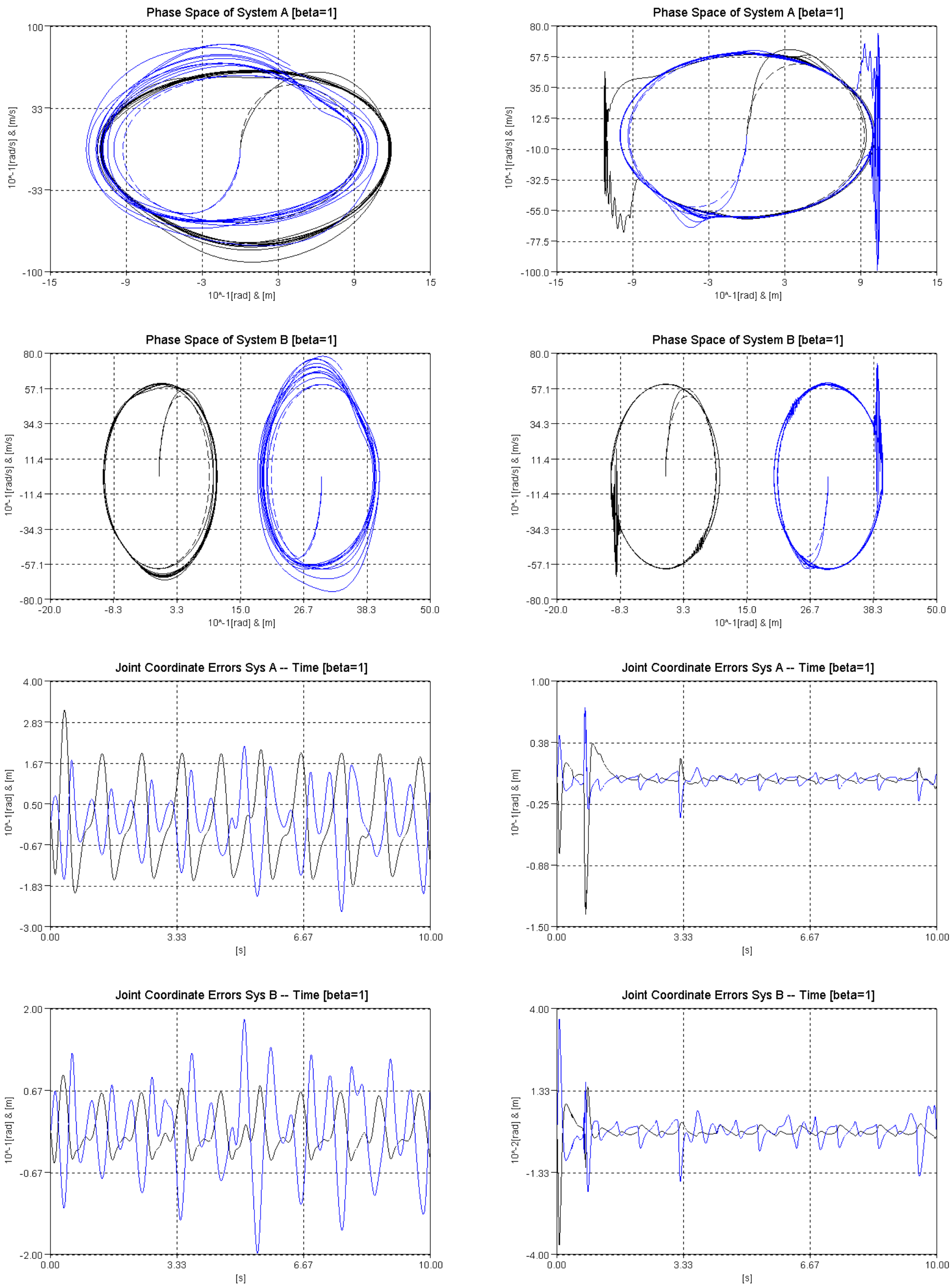


Figure A.5.2. Simulation results for the non-adaptive (LHS) and the "centralized adaptive" (RHS) control approaches (the notation "β=1" refers to the occurrence of integer order derivatives in the symplectic matrices)

To model the buffers two non-linear terms are applied that are very sharp near the $0.5\times L_0$ and $1.5\times L_0$ distances, while in the "internal points" they are very flat. They are described by two parameters, namely by the "strength" A=1000 [$N\times m^2$], and a small parameter $\varepsilon_{bump}=10^{-3}$ [m] determining the "nearness" of the singularity of these coupling forces. In the simulation the *rough initial system model* for both carts was $\mathbf{Q}=10\ddot{\mathbf{q}}+10[1,1,1]^T$ instead of (A.5.4). A PID-type kinematic trajectory tracking strategy was prescribed for the relaxation of the tracking error **h**=$\mathbf{q}^N$-**q** according to three oscillation-free (real) time-constants $\alpha_1=\alpha$, $\alpha_2=0.9\times\alpha$,$\alpha_3=0.8\times\alpha$ with $\alpha=20$ $[1/s]$:

$$\ddot{\mathbf{h}}^{Des} = -P\mathbf{h} - D\dot{\mathbf{h}} - I\int_0^t \mathbf{h}(\tau)d\tau, \quad P=\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_3\alpha_1, \quad D=\alpha_1+\alpha_2+\alpha_3, \quad I=\alpha_1\alpha_2\alpha_3 \cdot \tag{A.5.6}$$

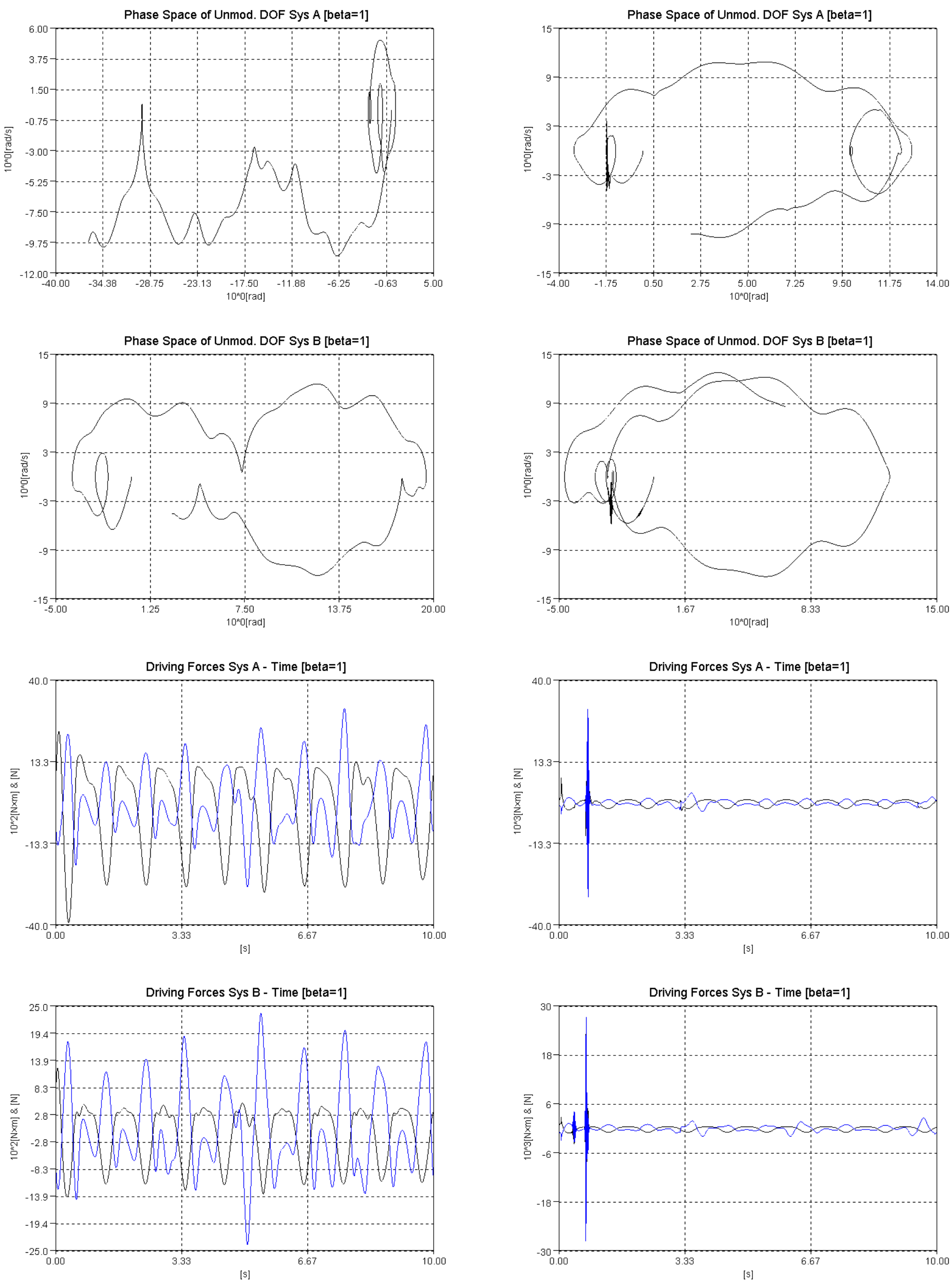


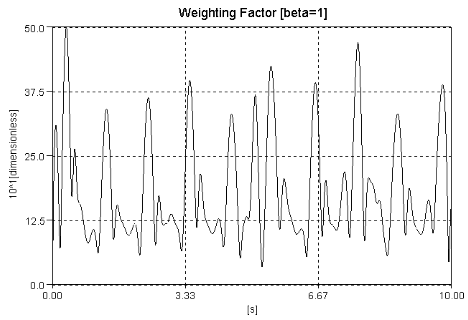
Figure A.5.3. Simulation results for the non-adaptive (LHS) and the "centralized adaptive" (RHS) control approaches [continued] (the notation "β=1" refers to the occurrence of integer order derivatives in the symplectic matrices)

In the control the "*Regulating Factor*" was calculated according to (7.2.19) with υ=0.5, ε_1=0.2, ε_2=10^{-5}. The finite element time-resolution for the control was δt=10^{-3} [s], the numerical integration happened according to the Euler formula with step length δt/10. For the "centralized" approach the special symplectic matrices of size 12×12 defined in (7.1.10) were applied with the "dummy parameter" d=80. The first column in the upper half of these matrices were defined as $\left[\tilde{\ddot{q}}_1^{DesA}, \tilde{\ddot{q}}_3^{DesA}, \tilde{\ddot{q}}_1^{DesB}, \tilde{\ddot{q}}_3^{DesB}, d, D\right]^T$, in which the symbol "tilde" denotes "weighted contributions" in each control cycle "*i*" as

$$w(i) = 0.92w(i-1) + (1-0.92)\sqrt{\ddot{\mathbf{q}}^{DesA^T}(i)\ddot{\mathbf{q}}^{DesA}(i) + \ddot{\mathbf{q}}^{DesB^T}(i)\ddot{\mathbf{q}}^{DesB}(i)}, \quad W(i) = 1 + w(i),. \tag{11.5.7}$$
$$\tilde{\ddot{\mathbf{q}}}^{DesA^T}(i) = \ddot{\mathbf{q}}^{DesA^T}(i)/W(i)$$

The function of this weighting is maintaining a proper relationship between b and the norm of the appropriate accelerations: the physically interpreted part of the column remains commensurate with b. (The former solutions that applied the accelerations without weighting with fixed dummy parameter b were found to be less precise.)

The simulation results obtained for the *non-adaptive* and the *"centralized adaptive"* approaches are given in Figs. A.5.2., A.5.3., A.5.4., and A.5.5. It is clear that both the trajectory and the phase trajectory tracking accuracy have been considerably improved by switching on the adaptive law. Following a sharp transient section the driving forces applied at both subsystems have been well stabilized. The same statement can be done in connection with the *"weighting factor"* in the case of the adaptive control. Figure A.5.5. also reveals that the variation of the *"regulating factor"* became *"canonical"*, and that the symplectic matrices applied by the control were really in the vicinity of the unit matrix.



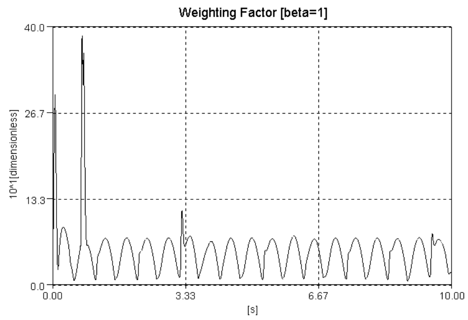


Figure A.5.4. Simulation results for the non-adaptive (LHS) and the "centralized adaptive" (RHS) control approaches [continued] (the notation "β=1" refers to the occurrence of integer order derivatives in the symplectic matrices); (these factors are not in use in the non-adaptive case)

The simulation results pertaining to the *"distributed approach"* are described in Figs. A.5.7.-A.5.11. It used two smaller symplectic matrices with the first columns in the upper half as $\left[\tilde{\ddot{q}}_1^{DesA}, \tilde{\ddot{q}}_3^{DesA}, d, D\right]^T$ and $\left[\tilde{\ddot{q}}_1^{DesB}, \tilde{\ddot{q}}_3^{DesB}, d, D\right]^T$, and also used two *"regulating factors"* and *"weighting factors"*, too. In this case slower motion was considered with α=10 [1/*s*].

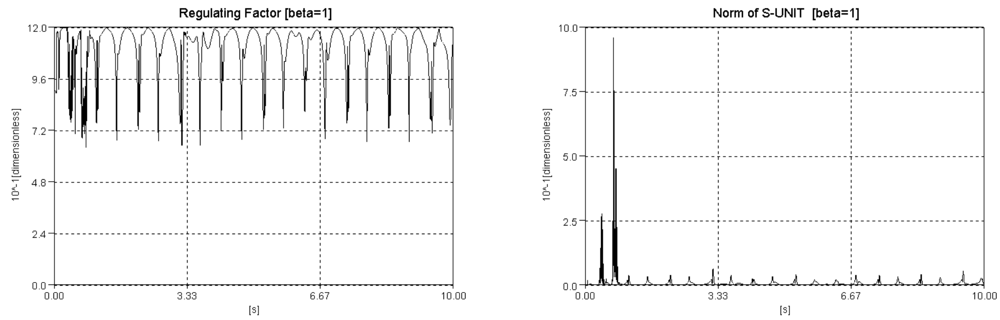


Figure A.5.5. Simulation results for the "centralized adaptive" control approach [continued] (the notation "β=1" refers to the occurrence of integer order derivatives in the symplectic matrices)

Switching on the adaptive law again considerably improved the precision of the trajectory- and phase-trajectory tracking. As in the case of the *"centralized approach"* the existence of a sharp initial transient section can be observed in which the appropriate symplectic matrices are not in the close vicinity of the unit matrix. In these regimes the use of the *"regulating factor"* plays important role in guaranteeing the convergence of the method. Following this transient phase stable control can be observed in which the transformation matrices remain in the close vicinity of the unit matrix, the weighting and regulating factors as well as the control forces and torques vary "regularly".

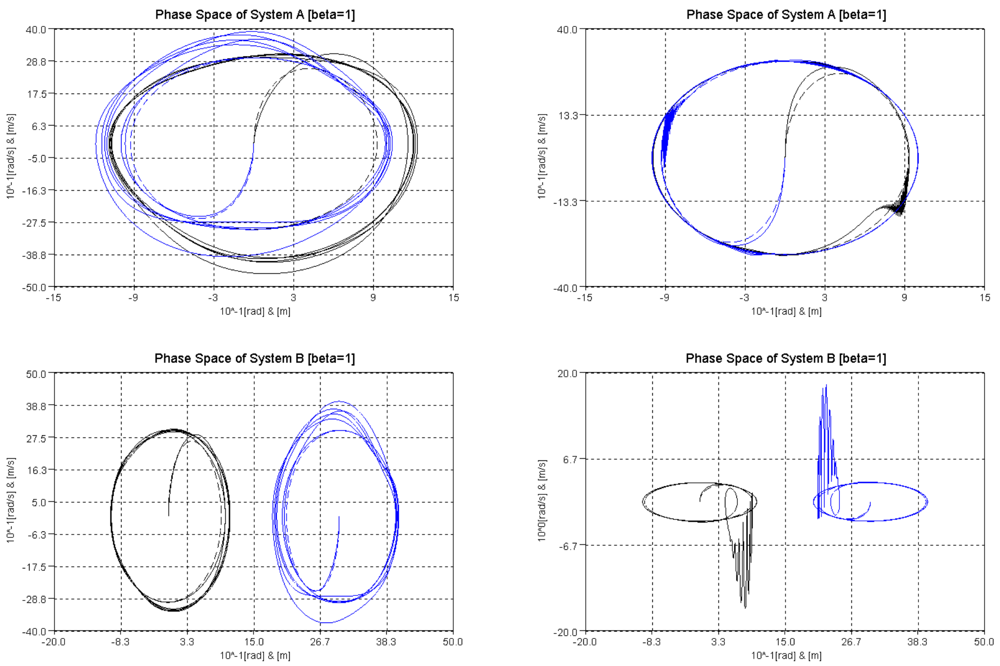


Figure A.5.6. Simulation results for the non-adaptive (LHS) and the "distributed adaptive" (RHS) control approaches (the notation "β=1" refers to the occurrence of integer order derivatives in the symplectic matrices)

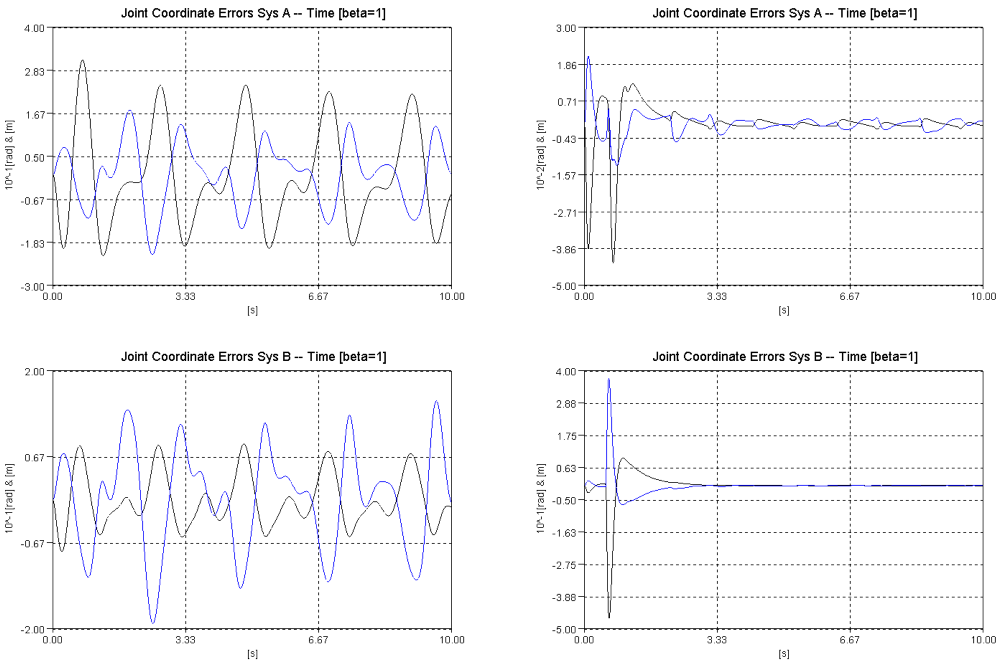


Figure A.5.7. (continued)Simulation results for the non-adaptive (LHS) and the “distributed adaptive” (RHS) control approaches (the notation “β=1” refers to the occurrence of integer order derivatives in the symplectic matrices)

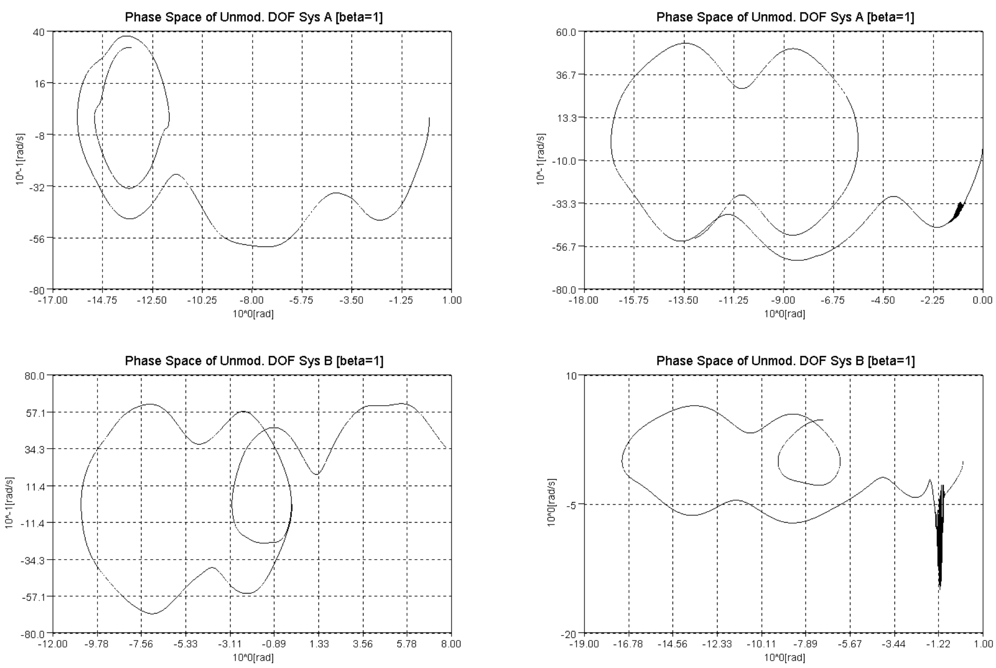


Figure A.5.8. Simulation results for the non-adaptive (LHS) and the “distributed adaptive” (RHS) control approaches [continued] (the notation “β=1” refers to the occurrence of integer order derivatives in the symplectic matrices)

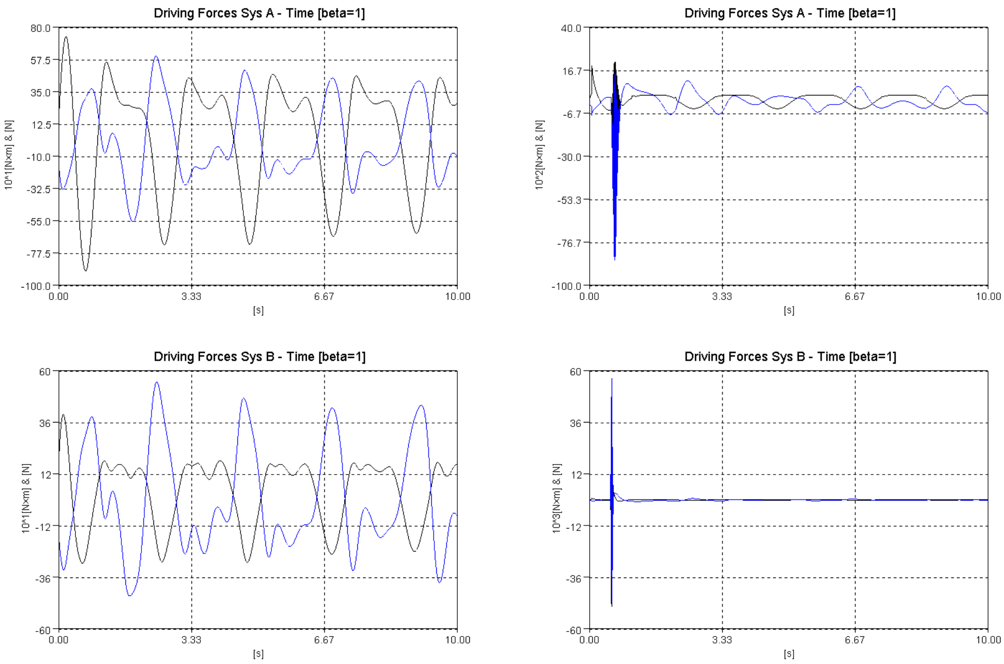


Figure A.5.9. (continued) Simulation results for the non-adaptive (LHS) and the "distributed adaptive" (RHS) control approaches [continued] (the notation "β=1" refers to the occurrence of integer order derivatives in the symplectic matrices)

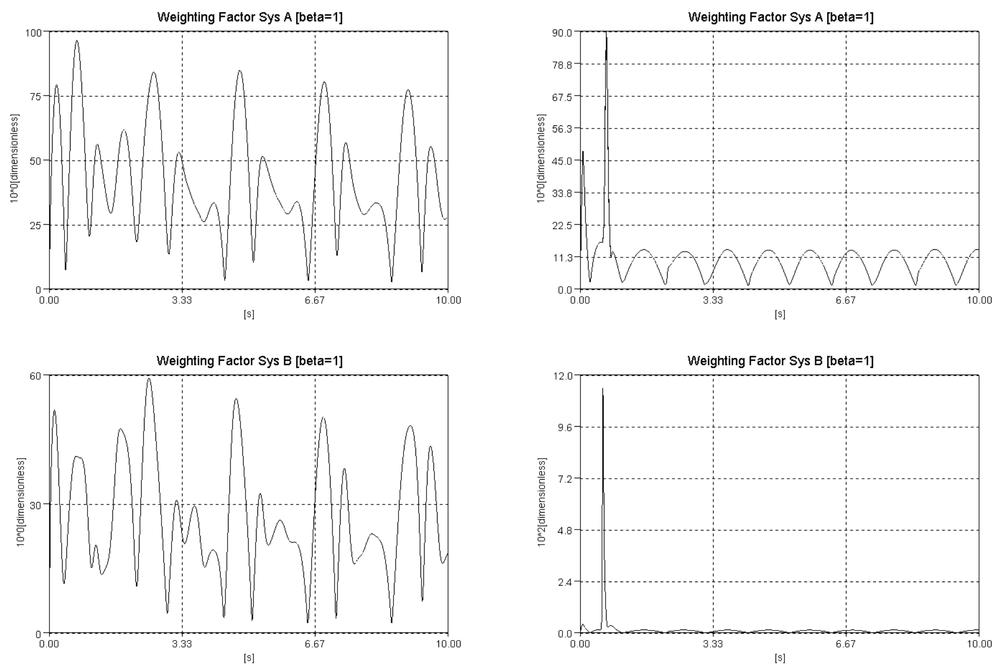


Figure A.5.10. Simulation results for the non-adaptive (LHS) and the "distributed adaptive" (RHS) control approaches [continued] (the notation "β=1" refers to the occurrence of integer order derivatives in the symplectic matrices); (these factors are not in use in the non-adaptive case)

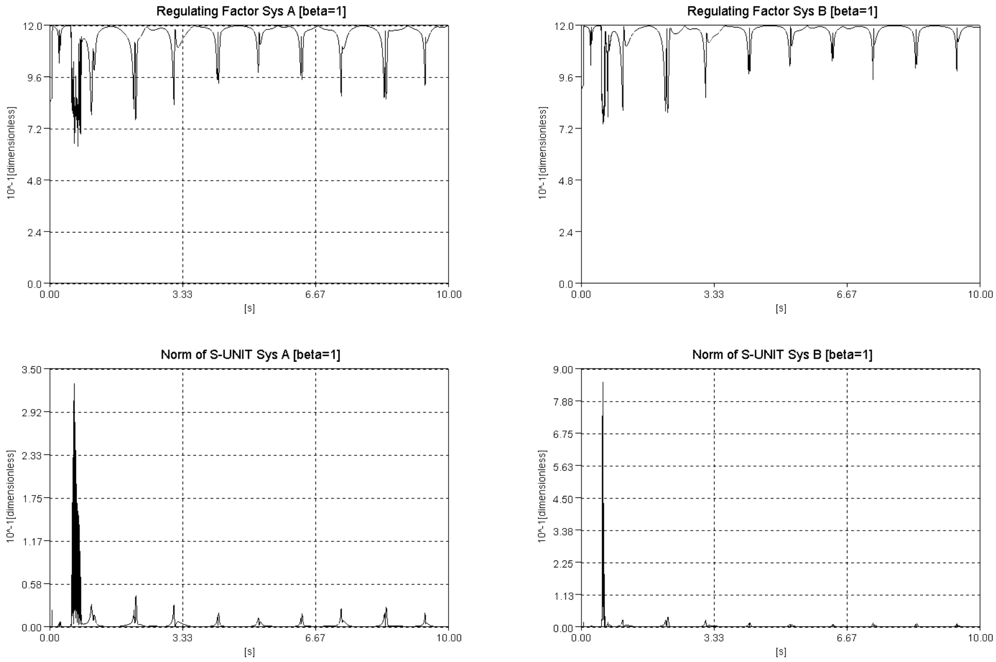


Figure A.5.11. Simulation results for the "distributed adaptive" control approach [continued] (the notation "β=1" refers to the occurrence of integer order derivatives in the symplectic matrices)

A.6. Illustrative Figures for Section "8.1. Fixed Point Transformations with a Few Parameters for "Increasing" and "Decreasing" SISO Systems"

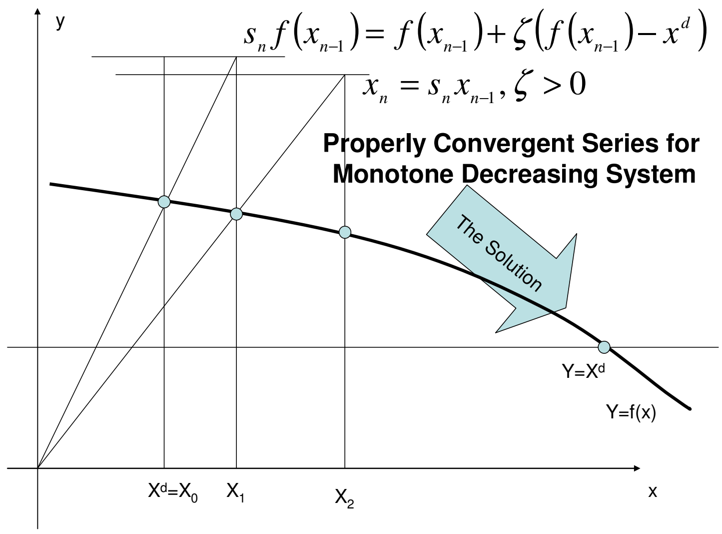


Figure A.6.1. Properly and improperly convergent sequences for "decreasing system"

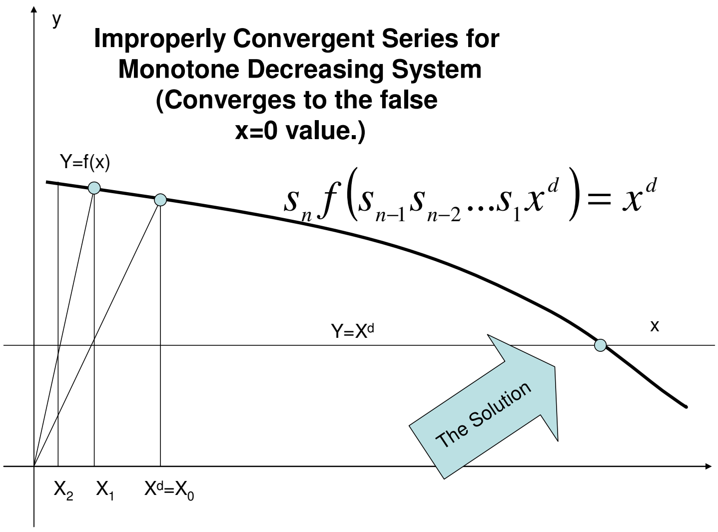


Figure A.6.2. (continmued) Properly and improperly convergent sequences for "decreasing system"

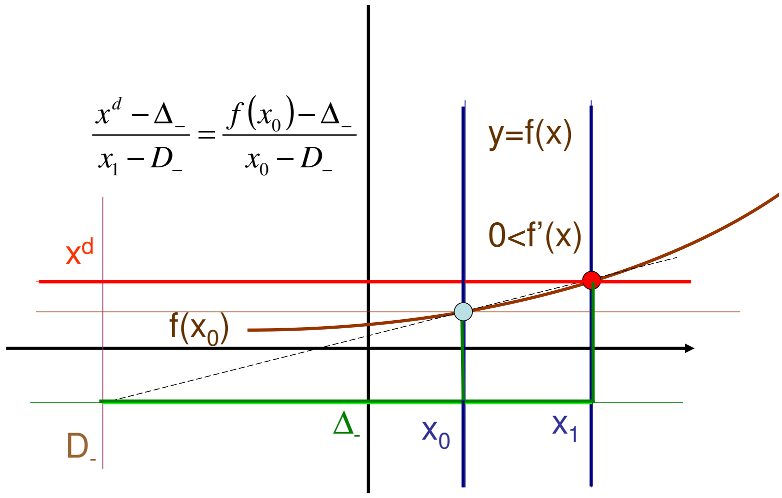


Figure A.6.3. Two parametric transformations for sequences for "increasing system"

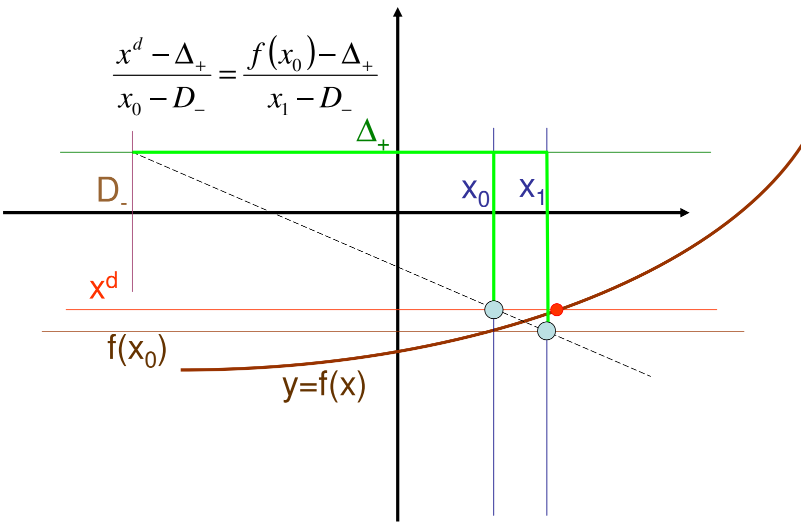


Figure A.6.4. (continued) Two parametric transformations for sequences for "increasing system"

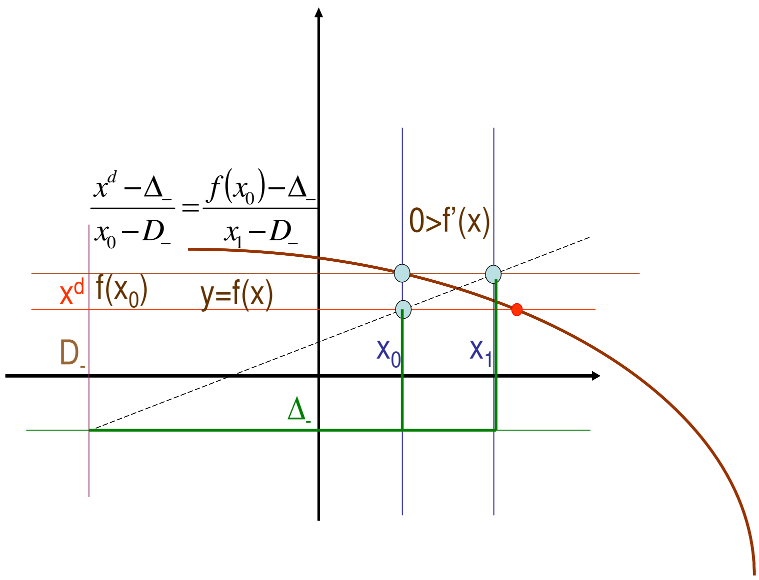


Figure A.6.5. Two parametric transformations for sequences for "decreasing system"

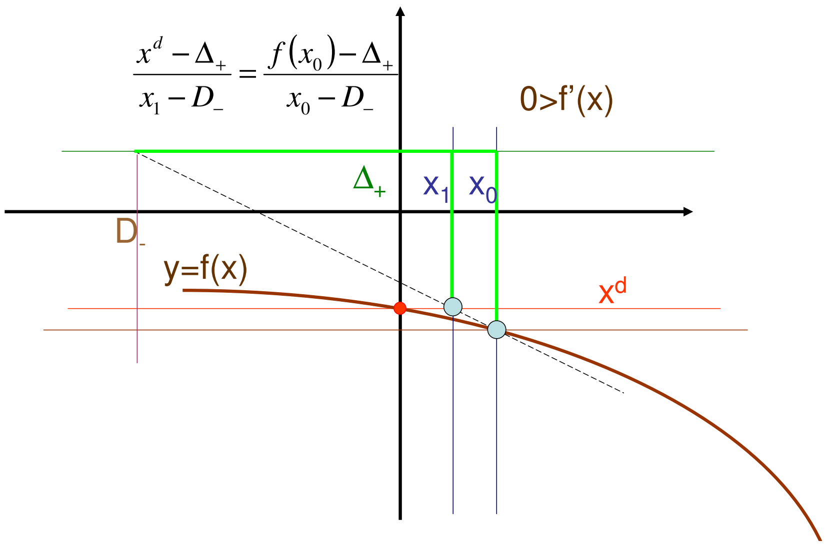


Figure A.6.6. (continued) Two parametric transformations for sequences for "decreasing system"

A.6.1. Further Details Belonging to Subsection "8.1.1. A Higher Order Application Example for Fixed Point Transformations of a Few Parameters"

In the control of this system a ball or cylinder can roll on the surface of a beam the tilting angle of which is driven by some actuator. The motion of the ball essentially is determined by the tilting angle and the force of gravitation. This means that even if we are in the possession of a very strong actuator, the acceleration of the ball along the beam is limited by the above two factors. Since the directly controllable quantity is the torque determining the 2^{nd} time-derivative of the angle tilting the beam, this system acts as a 4^{th} order one in the sense that the 4th time-derivative of the ball's position along the beam is determined by the tilting torque. It has the following parameters: the momentum of the beam Θ_{Beam}=2 (kg×m^2), the mass of the ball m_{Ball}=2 (kg), the radius of the ball r=0.05 (m), and the gravitational acceleration is g=9.81 (m/s^2). Via introducing the quantities $A=\Theta_{Beam}$, and $B=\Theta_{Ball}/r^2+m_{Ball}$, the following equations of motion are obtained as given in Fig. A.6.1.1. in which variable φ (rad) describes the rotation of the beam counter-clockwisely with respect to the horizontal position, and x (m) denotes the distance of the ball from the center of the beam where it is supported. Variable Q (N×m) describes the torque at the axis rotating the beam. This quantity consists of two different components: the torque directly exerted by the drive and the contribution by the friction forces acting at the surface of the axle. In the present investigations this latter component is unknown by the controller, only the consequences of its existence in the trajectory tracking can be observed. It is evident that only the 4^{th} time-derivative of x can be related to the 2^{nd} time-derivative of the tilting angle of the beam that is in direct relationship with the rotating torque taking part in tilting this angle. For making the model more realistic in the simulations it was assumed that the axle of the beam has considerable *dynamic friction* approximated by the LuGre model as follows.

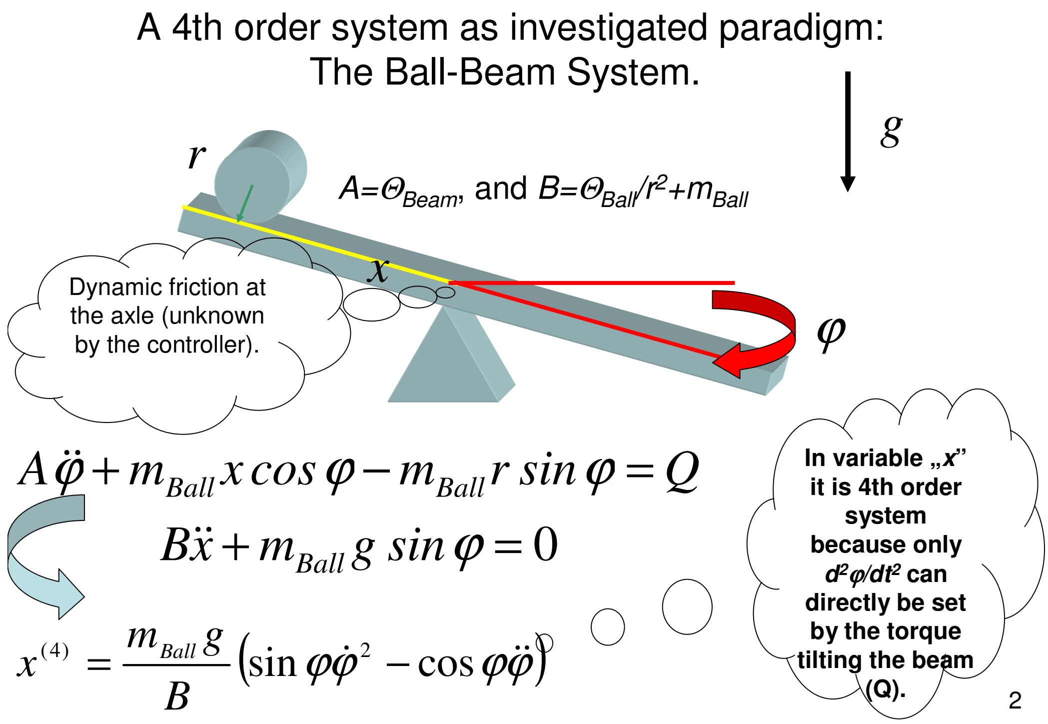


Figure A.6.1.1. The Ball-Beam System

Instead of the dubious "*velocity limit*" normally applied in simulations with static friction models (i.e. the limit value at which the relative motion of the contacted surfaces is practically zero) to describe the "*stick-slip phenomenon*" an "*internal degree of freedom*", z is introduced with the appropriate equations of motion as

$$\frac{dz}{dt}=v-\frac{\sigma_0|v|z}{F_C+F_S\exp\left(-|v|/v_s\right)},\qquad F_{fric}=\sigma_0 z+\sigma_1\frac{dz}{dt}+F_v v \tag{A.6.1.1}$$

in which σ_0=5000 (Nm/rad), σ_1=1000 (Nms/rad), Fv=100 (Nms/rad), F_C=10 (Nm), F_S=20 (Nm), v_s=0.05 (rad/s) are the friction model parameters, and v (rad/s) describes the rotational speed characteristic to the surfaces in contact at the axle. The kinematic tracking requirements were set by (A.6.1.2) with the order of differentiation m=4 and λ=10 s^{-1}

$$\left(\frac{d}{dt}+\lambda\right)^m\left[x^{Nom}-x\right]=0,\quad \lambda>0 \tag{A.6.1.2}$$

from which the desired 4th time-derivative $x^{(4)Des}$ can be computed. Since normally the beam must be in an almost horizontal position for stabilizing purposes it was expedient to limit its allowable rotational angle, and angular speed. For this purpose potential-like limiting terms were introduced in the calculation of the desired $\ddot{\varphi}^{Des}$ as follows:

$$\ddot{\varphi}^{Des} = \frac{-\tilde{B}x^{(4)^{Des}}}{\tilde{m}_{Ball}\, g \cos\varphi} + \tan\varphi\,\dot{\varphi}^2 - \Gamma_{\varphi}\frac{\partial}{\partial\varphi}\cosh\left(\frac{\beta_{Pot}\varphi}{1.5}\right) - \\ -\Gamma_{\dot{\varphi}}\frac{\partial}{\partial\dot{\varphi}}\cosh\left(\frac{\beta_{Pot}\dot{\varphi}}{3}\right) \qquad \text{(A.6.1.3)}$$

with $\Gamma_{\varphi} = \Gamma_{\dot{\varphi}} = 3$ and $\beta_{Pot} = 5$ that worked well setting limitation to the angle at 1.5 (rad) and angular velocity of 3 (rad/s). The parameters $\tilde{B}$ and $\tilde{m}_{Ball}$ mean the estimated model values. *The effects of the rough dynamic model data and the friction that were unknown by the controller could manifest themselves in the low accuracy of the non-adaptive control. The significance of adaptation can be measured by observing the improved tracking accuracy of the adaptive controller.* In the control approach applied for *negative realized* $x^{(4)}$ the $g(x|x^d, D_-, \Delta_+)$ function, for *positive realized values* the $h(x|x^d, D_-, \Delta_-)$ functions were used.

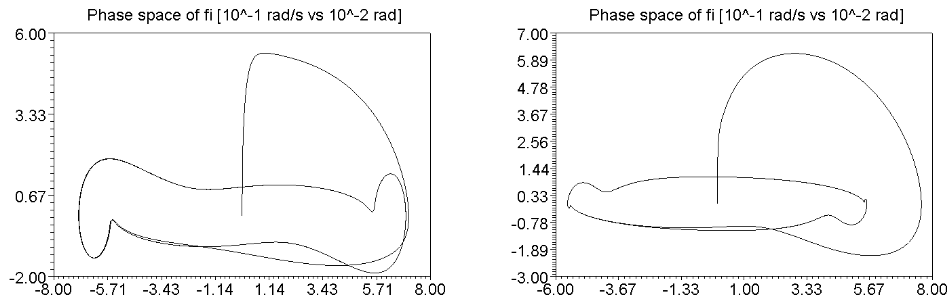


Figure A.6.1.2. The phase space of the tilting angle $\dot{\varphi}$ vs φ *: non-adaptive (LHS) and adaptive (RHS) solutions*

According to Fig. A.6.1.2. adaptivity considerably “regularizes” the motion of the beam.

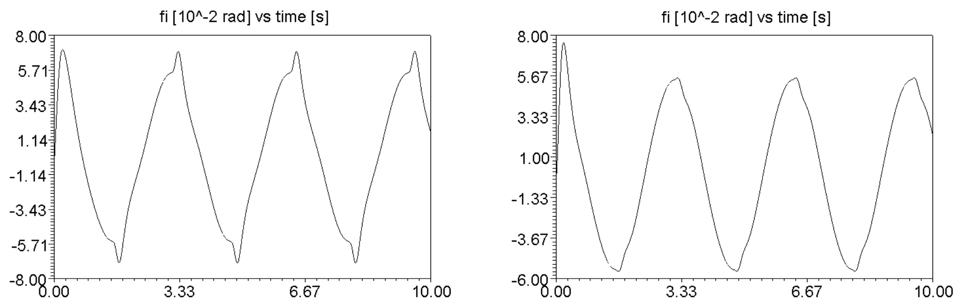


Figure A.6.1.3. The tilting angle φ *vs time: non-adaptive (LHS) and adaptive (RHS) solutions*

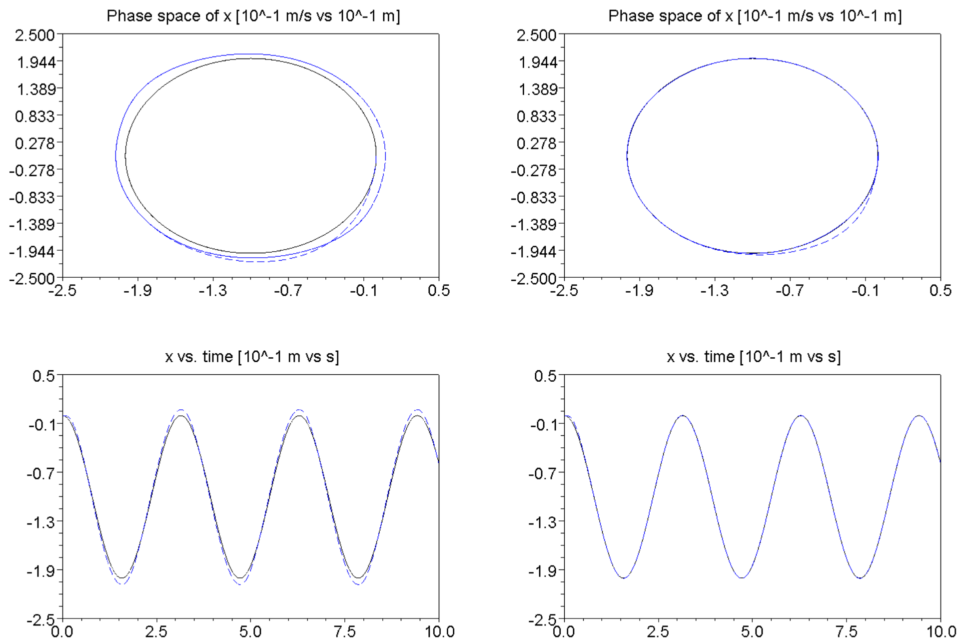


Figure A.6.1.4. The phase space and time dependence of the displacement of the cylinder along the beam: non-adaptive (LHS) and adaptive (RHS) solutions

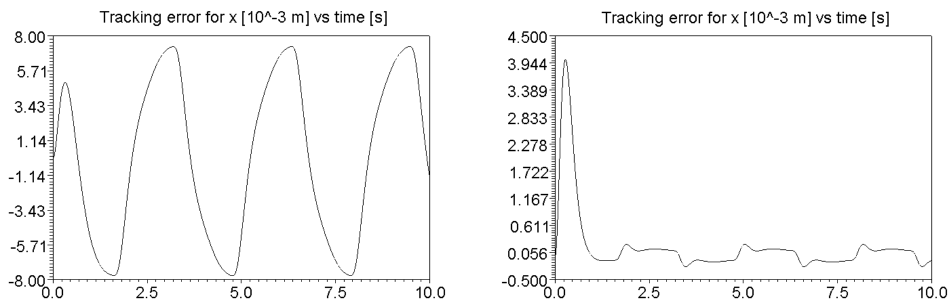


Figure A.6.1.5. The tracking error vs. time: non-adaptive (LHS) and adaptive (RHS) solutions

Similar can be stated for the displacement of the ball (cylinder) along the beam (Fig. A.6.1.4.). Adaptivity drastically improved the tracking accuracy (Fig. A.6.1.5.). In both the adaptive and the non-adaptive cases the effort of the feedback exerted for the compensation of the friction torque can be traced. In these figures the adaptive and the non-adaptive solutions show differences only nuances (Fig. A.6.1.6.), however, due to the integration according to time these nuances have significant effect on the tracking accuracy. Figure A.6.1.7. well reveals the essence of the adaptive method that realizes precise 4^{th} time-derivative of the coordinate x. To study the operation of the adaptive control further charts were made (Fig. A.6.1.8.) that displays when the functions g or h were used for realizing adaptivity.

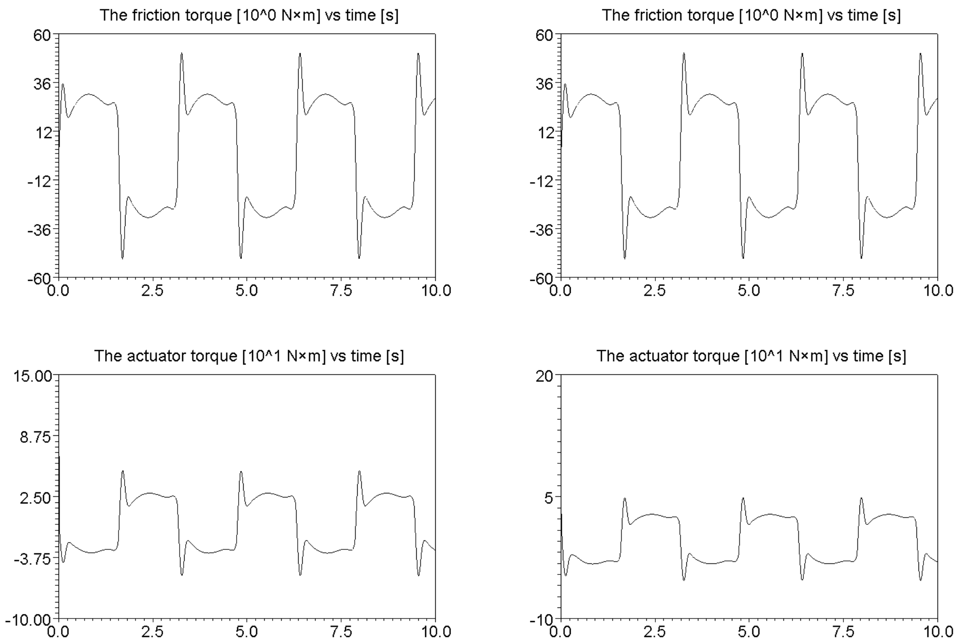


Figure A.6.1.6. Compensation of the friction torques: non-adaptive (LHS) and adaptive (RHS) solutions

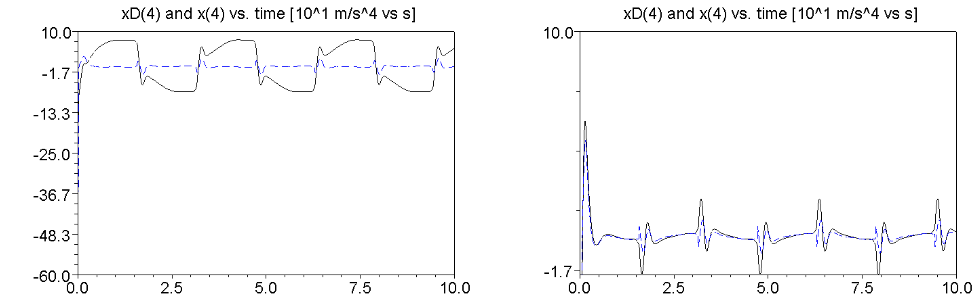


Figure A.6.1.7. Desired and realized 4th time-derivative of "x": non-adaptive (LHS) and adaptive (RHS) solutions

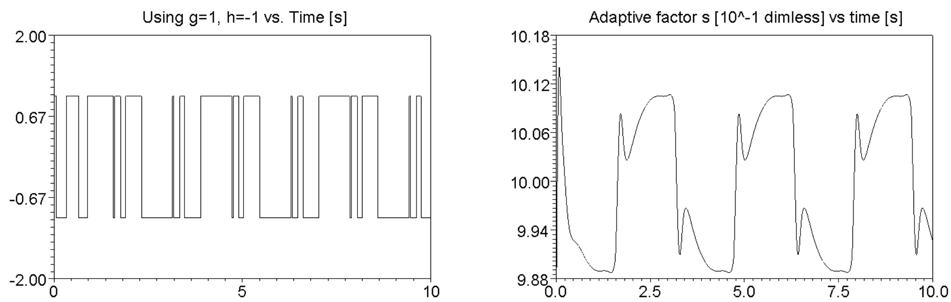


Figure A.6.1.8. The use of functions "g" and "h" versus time, and the "cumulative deformation factor" vs. time in the case of the adaptive control

According to (8.1.1) and (8.1.2) a *"cumulative deformation factor"* can be defined for functions h and g as follows: $s(t_k):=\prod_{i=0}^{k}\left(x^d(t_i)-\Delta_+\right)/\left(f(x(t_i))-\Delta_+\right)$ and $s(t_k):=\prod_{i=0}^{k}\left(f(x(t_i))-\Delta_-\right)/\left(x^d(t_i)-\Delta_-\right)$ that somehow are characteristic to the control (Fig. A.6.1.8.).

The consequences of the strongly nonlinear nature of the friction model applied can well be traced in Fig. A.6.1.8.

A.6.2. Further Details Belonging to Subsection "8.2.2. Application Example a): Precise Control of an AGV Equipped with Omnidirectional Wheels"

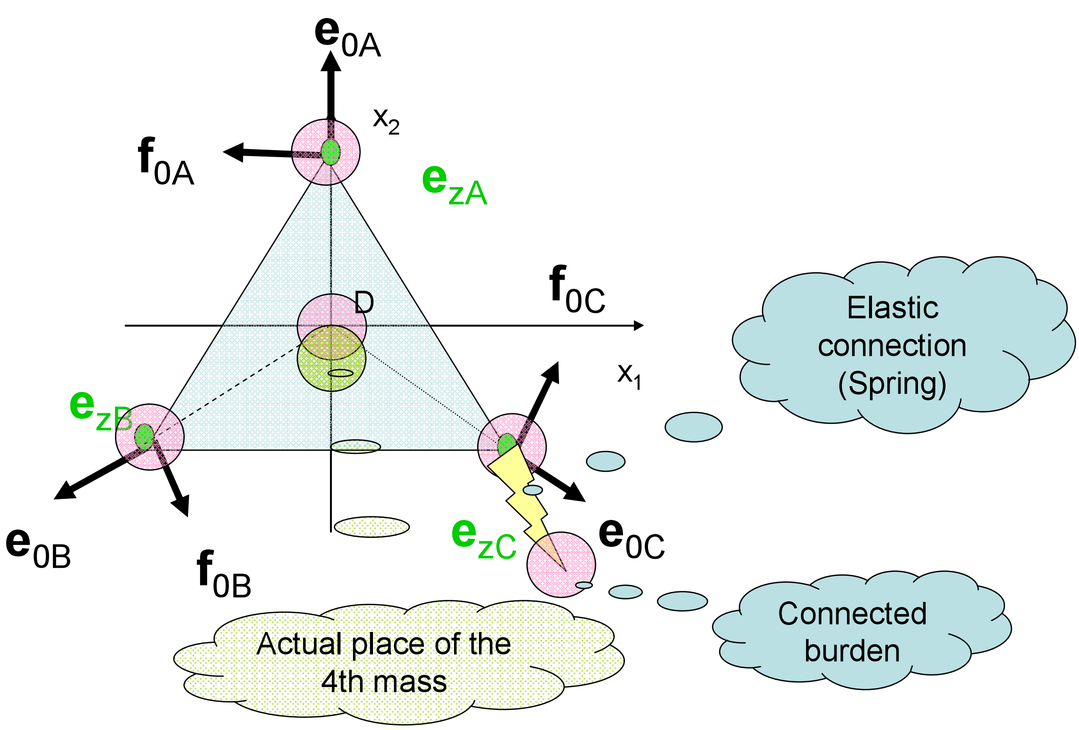


Figure A.6.2.1. The sketch of the triangular cart considered

The cart was supposed to have canonical triangular shape of side length L=2 (m). The orientation of the active forces were supposed to be described by the orthogonal unit vectors $\mathbf{e}_A$, $\mathbf{f}_A$, $\mathbf{e}_B$, $\mathbf{f}_B$, $\mathbf{e}_C$, and $\mathbf{f}_C$ at the wheels A, B, and C in the (x,y) plane in which the direction of the appropriate $\mathbf{e}$ vectors was identical to that of the straight line connecting the geometric center of the triangle to the appropriate vertices. These vectors were assumed to rigidly rotate around the axis z with angle q_3. Each wheel had the common constant vector component in the z direction $\mathbf{e}_z$ along which the contact constraint forces originating from the ground acted. It was assumed that the plane of motion was exactly horizontal, so the vector of the gravitational acceleration in the reality had a component only in the z direction. At the vertices of the triangle three heavy wheels and drive systems were located, each of them had the mass M=30 (kg). It was assumed that further $2M$ mass was located over the geometric center of the triangle at the height of h_D=0.5 (m). The vehicle was assumed to move on the (x,y) plane with prescribed nominal location of the projection of its hypothetical mass center point $\mathbf{S}^{(m)N}$ (m) and nominal rotational pose q_3^{N} (rad) around the axis z. According to Fig. A.6.2.1. the "*not modeled degree of freedom*" in this system was a mass-point connected to wheel C by an elastic connection, a spring. In this case it had the mass of $0.45M$ attached to the spring of stiffness k=1000 (N/m) and zero force length L_0=1 (m). It was assumed to move along the (x,y) plane with a viscous friction coefficient μ=5 (Ns/m).

Utilizing the well known fact that the acceleration of the mass center point of a rigid body multiplied by its full mass is equal to the sum of the external forces acting on that system, and that the time-derivative of momentum of the system computed with respect to the actual mass center point is equal to the momentum of the external forces (torque) with respect to this point, the required active driving force components F_{AeA}, F_{AfA}, F_{BeB}, F_{BfB}, and F_{CeC}, F_{CfC}, as well as the hypothetical vertical constraint force components F_{Az}, F_{Bz}, and F_{Cz} can be calculated. (According to Fig. A.6.2.1., if the small wheels do not have drives in the horizontal $\mathbf{e}$ directions no any forces can be exerted.) The rough dynamic model available for the controller is given as follows:

$$\begin{bmatrix} 5M^{(m)}\left(\ddot{\mathbf{S}}^{(m)}+\mathbf{g}\right) \\ \dot{\mathbf{P}}^{(m)} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^{(m)} & \mathbf{B}^{(m)} & \mathbf{C}^{(m)} \\ \mathbf{D}^{(m)} & \mathbf{E}^{(m)} & \mathbf{F}^{(m)} \end{bmatrix} \begin{bmatrix} \mathbf{F}_e \\ \mathbf{F}_f \\ \mathbf{F}_z \end{bmatrix} \tag{A.6.2.1}$$

in which $\mathbf{A}^{(m)}=[\mathbf{e}_A,\mathbf{e}_B,\mathbf{e}_C]$, $\mathbf{B}^{(m)}=[\mathbf{f}_A,\mathbf{f}_B,\mathbf{f}_C]$, $\mathbf{C}^{(m)}=\left[\mathbf{0},\mathbf{0},[1,1,1]^T\right]$, $\mathbf{D}^{(m)}=\left[\mathbf{e}_A\times\mathbf{x}_A^{(m)},\mathbf{e}_B\times\mathbf{x}_B^{(m)},\mathbf{e}_C\times\mathbf{x}_C^{(m)}\right]$, $\mathbf{E}^{(m)}=\left[\mathbf{f}_A\times\mathbf{x}_A^{(m)},\mathbf{f}_B\times\mathbf{x}_B^{(m)},\mathbf{f}_C\times\mathbf{x}_C^{(m)}\right]$, and $\mathbf{F}^{(m)}=\left[\mathbf{e}_z\times\mathbf{x}_A^{(m)},\mathbf{e}_z\times\mathbf{x}_B^{(m)},\mathbf{e}_z\times\mathbf{x}_C^{(m)}\right]$, $\mathbf{g}=[0,0,-g]^T$, $\mathbf{F}_e=[F_{AeA},F_{BeB},F_{CeC}]^T$, $\mathbf{F}_f=[F_{AfA},F_{BfB},F_{CfC}]^T$, and $\mathbf{F}_z=[F_{Az},F_{Bz},F_{Cz}]^T$, and the $\mathbf{x}_A^{(m)},\mathbf{x}_B^{(m)},\mathbf{x}_C^{(m)}$ vectors connect the assumed mass center point with the appropriate vertices at the wheels A, B, and C, while $\mathbf{P}^{(m)}$ denotes the model value of the momentum of the rotating system. The "actual system's" equation of motion that can be used for calculating the "realized accelerations" and "realized contact forces in the z direction" is similar to (A.6.2.1), but it contains the acceleration of the actual mass center point $\mathbf{S}$ and the actual momentum calculated with respect to that ($\mathbf{P}$). Fortunately $\mathbf{S}^{(m)}$ and $\mathbf{S}$ have simple geometric connection. Beside that it contains the $\mathbf{x}_A$, $\mathbf{x}_B$, and $\mathbf{x}_C$ vectors that connect the actual mass center point with the appropriate vertices at the wheels A, B, and C. Furthermore, the equation has to be rearranged since in it in the "input side" we have $\mathbf{F}_e$ and $\mathbf{F}_f$, and the unknown quantities are $\mathbf{F}_z$, $\ddot{\mathbf{S}}$ and $\ddot{q}_3$. By expressing $\dot{\mathbf{P}}$ with $q_3,\dot{q}_3,\ddot{q}_3$ it is obtained that

$$\begin{bmatrix} 5M\ddot{\mathbf{S}} \\ \dot{\mathbf{P}} \end{bmatrix} = \begin{bmatrix} -5M\mathbf{g} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{D} & \mathbf{E} \end{bmatrix} \begin{bmatrix} \mathbf{F}_e \\ \mathbf{F}_f \end{bmatrix} + \begin{bmatrix} \mathbf{C} \\ \mathbf{F} \end{bmatrix} [\mathbf{F}_z] \tag{A.6.2.2}$$

in which $\dot{\mathbf{P}}$ contains certain elements of the actual inertia matrix $\mathbf{\Theta}$, and the array $\mathbf{H}=[\mathbf{e}_z\times\mathbf{x}_A,\mathbf{e}_z\times\mathbf{x}_B,\ \mathbf{e}_z\times\mathbf{x}_C,\ -\Theta^{(3)};\ 1,1,1,0]$. From the fact that certain kinematic data can be exactly known it concludes that $\mathbf{A}=\mathbf{A}^{(m)}$, $\mathbf{B}=\mathbf{B}^{(m)}$, and $\mathbf{C}=\mathbf{C}^{(m)}$. In the calculations it was taken into account that the full momentum of the gravitational forces with respect to the actual mass center point is zero, and that no acceleration component may exist in the z direction (supposing that the vehicle does not turn over). So the appropriate component of the gravitational forces must be compensated by the contact forces in the direction z. In the solution of the "actual system's equations" it can be utilized that they are decoupled to some extent: $\dot{\mathbf{S}}$ has only x and y components, as well as the arrays **A** and **B**, while the array **C** does not have 1st and 2nd components. On this reason the two nontrivial nonzero components of $\ddot{\mathbf{S}}$ can be determined independently of the $\mathbf{F}_z$ values, while its zero 3rd component yields some restriction for the sum of the components of $\mathbf{F}_z$. This can be associated with the three equations pertaining to $\dot{\mathbf{P}}$, therefore we obtain 4 equations for 4 unknown quantities in (A.6.2.3):

$$\begin{bmatrix} -\dot{q}_3^2\Theta_{23} \\ \dot{q}_3^2\Theta_{13} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\mathbf{F}_{ABCef} \\ 5Mg \end{bmatrix} = \mathbf{H} \begin{bmatrix} F_{Az}^{\mathrm{Real}} \\ F_{Bz}^{\mathrm{Real}} \\ F_{Cz}^{\mathrm{Real}} \\ \ddot{q}_3^{\mathrm{Real}} \end{bmatrix}. \tag{A.6.2.3}$$

Figure A.6.2.2. The sketch of the omnidirectional wheel

It is worth noting that according to Fig. A.6.2.2. the omnidirectional wheels normally are driven to rotate around the axle of the "big wheel", and normally can freely roll in the direction of this axle due to the "small wheels". In Fig. A.6.2.1. the axles of the big wheels were denoted by the vectors $\mathbf{e}_A$, $\mathbf{e}_B$, and $\mathbf{e}_C$, therefore in (A.6.2.1) $\mathbf{F}_e=\mathbf{0}$, that means that the arrays $\mathbf{A}^{(m)}$ and $\mathbf{D}^{(m)}$ do not play role in this equation. Horizontal driving forces can be exerted only in the actual directions of the vectors $\mathbf{f}_A$, $\mathbf{f}_B$, and $\mathbf{f}_C$. Equations (A.6.2.1) and (A.6.2.3) do not contain the model of the connected subsystem the existence of which can be taken into account by calculating the contact forces (and their momentum) that appear due to the dynamic coupling of these subsystems. (The equations of motion of the mass point were solved separately.)

In the simulations the trajectory tracking strategy was prescribed on purely kinematical basis as a PID-type control to obtain the "desired accelerations" as

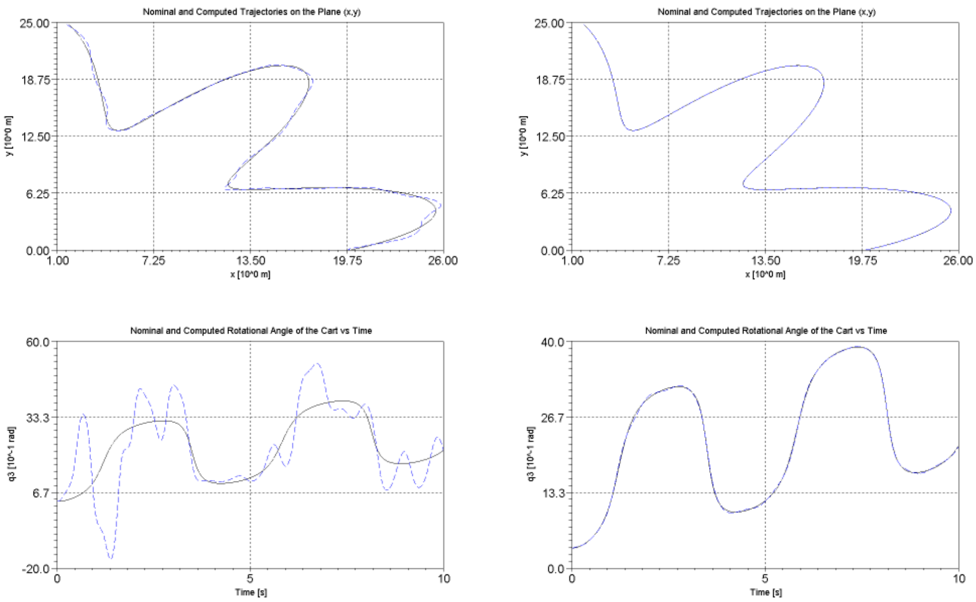


Figure A.6.2.3. The location of the "hypothetical mass center point" on the (x,y) plane (upper row, in [m] units), and the rotational orientation of the cart (lower row in 10^{-1} [rad] units)vs. time (in [s] units) for the non-adaptive control (LHS) and the adaptive one (RHS) [Nominal trajectory: black solid line, simulated trajectory: blue dashed line]

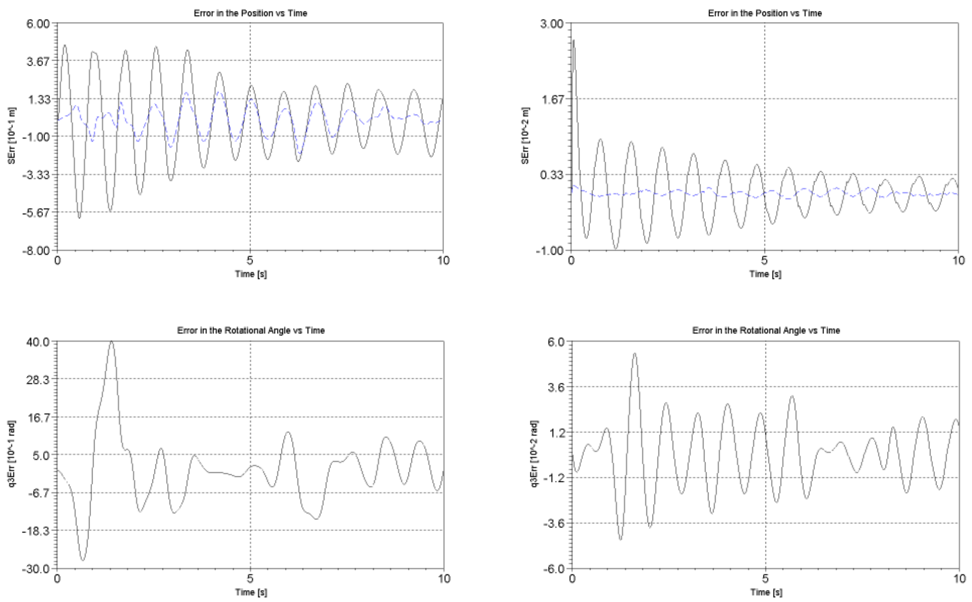


Figure A.6.2.4. The trajectory tracking error vs. time (upper row): for the non-adaptive control in 10^{-1} [m] units [LHS], and for the adaptive control in 10^{-2} [m] units [RHS] (black solid line: for "x", blue dashed line: for "y"), and the orientation tracking error vs. time (lower row) for the non-adaptive control in 10^{-1} [rad] units [LHS], and for the adaptive control in 10^{-2} [rad] units [RHS], time is given in [s] units

$$\ddot{\xi}^{Des}(t)=\ddot{\xi}^{N}(t)+P_{\alpha}\left(\xi^{N}(t)-\xi(t)\right)+D_{\alpha}\left(\dot{\xi}^{N}(t)-\dot{\xi}(t)\right)+I_{\alpha}\int_{0}^{t}\left(\xi^{N}(\tau)-\xi(\tau)\right)d\tau \quad \text{(A.6.2.4)}$$

in which P_α, D_α, and I_α are appropriate positive constants [actually P_α=50 (s^{-2}) D_α=10 (s^{-1}) and I_α=5 (s^{-3})]. The superscript N refers to the nominal accelerations determined by the trajectory to be traced. Simulation results are presented in the sequel. The adaptive parameters were: K_{ctrl}= -4000, B_{ctrl}=1, A_{ctrl}=1×10^{-4}.

In Figs. A.6.2.3. and A.6.2.4. the *trajectory* and orientation *tracking* is displayed for the non-adaptive and the adaptive controls. They reveal that the proposed adaptivity well compensates the simultaneously occurring modeling errors and the dynamic interaction with the unmodeled sub-system.

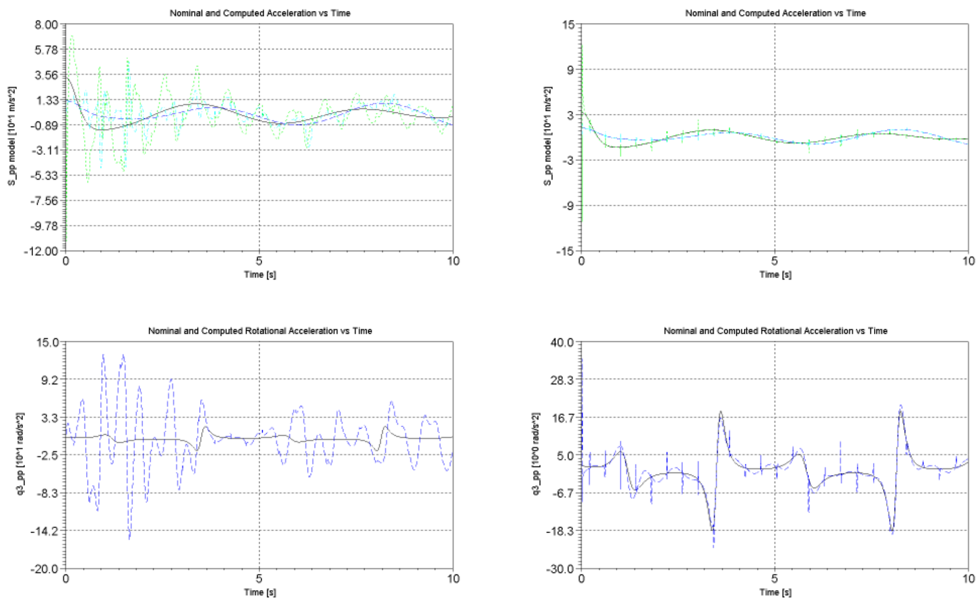


Figure A.6.2.5. The linear acceleration (upper row in 10 [m/s^2] units, black solid: for desired "x", blue dashed for desired "y", green dense dashes for simulated "x", light blue dash-dot for "y"), and the rotational acceleration (lower row in [rad/s^2] units, black solid: desired values, blue dashed: simulated values) vs time (in [s]) for the non-adaptive control [LHS], and the adaptive one [RHS]

In Fig. A.6.2.5. the linear and rotational accelerations are described. It well reveals that the adaptive law far better approximates the "*desired*" acceleration values than the non-adaptive one, that is the essence of the control idea is well realized. By the help of Fig. A.6.2.6. it can be seen that the unmodeled dynamics of the coupled sub-system completely "destroys" the "canonical" form of the velocity pattern that is well saved by the adaptive version.

Figure A.6.2.7. indicates that the "high frequency" of the force pattern mainly originates from the "chaotic" motion of the directly uncontrolled, unmodeled coupled subsystem.

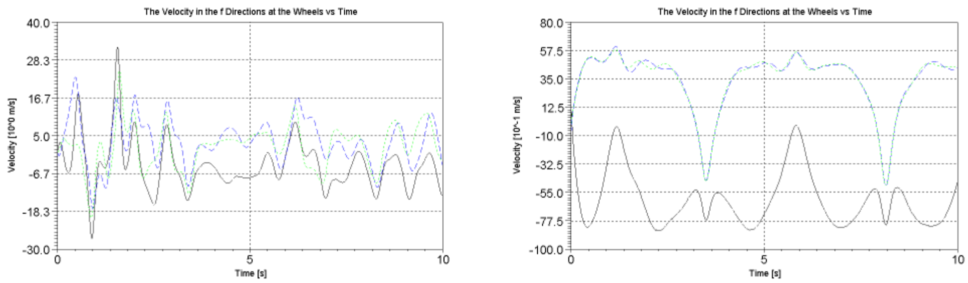


Figure A.6.2.6. The velocities at the wheels (A: black solid, B: blue dashed, C: green dense dashes) vs. time (in [s] units) for the non-adaptive controller (LHS in [m/s] units), and the adaptive one (RHS in 10^{-1} [m/s] units)

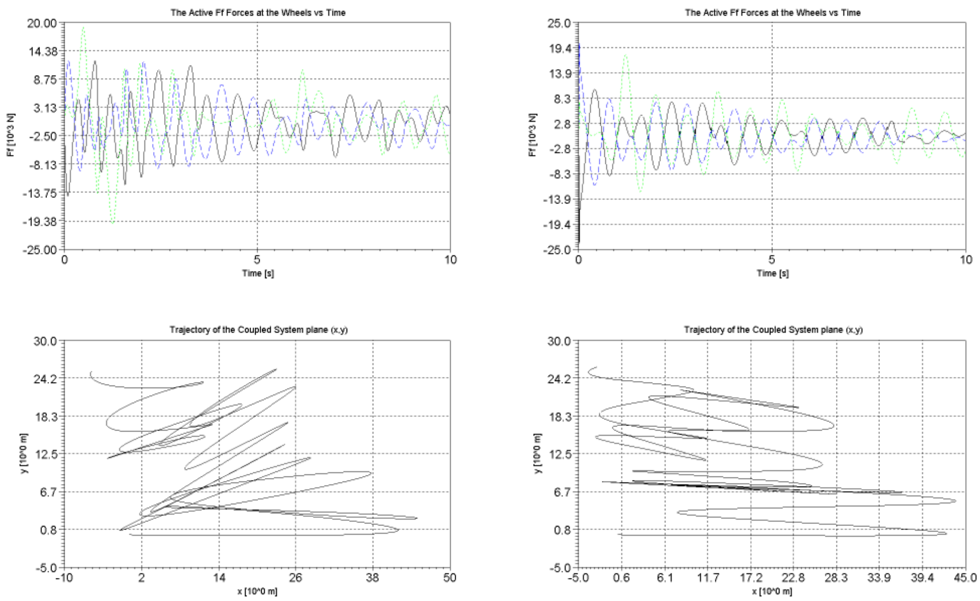


Figure A.6.2.7. The active driving force components in the direction of the appropriate **f** *vectors at the wheels (A: black solid, B: blue dashed, C: green dense dashes in 10^3 [N] units) vs. time (in [s] units) for the non-adaptive controller (LHS in [m/s] units), and the adaptive one (RHS in 10^{-1} [m/s] units) (upper row); The location of the coupled burden in the (x,y) plane in [m] units for the non-adaptive controller [LHS] and the adaptive one (lower row)*

A.6.3. Further Details Belonging to Subsection "8.2.3. Application Example b): Precise Control of the Cart-Beam-Hamper System"

The adaptive control parameters (naturally only in the adaptive case) took the following constant values: K_{ctrl}= -32000, B_{ctrl}=1, A_{ctrl}=2×10^{-6}. The simulation results are presented in Fig. A.6.3.1. revealing how efficient this simple approach is in comparison with the more sophisticated but more restricted methods (Figs. A.1.2.-A.2.6.). For demonstrating the robustness of the adaptive method two of the most significant control parameters K_{ctrl} and A_{ctrl} were tuned in real-time using the SCILAB's real-time package and the figure handling properties. The acceleration error was plotted as a function of the actual parameter values (Fig. A.6.3.2.). Other details were published in [C116].

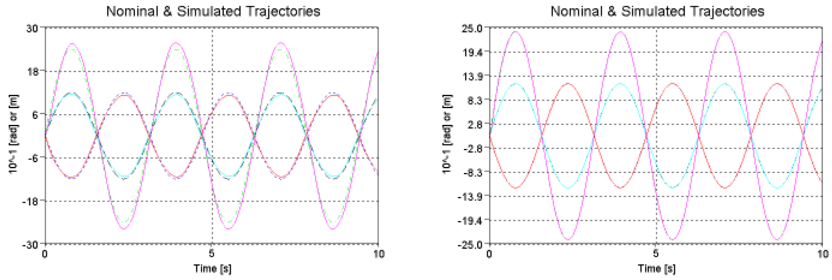


Figure A.6.3.1. Comparison of the non-adaptive (LHS) and the adaptive (RHS) controllers' operations for the cart-beam-hamper system with friction and external disturbance forces for $\boldsymbol{\Lambda}=10\times\mathbf{I}$ *[1/s],* $K_{ctrl}=-32000$, $B_{ctrl}=1$, $A_{ctrl}=2\times10^{-6}$.

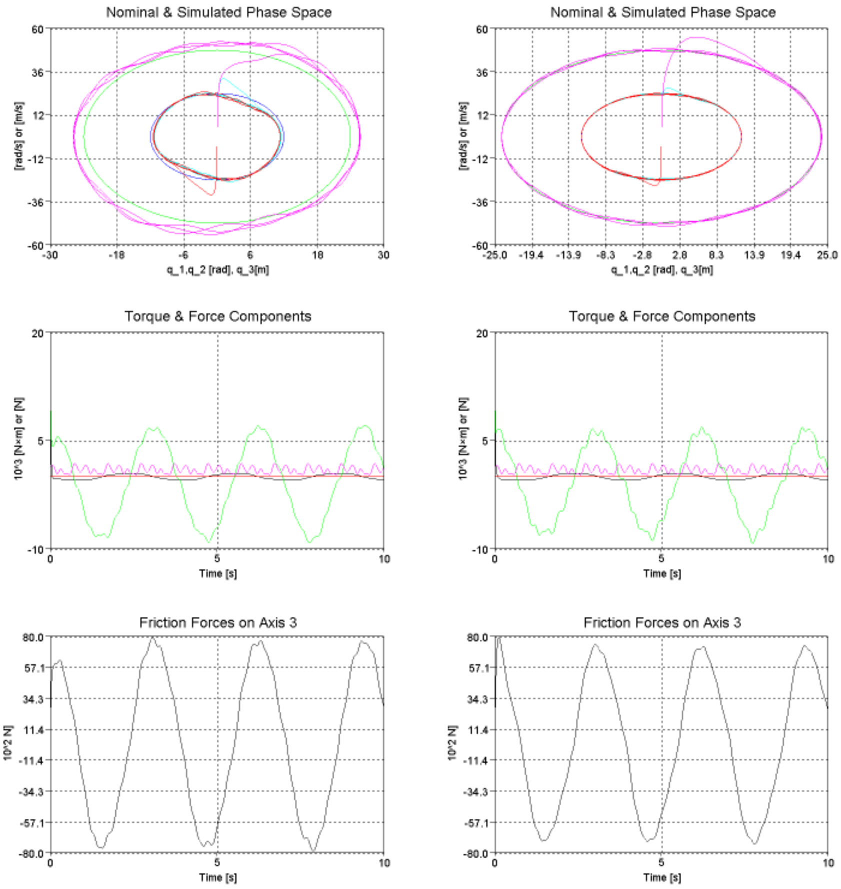


Figure A.6.3.1. (continued) Comparison of the non-adaptive (LHS) and the adaptive (RHS) controllers' operations for the cart-beam-hamper system with friction and external disturbance forces for $\boldsymbol{\Lambda}=10\times\mathbf{I}$ *[1/s],* $K_{ctrl}=-32000$, $B_{ctrl}=1$, $A_{ctrl}=2\times10^{-6}$.

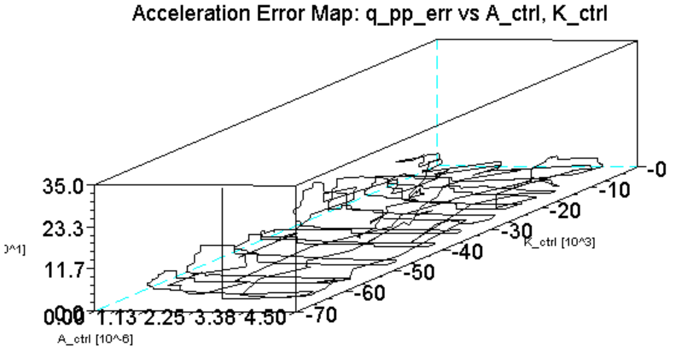


Figure A.6.3.2. Real-time modification of the adaptive control parameters A_{ctrl} and K_{ctrl} reveals the very robust environment around $A_{ctrl}=4\times 10^{-6}$ and $K_{ctrl}= -60\times 10^{3}$

A.6.4. Simulation Results Belonging to Subsection "8.3.1. Possible Application:Control of the Cart and Double Pendulum System"

In these examples the cart plus double pendulum system of Fig. A.5.1. and equations of motion (A.5.4) was considered as an *underactuated system* that means the the linear degree of freedom (q_3) was left without own drives, i.e. $Q_3 \equiv 0$ was assumed. The motion in the linear direction was controlled through the dynamic coupling between the linear axis and the two rotary ones. That is the reaction forces needed for moving the two "counterweights" m_1 and m_2 were used for generating acceleration along q_3.

So let us assume that we prescribe two "desired" second time derivatives as $\ddot{q}_3^{Des}$ according to the trajectory along which we wish to move the cart, and an "ancillary acceleration", $\ddot{q}_1^{Des}$. Via substituting these values into the 3rd equation of the set (A.5.4) the necessary $\ddot{q}_2^{Des}$ can be determined. Following that, by substituting these components of $\ddot{\mathbf{q}}^{Des}$ into the 1st and 2nd equations of (A.5.4) Q_1 and Q_2 can be determined. For this calculation we can use the available approximate values of the dynamic parameters $\hat{m}_1, \hat{m}_2, \hat{M}$ (it can be assumed that the gravitational acceleration is precisely known as well as the lengths of the arms of the cart, that is $\hat{L}_1 = L_1, \hat{L}_2 = L_3, \hat{g} = g$). The so calculated generalized forces then can be exerted to the actual system, and cause the "realized accelerations" $[\ddot{q}_1, \ddot{q}_2, \ddot{q}_3]^T$ according to (A.5.4). Then by observing the response error, viz. the differences between $\left[\ddot{q}_1^{Des}, \ddot{q}_3^{Des}\right]^T$ and $[\ddot{q}_1, \ddot{q}_3]^T$ the adaptive can be applied to achieve precise tuning. (In this approach the realization of the appropriate $\ddot{q}_2^{Des}$ is out of any interest.)

While implementing the above program the following difficulties arise: for determining $\ddot{q}_2$ we have to make a division with its coefficient in the last row of (A.5.4) which is $\hat{m}_2 L_2 \sin q_2$. It evidently is singular around $q_2=\pm\pi$, 0. Similar problems arise around $q_1=\pm\pi$, 0, too. To avoid this situation we can combine the reaction forces of the masses m_1 and m_2 in the following manner: a) both angles are started from the "best position", i.e. from $\pi/2$; if q_1 is within a "safe region" (i.e. $q_1 \in [\pi/4, 3\pi/4]$) then the particular value of $\ddot{q}_1$ is not important, but q_2 can be forced to move into the direction of $\pi/2$ by the control law $\ddot{q}_2^{Des} = -C^2(q_2 - \pi/2) - C\dot{q}_2$ (C>0), and $\ddot{q}_1^{Des}$ can be determined accordingly; if q_1 is outside of the safe region then $\ddot{q}_1^{Des} = -C^2(q_1 - \pi/2) - C\dot{q}_1$ can be prescribed and $\ddot{q}_2^{Des}$ can be determined accordingly. (More sophisticated compromises can be invented according to the ideas used e.g. in optimal control or by a fuzzy-type mixing of the above “crisp”, *if … then* type application of the acceleration of the counterweights.) The prescribed tracking error relaxation used the following PID setting: $\ddot{q}_3^{Des}(t)=\ddot{q}_3^{N}(t)+3\Lambda^2 e(t)+3\Lambda\dot{e}(t)+\Lambda^3\int_{t_0}^{t} e(\xi)d\xi$, in which $e(t):=q_3^N(t)-q_3(t)$ denotes the trajectory tracking error, $\Lambda>0$ is the reciprocal of a time constant. Actually Λ=12/s, C=10/s, $m_1=8\,kg, m_2=8\,kg, M=20\,kg$, $\hat{m}_1=4\,kg, \hat{m}_2=6\,kg, \hat{M}=18\,kg$, L_1=L_2=2 m, g=9.81 m/s^2 were used in the simulations detailed in the next section that also contains information on the adaptive control parameters. The nominal trajectory was a 3rd order spline function of time consisting of consecutive intervals of linear variation of the 2nd time derivative.

For simulation purposes the SCILAB 5.1.1 version and its SCICOS ver. 4.2 co simulator package were applied that can freely be used for research purposes. For the simple SCILAB program representative results are given in Figs. A.6.3.1. and A.6.3.2. and A.6.3.3.

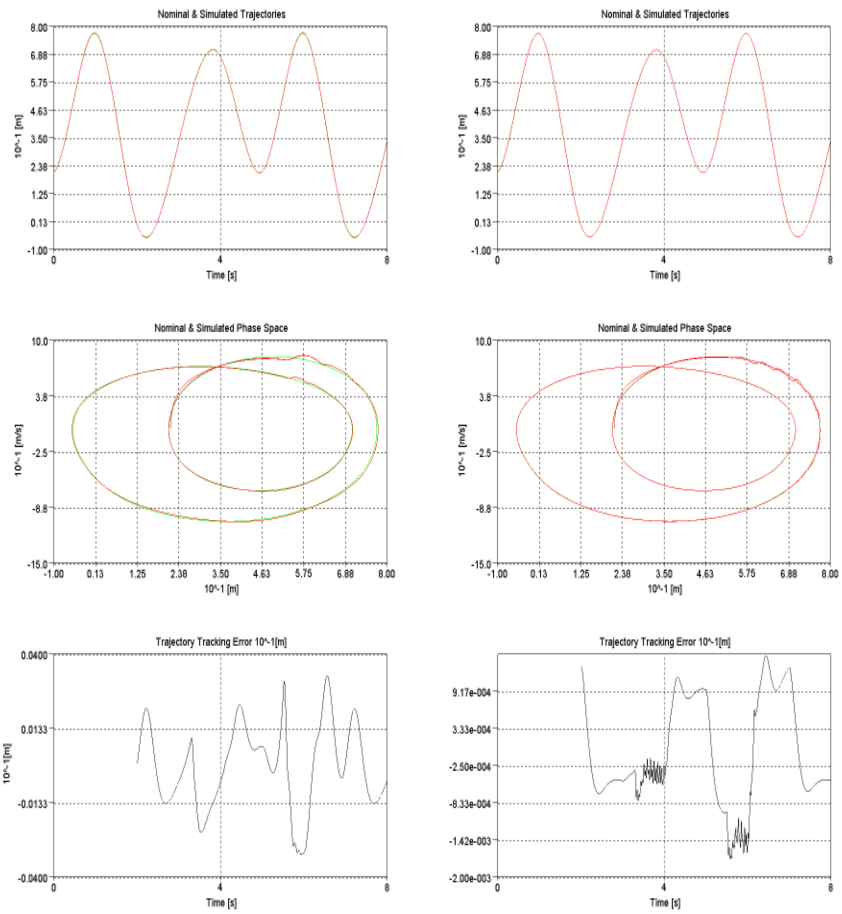


Figure A.6.3.1. Simulation results obtained by the simple SCILAB program: nominal (green line) and simulated (red line) trajectories [10^{-1} m] (upper chart), phase trajectories i.e. dq^N_3/dt [10^{-1} m/s] vs. q^N_3 [10^{-1} m] (green, “canonical” line) and dq_3/dt [10^{-1} m/s] vs. q3 [10^{-1} m] (red, “less canonical” line), and trajectory tracking error [10^{-1} m] vs. time [LHS: simple non-adaptive PID controllers, RHS: adaptive controller tuning parameter A according to (8.3.5) with α=6/s];

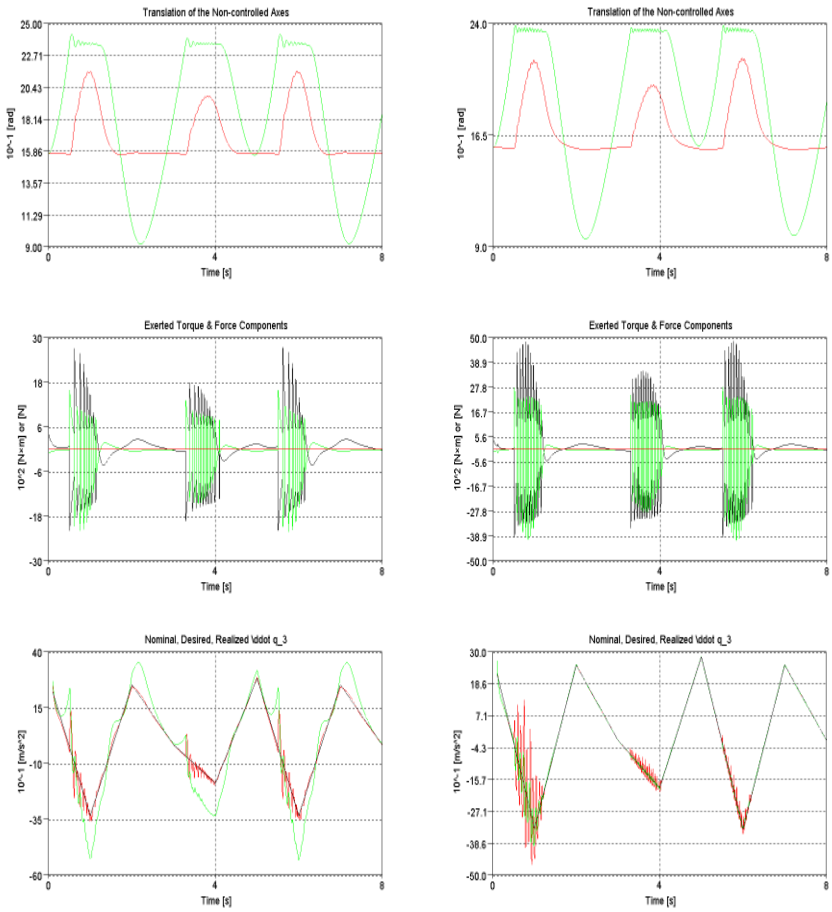


Figure A.6.3.2. Simulation results obtained by the simple SCILAB program: variation of the ancillary axes [10^{-1} rad] (upper chart, q_1 green "upper in the beginning" line, q_2 red line), the exerted generalized forces (chart in the middle, Q_1 black "upper in the beginning" line, Q_2 green "lower in the beginning" line [10^2 N×m], $Q3 \equiv 0$ [10^2 N] red "middle in the beginning" line), and the second time derivatives for q_3 [10^{-1} m/s^2] (lower chart, nominal: black "canonical" line, desired: green "slowly varying" line , realized: red "more hectic" line) [LHS: simple non-adaptive PID controllers, RHS: adaptive controller tuning parameter A according to (8.3.5) with α=6/s];

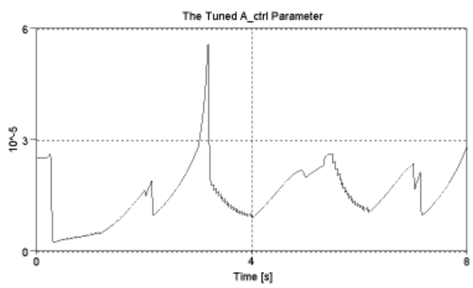


Figure A.6.3.3. Simulation results obtained by the simple SCILAB program for the adaptive controller tuning parameter A according to (8.3.5) with α=6/s: parameter A vs. time

To investigate the “reality” of this “ideal” SCILAB-based solution the built in integrator of the SCICOS simulator was used. It was assumed that we had a digital controller of discrete time-resolution Δ_{cycle}=1 *ms*. This means that the controller yields constant torque/force command signals for a duration of Δ_{cycle}, and the drive system has so fast response (i.e. small time-constant) that Q can well be approximated as a step function. In similar manner, it can be assumed that the system's response is observed in discrete steps and the controller is provided with constant observed quantities within the steps of duration Δ_{cycle}. For this purpose small buffers and sample holders can be used as typical electronic / software components. In principle the sample holders should be set to $\Delta_{delay}=\Delta_{cycle}$. However, in this construction the sampling practically would be indefinite (i.e. depending on the accuracy of the electronic components) in the steps of the command and observed quantities. To make the situation definite the $\Delta_{delay}=0.999\Delta_{cycle}$ choice was used. In this case in the beginning of a new control cycle for a very short duration the values just preceding the previous values are sampled. The SCICOS numerical co-simulator had the following parameter settings: “Integrator absolute tolerance = 0.0001”, “Integrator relative tolerance = 0.000001”, “Tolerance on time = 1.000D-10”, “Maximum step size in integration = 0.0001 s”.

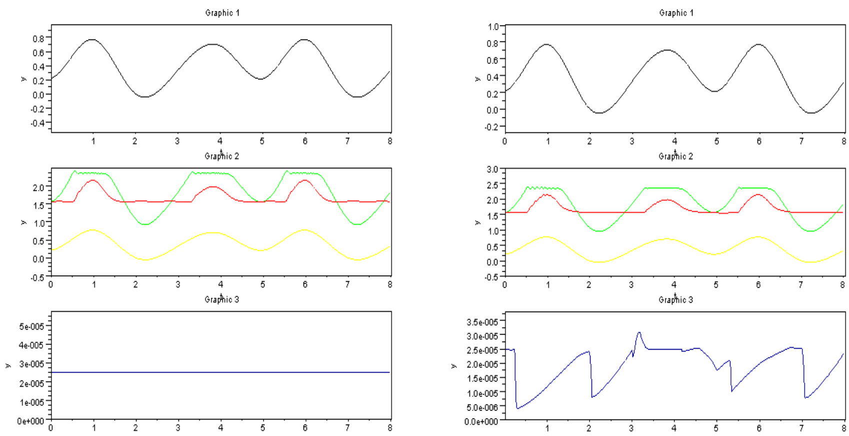


Figure A.6.3.4. Simulation results obtained by the SCICOS program: the nominal trajectory (upper chart), the simulated trajectories (chart in the middle, q1 green “upper in the beginning” line, q_2 red “middle in the beginning” line, q_3 yellow “lower” line), and the control parameter A (lower chart) [LHS: simple non-adaptive PID controllers, RHS: adaptive controller tuning parameter A according to (8.3.5) with α=6/s];

Representative simulation results are given for the SCICOS program in Figs. A.6.3.4., A.6.3.2., and A.6.3.3. It can well be seen that the common SCILAB programs with the simplest Euler integration and the far more sophisticated SCICOS simulations provided comparable results and that the main qualitative/quantitative features are reliable in the case of the simple SCILAB programs, too. [Also consider Fig. A.6.3.7.] For Fig. A.6.3.7. it can be noted in the simulations the function $\sigma(x):=x/(1+|x|)$ was used, so the region of convergence is $\sigma\in[\sigma(0),\sigma(-1)]=[0,-0.5]$. For the ideal case $\sigma(\varepsilon_{goal})=\sigma(-0.5)=-0.333\ldots$. The SCICOS-based program also took it into account that following a tuning act for α the controller must wait $3\times\Delta_{cycle}$ time in order to obtain quite relevant data for tuning since it is based on past information.

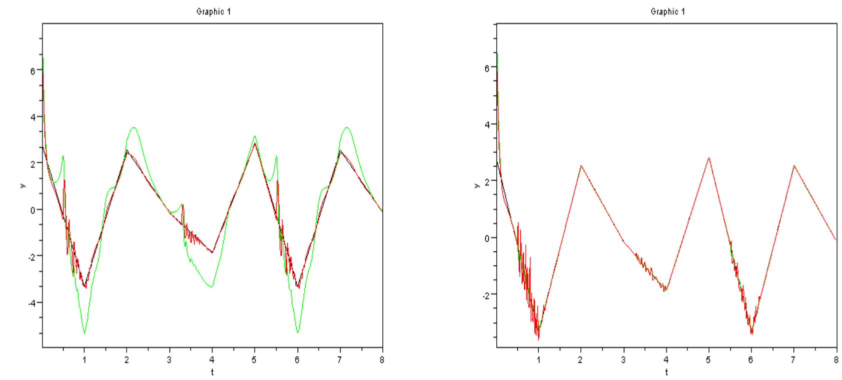


Figure A.6.3.5. Simulation results obtained by the SCICOS program: the 2nd time derivatives of the controlled axis: nominal (black "canonical" line), desired (green "slowly varying" line), and simulated (red "more hectic" line) [LHS: simple non-adaptive PID controllers, RHS: adaptive controller tuning parameter A according to (8.3.5) with α=6/s];

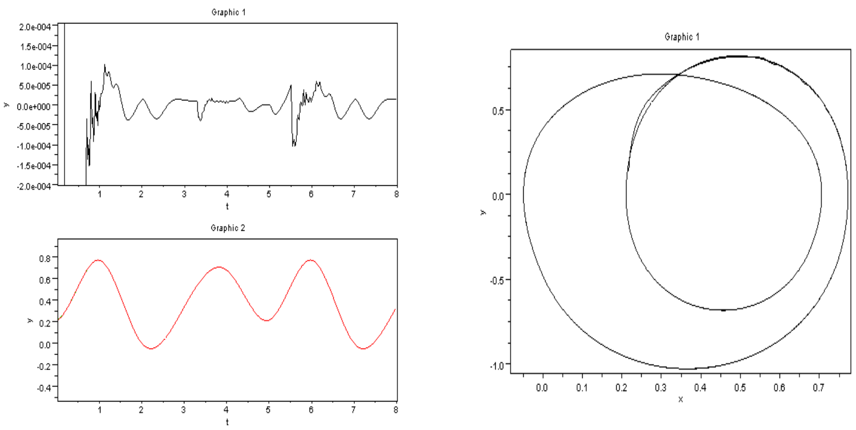


Figure A.6.3.6. Simulation results obtained by the SCICOS program for the adaptive controller tuning parameter A according to (8.3.5) with α=6/s: the trajectory tracking error (LHS upper chart), the nominal and simulated trajectories (LHS lower chart, green and red lines), and the nominal and simulated phase trajectories (RHS);

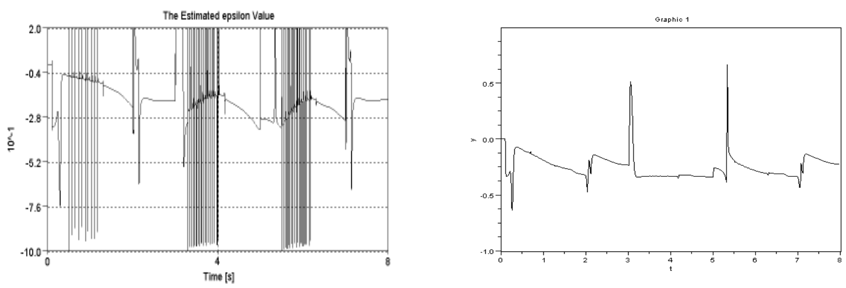


Figure A.6.3.7. Variation of $\sigma(\varepsilon_{est})$ obtained by the simple SCILAB (LHS) and the SCICOS (RHS) programs for the adaptive controller tuning parameter A according to (8.3.5) with α=6/s

A.7.1. Simulation Results Belonging to Subsection "9.1.1. Possible Applications: a) MRAC Control of the Cart + Beam + Hamper System"

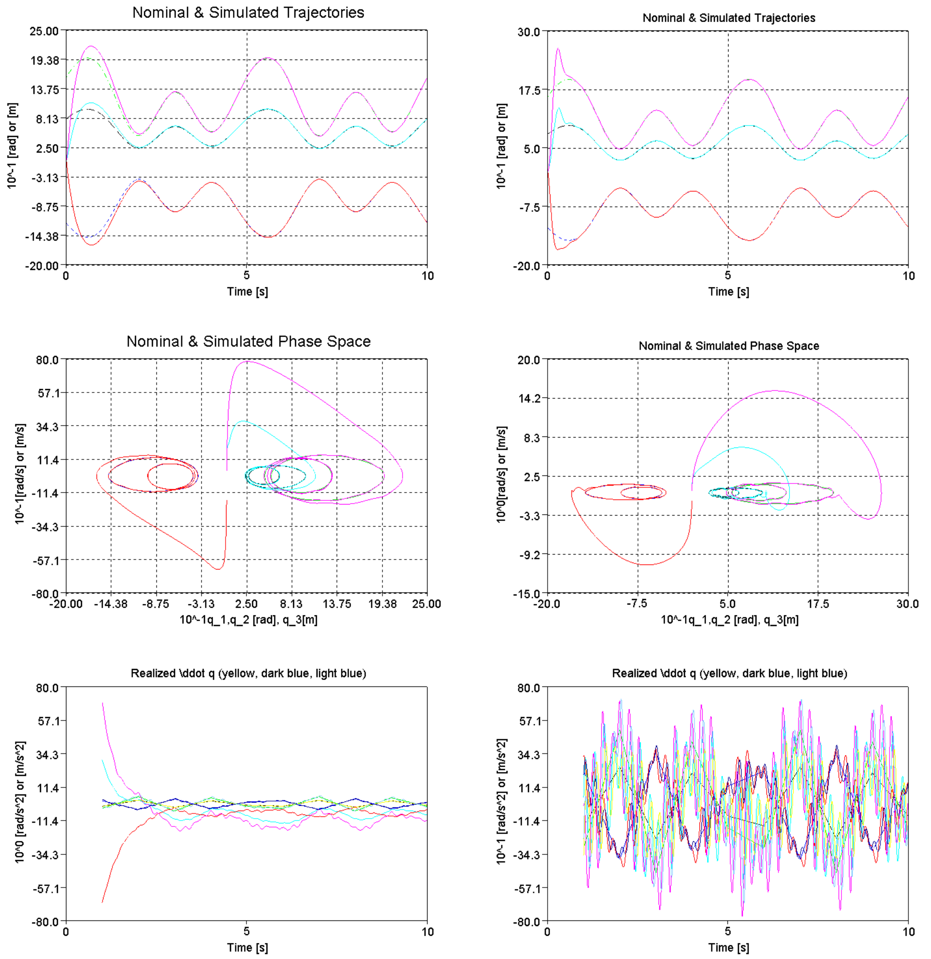


Figure A.7.1.1. The operation of the "traditional" (LHS) and the "novel" (RHS) MRAC controllers: tracking for a nominal trajectory generated by a 3rd order spline function: trajectory tracking (1st row), phase trajectory tracking (2nd row); 2nd derivetives (3rd row), color coding: $q^N{}_1$=black, $q^N{}_2$=blue, $q^N{}_3$=green (for the nominal values), $q^D{}_1$=bright blue, $q^D{}_2$=red, $q^D{}_3$=magenta (for the "desired" values), and q_1=yellow, q_2=dark blue, q_3=light blue (for the realized values)

The main results of the comparative analysis are given in Fig. A.7.1.1. The figures reveal that both methods resulted in acceptable control. However, the novel controller resulted in far more precise "acceleration tracking" than the traditional one, in spite of the drastic disturbance forces applied. The zoomed excerpts in Fig. A.7.1.3. reveals that the *nominal*, *desired*, and *realized* joint coordinate accelerations are in each other's vicinity. Really, the *black*, *bright blue* and *yellow* lines belonging to q_1, the *blue*, *red*, and *dark blue* lines belonging to q_2, and the *green*, *magenta*, and *light blue* lines belonging to q_3 keep together in the three groups. Actually the desired and the realized values fluctuate around the nominal values (straight lines in the case of 3rd order spline trajectories). The fluctuation is caused by the external disturbance forces.

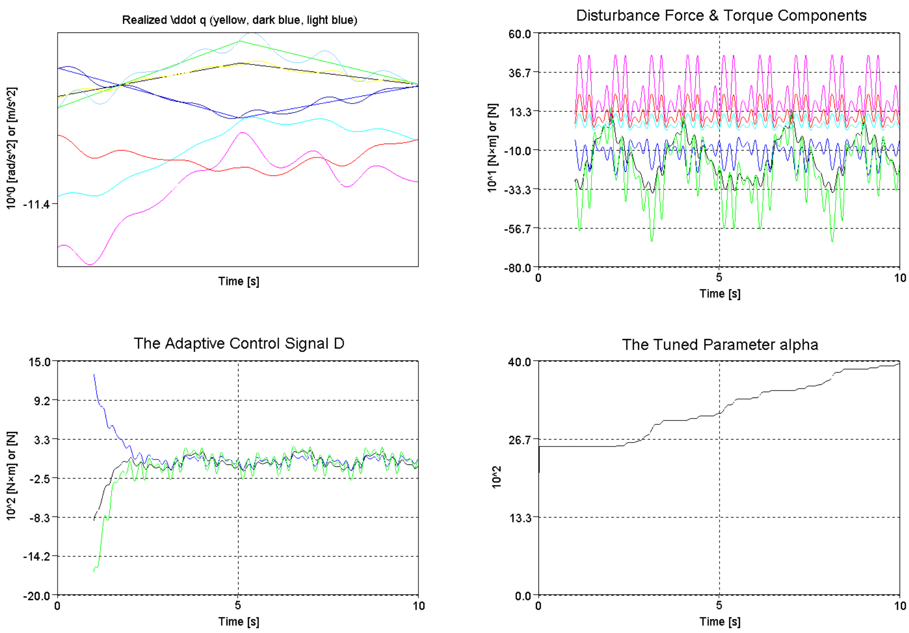


Figure A.7.1.2. Other characteristics of the "traditional" MRAC controller: tracking for a nominal trajectory generated by a 3rd order spline function: zoomed excerpt of the 2nd derivatives (color coding as in Fig. A.7.1.1.) [LHS, 1st line], exerted generalized forces Q_1 [Nm] (black), Q_2 [Nm] (blue), Q_3 [N] (green), and disturbance components Q_1^{Dist} [Nm] (bright blue), Q_2^{Dist} [Nm] (red), Q_3^{Dist} [N] (magenta) [RHS, 1st line]; The additive adaptive component **D**: *D_1 [Nm] (black), D_2 [Nm] (blue), D_3 [N] (green) [LHS, 2nd line]; the tuned parameter α vs. time [RHS, 2nd line]*

Furthermore, the desired 2nd derivatives only slightly differ from the nominal ones, that in the case of a purely kinematically designed trajectory tracking policy means that only small PID corrections were necessary. The desired ant the realized accelerations are in each other's vicinity, too, that proves the operation of the adaptation. The situation is far less elegant in the case of the traditional solution using a Lyapunov function (Fig. A.7.1.2.).

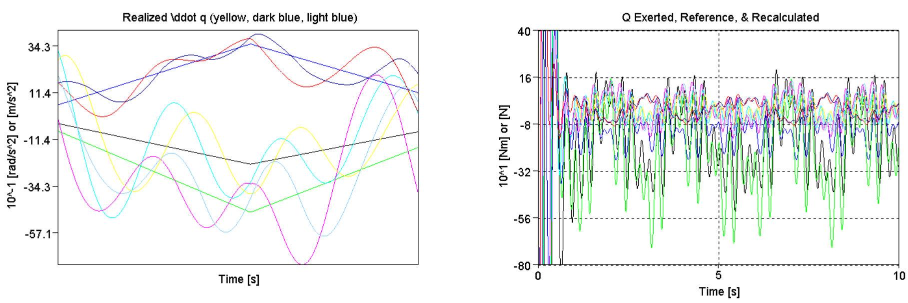


Figure A.7.1.3. Other characteristics of the "novel" MRAC controller: tracking for a nominal trajectory generated by a 3rd order spline function: zoomed excerpt of the 2nd derivatives (color coding as in Fig. A.7.1.1.) [LHS],], exerted generalized forces Q_1 [Nm] (black), Q_2 [Nm] (blue), Q_3 [N] (green), the "desired" Q_1^{Des}[Nm] (bright blue), Q_2^{Des}[Nm] (red), Q_3^{Des}[N] (magenta), and the "recalculated" torque/force components Q_1^{Recalc}[Nm] (yellow), Q_2^{Recalc}[Nm] (dark blue), Q_3^{Recalc}[N] (light blue) [RHS]

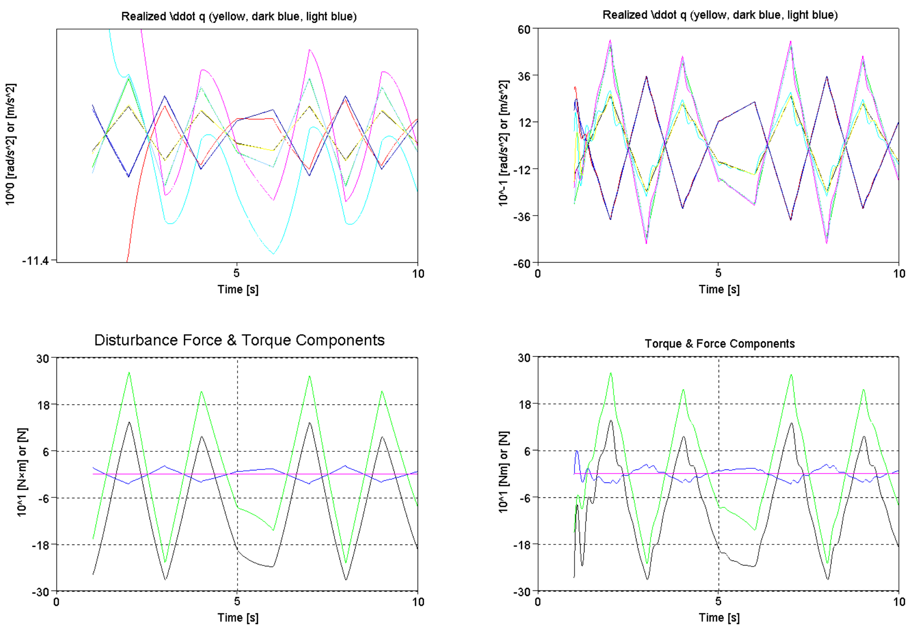


Figure A.7.1.4. Comparison of the "traditional" [LHS] and "novel" [RHS] MRAC controllers without external disturbances: zoomed excerpt of the 2nd derivatives (color coding as in Fig. A.7.1.1.) [1st row], exerted generalized forces Q_1 [Nm] (black), Q_2 [Nm] (blue), Q_3 [N] (green), and disturbance components Q_1^{Dist} [Nm] (bright blue), Q_2^{Dist} [Nm] (red), Q_3^{Dist} [N] (magenta) [2nd row]

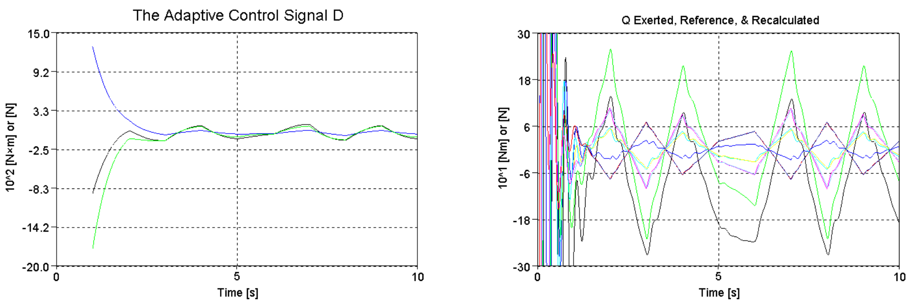
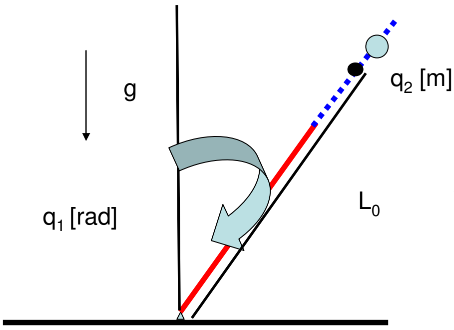


Figure A.7.1.5. The additive adaptive component of the "traditional" MRAC without external disturbances **D**: *D_1 [Nm] (black), D_2 [Nm] (blue), D_3 [N] (green) [LHS]; The generalized forces of the "novel" MRAC without external disturbances: the "exerted" Q_1 [Nm] (black), Q_2 [Nm] (blue), Q_3 [N] (green), the "desired" Q_1^{Des}[Nm] (bright blue), Q_2^{Des}[Nm] (red), Q_3^{Des}[N] (magenta), and the "recalculated" torque/force components Q_1^{Recalc}[Nm] (yellow), Q_2^{Recalc}[Nm] (dark blue), Q_3^{Recalc}[N] (light blue) [RHS]*

The superiority of the novel approach is even more evident in the case when no external disturbances were present. The 1st row of Fig. A.7.1.4. reveals that the *nominal*, *desired*, and *realized* 2nd time-derivatives are almost identical to each other. Furthermore, according to Fig. A.7.1.5. the *desired* torque components deduced from the reference model very precisely agree with the *recalculated* values.

A.7.2. Simulation Results Belonging to Subsection "9.1.2. Possible Applications: b) Novel MRAC Control of a Pendulum of Uncertain Mass Center Point"

The Dynamic Model of the Paradigm



The Euler-Lagrange Equations of Motion

$$T=\frac{1}{2}\Theta\dot{q}_1^2+\frac{1}{2}m\left\{(L_0+q_2)^2\dot{q}_1^2+\dot{q}_2^2\right\}$$

$$V=[C+mg(L_0+q_2)]\cos q_1+\frac{1}{2}kq_2^2$$

$$L:=T-V$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i}-\frac{\partial L}{\partial q_i}=Q_i$$

For determining the „actual" joint accelerations:

$$\begin{bmatrix}Q_1\\Q_2\end{bmatrix}=\begin{bmatrix}\Theta+m(L_0+q_2)^2 & 0\\ 0 & m\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}+\begin{bmatrix}2m(L_0+q_2)\dot{q}_1\dot{q}_2-[C+mg(L_0+q_2)\sin q_1]\\ -m(L_0+q_2)\dot{q}_1^2+kq_2+mg\cos q_1\end{bmatrix}$$

$$Q_2\equiv 0$$

Figure A.7.2.1. The dynamic model of the pendulum of uncertain mass center point

The dynamic model of the pendulum of mass center point of uncertain location is given in Fig. A.7.2.1. It has an extra degree of freedom that behaves as a not controllable axis completely hidden for the controller. It is a mass-point that can move along the rod of the pendulum against *viscous friction* and *elastic bounding forces*. In the case of the presence of viscous friction along the linear axis the model given in Fig. A.7.2.1. has to be completed by the friction force as $Q_2=-\mu\dot{q}_2$. The precise parameters of the dynamic model were: Θ=30 kg×m^2, C=50 kg, m=50 kg, k=3000N/m, g=9.81 m/s^2, μ=5 Ns/m, and L_0=2 m. The appropriate approximate values used by the controller were as follows (in the same measuring units, respectively): Θ^m=50, C^m=70, m^m=20, g^m=10, μ^n=0.01, and $L^m{}_0$=L_0. (The model value of the *spring stiffness* k and the *viscous friction coefficient* did not play any role, the controller assumed that no 2nd axis exists in the system.)

In the sequel simulation results will be provided for the novel MRAC control of this system. The adaptive control parameters initially were set as K= -2×10^5, B=1, A_0=10^{-5}. In this case parameter A had an aggressive, agile tuning quite different to that used in the case of the cart + double pendulum system in (8.3.5). This tuning has some "exponential nature" and can be described as follows:

- let us start with a roughly estimated initial value A_0;
- the estimated value of the partial derivative of the actual ε_{est} is calculated according to equation (9.1.2.1);
- from its present value it can be determined whether A must be increased or decreased;
- since for the estimation various delayed ("past") values are used, for proper modification of A we have appropriate information in 3Δ_{cycle} time steps;

- let us prescribe a “quasi-exponential” variation for decreasing A in a discrete approximation by using a parameter $0<\gamma<1$ as $A_{n+1}=\gamma A_n$; if the discrete time step is of duration Δ_{cycle} this corresponds to the derivative $dA_n/dt\approx[A_{n+1}-A_n]/(3\Delta_{cycle})=[(\gamma-1)/(3\Delta_{cycle})]A_n$ that roughly corresponds to the exponent $\tau=(\gamma-1)/(3\Delta_{cycle})$ of an exponential function $A(t)=A_0 exp(\tau t)$ leading to the estimation $\gamma=\tau\Delta_{cycle}+1$;
- for increasing A the $A_{n+1}=A_n/\gamma$ operation can be applied;
- for decreasing A the $A_{n+1}=\gamma A_n/c_{factor}$ operation can be applied;
- to avoid numerically achieving 0 by decreasing A that could result in constant $A\equiv 0$, if A_{n+1} becomes smaller than $A_0/1000$, we restart the tuning from $A_{n+1}=A_0$;
- also, for critically small denominator $|r_n-r_{n-1}|<=$%eps (%eps means the “small value” in the SCILAB program) we again use the initial $A_{n+1}=A_0$ estimation.

Simulation examples are presented in Figs. A.7.2.2.-A.7.2.6. that reveal that the MRAC idea works well in this case, too. The tuned parameter suffers from drastic variation.

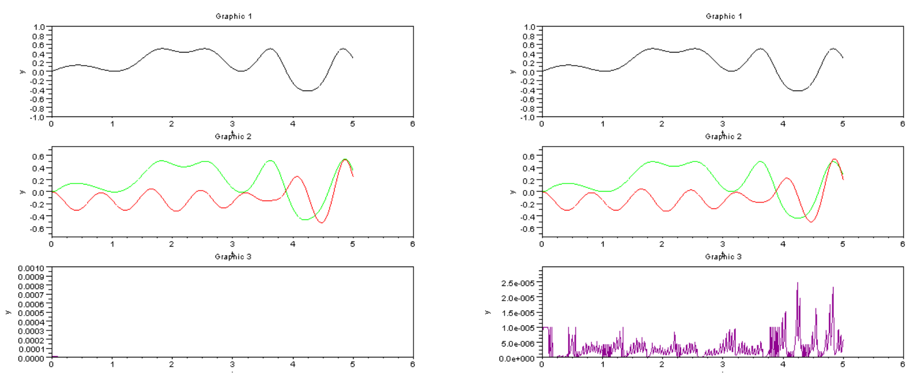


Figure A.7.2.2. The results for the non-adaptive simple PID controller (LHS) and the adaptive one (RHS): the nominal trajectory of the rotary joint (joint #1) [rad] vs time [s] (1st chart); the simulated trajectory of the rotary joint #1 (green line) [rad] and the swinging of the not controlled linear joint (#2) (red line) [m] vs time [s] (2nd chart); the tuned control parameter A [s^2/rad] vs. time [s] (3rd chart, purple line)

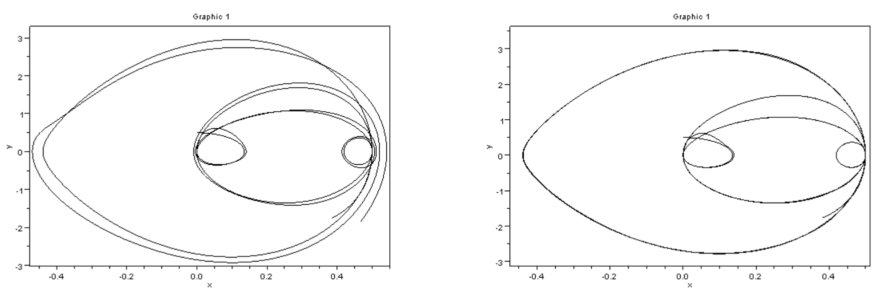


Figure A.7.2.3. The results for the non-adaptive (LHS) and the adaptive (RHS) controllers: the nominal and simulated phase trajectories of the rotary joint (joint #1) dq1/dt [rad/s] vs. q1 [rad]

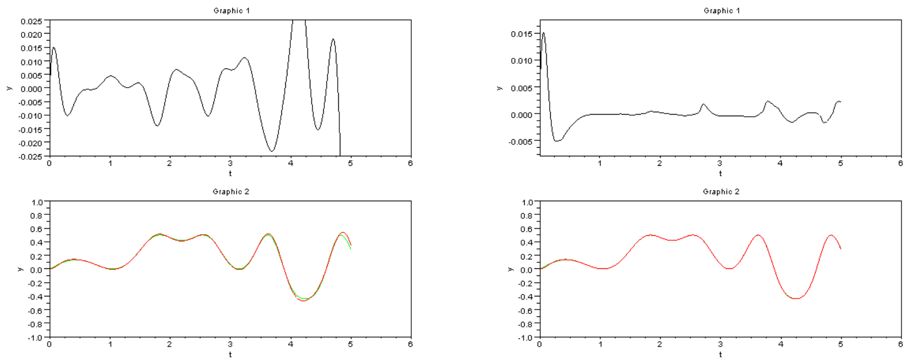


Figure A.7.2.4. The results for the non-adaptive simple PID (LHS) and the adaptive (RHS) controllers: the trajectory tracking error of the controlled rotary joint (joint #1) [rad] vs. time [s] (1st chart); the nominal (green line) and simulated (red line) trajectories of the controlled rotary joint (2nd chart)

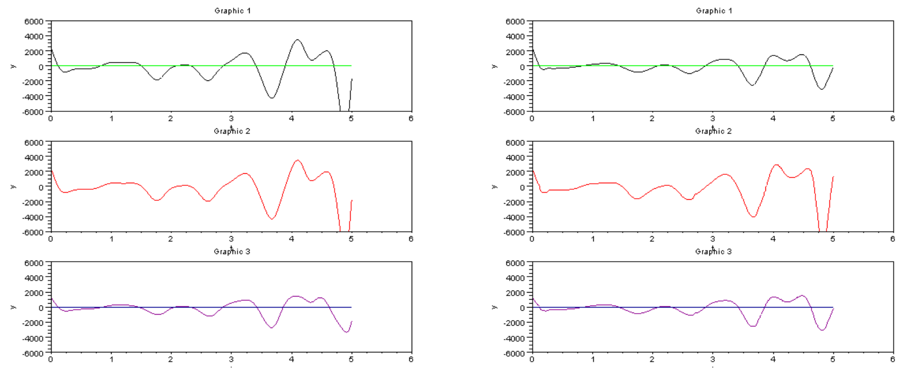


Figure A.7.2.5. The results for the non-adaptive (LHS) and the adaptive (RHS) controllers: the Q components to be exerted according to the reference model at the rotary joint (joint #1) [Nm] (black line) and the zero force for the not controlled linear joint (green line) vs. time [s] (upper graph), the exerted torque at (joint #1) (red line, central graph), and the Q components of the reference model recalculated from the actual system's response (purple line, lower graph)

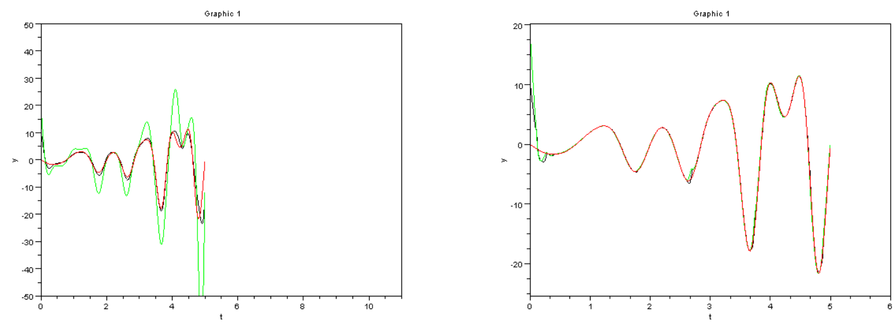


Figure A.7.2.6. The results for the non-adaptive (LHS) and the adaptive (RHS) controllers: the realized acceleration of the rotary joint (joint #1) [rad/s^2] vs. time [s] (black line), the desired acceleration computed from the PID block (green line), and the nominal acceleration (red line)

A.8. Simulation Results for Section "10.2. Application Example: Adaptive Control of the Cart plus Double Pendulum System"

The determinant of the inertia matrix in (A.5.4) has the form of

$$\det \mathbf{M} = m_1 L_1^2 m_2 L_2^2 \left(M + m_1 + m_2 - m_1 \sin^2 q_1 - m_2 \sin^2 q_2\right) \qquad \text{(A.8.1)}$$

It can well be seen from (A.8.1) that the minimum value of this determinant is equal to

$$\min(\det \mathbf{M}) = m_1 L_1^2 m_2 L_2^2 M \qquad \text{(A.8.2)}$$

and this situation happens whenever $q_1,\ q_2 = \pm\pi/2$ simultaneously. If $M << m_1,\ m_2$ these points correspond to near singular or badly conditioned inertia matrix that may cause problems in the control and simulation. On the basis of (A.5.4) it is easy to express the inverse dynamical equations of motion in closed analytical form used for simulation purposes. For making the simulation tests more realistic the purely conservative mechanical model in (A.5.4) was completed by dissipative *Dynamic Friction* terms yielding an additional contribution to the array **Q**. (This term was used only in the equations applied for representing the results of real time measurement, but is was "unknown" by the controller.) For numerical description a variant of the *Lund-Grenoble (LuGre) Model* was used in which the deformation of the bristles of some "brushes" are applied to describe the deformation of the surfaces in dynamic contact, so friction is described as a dynamic coupling between two subsystems having their own equations of motion as

$$\frac{dz}{dt} = v - \frac{\sigma_0 |v| z}{F_C + F_S \exp\left(-|v|/v_s\right)},\ F = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \mu v \qquad \text{(A.8.3)}$$

for which the proper direction of F has to be set in the applications, μ describes the usual viscous friction coefficient that dominates at "*higher velocity*" of the relative motion of the surfaces in contact "v" (this term is to be understood as a comparison between $|v|$ and v_s>0 since v_s represents the limit of the low velocity region), σ_0 corresponds to some elastic deformation of the surfaces in contact, "z" is the hidden internal degree of freedom, and σ_1 is a new parameter pertaining to the effect of the bending bristles. To clarify the role of the positive F_S and F_C parameters observe that the 1st equation in (A.8.3) pulls z in the direction of v if $|z|$ is small (in this case the 1st term dominates in the right hand side of the equation). For big $|z|$ values the dominating term is the 2nd term that tries exponentially damp z. The z variable stops varying when the limit for it $z_{\lim} := \mathrm{sgn}(v)\left[F_C + F_S \exp\left(-|v|/v_s\right)\right]/\sigma_0$ is achieved that corresponds to the contribution of $\sigma_0 z_{\lim} = \mathrm{sgn}(v)\left[F_C + F_S \exp\left(-|v|/v_s\right)\right]$. From it follows that for near zero velocities and stabilized z values big contribution (F_C+F_S) is obtained (the so called "sticking" phenomenon), while for "big" velocities it is reduced to F_C, therefore this model is able to describe the "slipping" phenomenon, too. This model is physically complete in the sense that no any velocity limit of dubious interpretation must be introduced for its use, in contrast to the static friction models that cannot yield definite friction force for v=0, and also leave the question open how to use this equation in numerical simulations. The behavior of the whole system is described by the dynamic coupling between the hidden internal and the observed degrees of freedom. Though the appropriate quantities in (A.8.3) were developed for linear motion and forces, it easily can be generalized for rotary motion in which torques appear in the role of the forces, and rotational velocities are present instead of the linear motion's velocity.

For control purposes the "*very rough model*" used instead of (A.5.4) was defined as

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix}, \tag{A.8.4}$$

and the *approximate model exact in its form but imprecise in its parameters* (just as in the case of the *Adaptive Inverse Dynamics Control* or the *Adaptive Slotine-Li Control*) had the same form as (A.5.4) with the appropriate model parameters $M^M=0.7\times M$, $L_1^M=0.9\times L_1$ and $L_2^M=0.8\times L_2$, $m_1^M= 0.6\times m_1$ and $m_2^M=0.5\times m_2$. Regarding the friction parameters, the appropriate values defined in (A.8.3) were chosen for each axis as follows: $\sigma_{10}=10$, $\sigma_{11}=156$, $\mu_1=1$, $F_{C1}=100$, $F_{S1}=200$, $v_{s1}=0.1$ for the 1st axis, $\sigma_{20}=20$, $\sigma_{21}=300$, $\mu_2=2$, $F_{C2}=200$, $F_{S2}=400$, $v_{s2}=0.2$ for the 2nd axis, and $\sigma_{30}=30$, $\sigma_{31}=450$, $\mu_3=3$, $F_{C3}=300$, $F_{S3}=300$, $v_{s3}=0.3$ for the 3rd one (each in appropriate physical dimensions). For better testing the control method additional disturbance force components were added that had the same numerical value and had the dimension of torque for Q_1 and Q_2, and force for Q_3.

The parameters of the adaptive controller in (10.1.5) were $K=200$, $n=3$ (this paradigm has 3 *DOF*), $C_{cut}=0.5$, and the *kinematically prescribed trajectory tracking* resulted in the *desired* 2nd time-derivatives of the generalized coordinates as follows:

$$\ddot{q}_i^d(t) = \ddot{q}(t)_i^N + 3\Lambda\left(\dot{q}(t)_i^N - \dot{q}(t)_i\right) + 3\Lambda^2\left(q(t)_i^N - q(t)_i\right) + \frac{\Lambda^3}{2}\int_0^t \left[q(\tau)_i^N - q(\tau)_i\right]d\tau, \tag{A.8.5}$$

that corresponds to some PID-type controller resulting in the convergence of the tracking error to zero if it is precisely implemented. (superscript N refers to the *nominal* motion). This convergence is roughly exponential with the exponent of $-\Lambda$. In the simulations $\Lambda=15/s$ value was used. As further refinement of the control instead of the desired accelerations prescribed by (A.8.5) a *reduced desired acceleration* was applied as

$$\begin{aligned} &\xi := \frac{\left\|\ddot{\mathbf{q}}^d(t) - \ddot{\mathbf{q}}(t-\delta t)\right\|}{1+\max\left(\left\|\ddot{\mathbf{q}}^d(t)\right\|, \left\|\ddot{\mathbf{q}}(t-\delta t)\right\|\right)} \\ &\lambda := (1+\varepsilon_1) + (\varepsilon_2 - 1 - \varepsilon_1)\frac{a_{shape}\xi}{1+a_{shape}\xi} \\ &\tilde{\ddot{\mathbf{q}}}(t)^d := (1-\lambda)\ddot{\mathbf{q}}(t-\delta t) + \lambda\ddot{\mathbf{q}}(t)^d \end{aligned} \tag{A.8.6}$$

where δt corresponds to the assumed cycle time of the control (10^{-3} s in the simulations) with small positive ε_1, ε_2, and some positive a_{shape} parameters. Equation (A.8.6) corresponds to some linear interpolation between the actual desired and the past realized accelerations in which the parameters ξ and a_{shape} measure the significance of their difference. These parameters can be set according to the order of magnitude of the signals occurring in the particular application. For zero ξ it practically corresponds to insignificant modification, for $\xi >> 1/a_{shape}$ it results in $\lambda=\varepsilon_2$ that means drastic reduction. In the simulations we had $\varepsilon_1=0.2$, $\varepsilon_2=10^{-5}$, and $a_{shape}=0.5$. The control signal was supposed to be constant during δt, and the integration of the equations of motion happened with the time-resolution of $\delta t/10$ with the simple method proposed by Euler. Since the simulations revealed that the direct application of (10.1.5) still resulted in very small fluctuation of the value of α_{max}, instead of it a smoothed value was used as

$$\alpha(t) = \alpha_{\max}(t)\tanh\left(\frac{\ddot{\mathbf{q}}(t-\delta t)^d - \ddot{\mathbf{q}}(t-\delta t)}{K}\right) \tag{A.8.7}$$

That reduced the relative significance of the fluctuation in α for small values. Finally, the torque / force components that should have been exerted according to $\alpha(t)$, (A.8.6) and (A.8.4) (i.e. the actual proposal) was smoothed according to its past proposed values by a forgetting filter

$$\mathbf{Q}(t)^{Actual} = \frac{\sum_{l=0}^{\infty} \beta^l \mathbf{Q}(t - l\delta t)^{Prop}}{\sum_{l=0}^{\infty} \beta^l} \tag{A.8.8}$$

that can be realized very easily by multiplying the content of a buffer by $0<\beta<1$ and adding to it the new contribution (the normalizing factor can be computed in closed form). In the simulations β=0.5 was applied.

It is worthy of note that the *SVD* was not executed within the control cycle. Instead of that, by the use of the *very rough* and the *approximate model* it was calculated in advance over a grid of dimensions 5×5 in the $[-\pi,+\pi]\times[-\pi,+\pi]$ grid in advance, and the appropriate diagonal and the orthogonal matrices were stored in memory. During the calculations these grid points served as the supports of a *Support Vector Machine (SVM)* of cylindrical function with Gaussian shape, and within the cycle only a simple interpolation happened by calculating "distance dependent averages" with the "distance functions" $d_k(\mathbf{q}) := \exp\left(-\gamma\|\mathbf{q}_k - \mathbf{q}\|^2\right)$ with γ=0.2 in which $\mathbf{q}$ denotes the actual state, and $\mathbf{q}_k$ means the k^{th} grid point.

In the 1st series of simulations the effect of the modeling errors (without friction and external disturbances) were studied in the case of the non-adaptive and the adaptive controller, respectively.

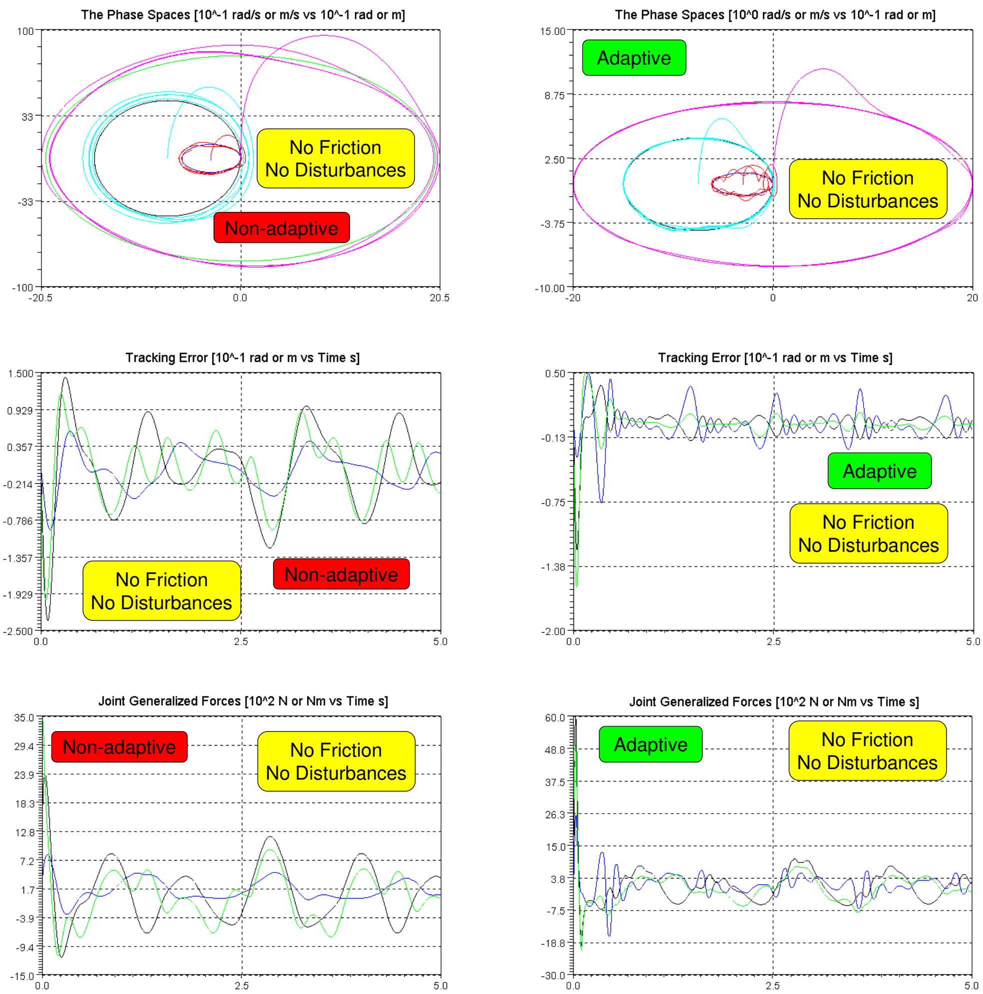


Figure A.8.1. The phase trajectories (1st row), the tracking error (2nd row), and the exerted generalized forces (3rd row) for the non-adaptive (LHS) and the adaptive (RHS) control (for the nominal motion: q_1: black, q_2: blue, q_3: green, for the simulated motion: q_1: light blue, q_2: red, q_3: magenta line in the phase trajectories, and q_1: black, q_2: blue, q_3: green for the rest)

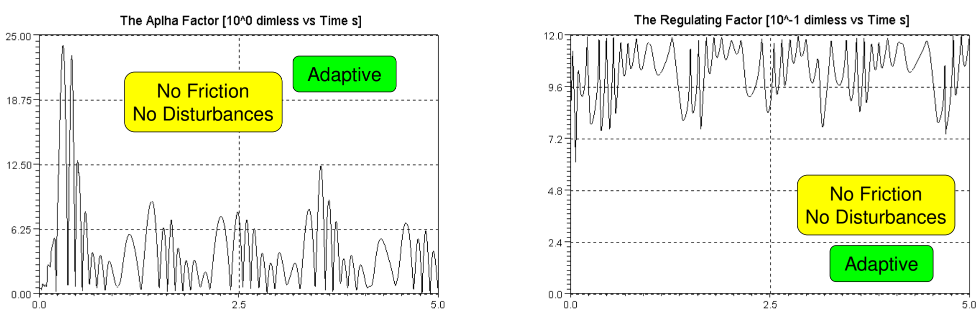


Figure A.8.2. The variation of the adaptive factors α and λ versus time

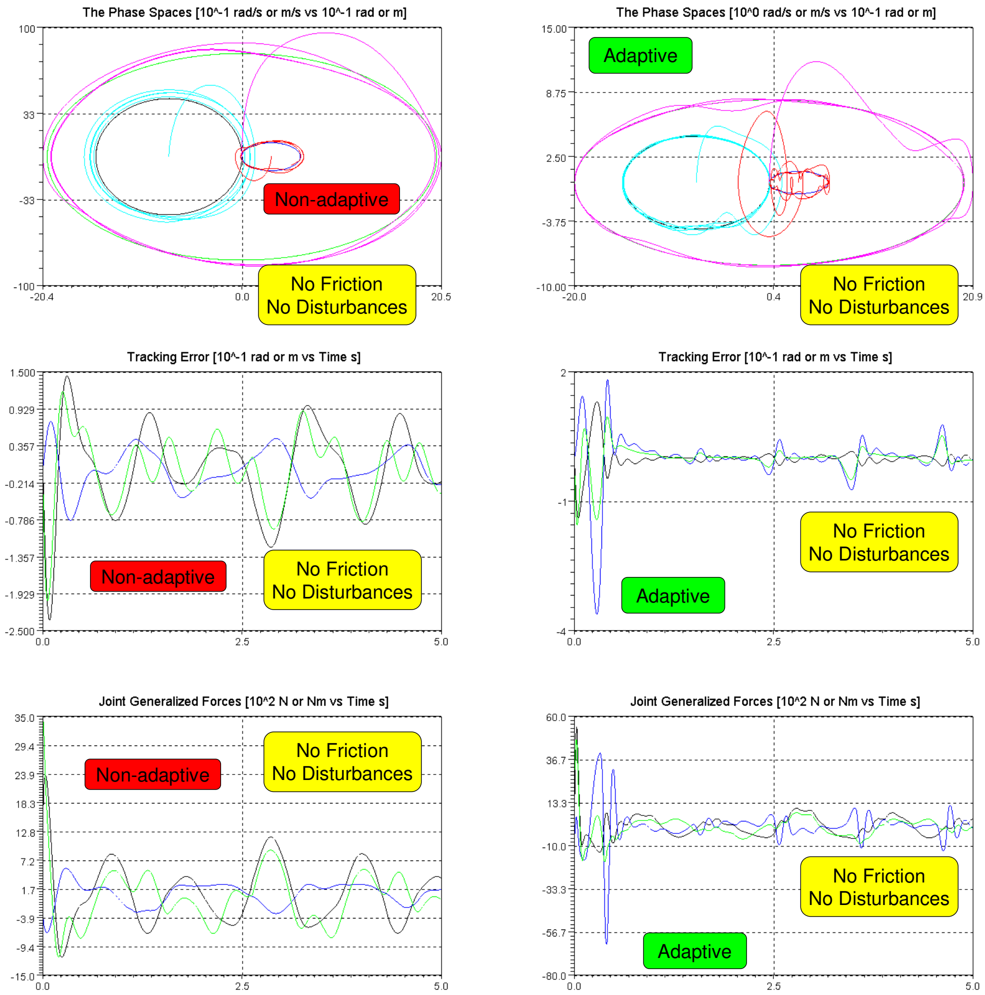


Figure A.8.3. The phase trajectories (1st row), the tracking error (2nd row), and the exerted generalized forces (3rd row) for the non-adaptive (LHS) and the adaptive (RHS) control for balls moving in the opposite directions [counterpart of Fig. A.8.1.] (for the nominal motion: q_1: black, q_2: blue, q_3: green, for the simulated motion: q_1: light blue, q_2: red, q_3: magenta line in the phase trajectories, and q_1: black, q_2: blue, q_3: green)

The appropriate phase trajectories and the tracking errors (Fig. A.8.1.) well exemplify the superiority of the adaptive control. The difference in the variation of the generalized forces exerted by the controller is significant and informative, too. Fig. A.8.2. reveals the fast variation of the adaptive variables α and λ versus time. It is worthy of note that the initial velocities considerably differ from the nominal ones, therefore in the beginning a “shock” was defied by the controller thank to the detailed interpolation and smoothing techniques. To demonstrate that the method worked at different regions of the state space the counterparts of Figs. A.8.1. and A.8.2. were calculated for a different nominal motion in which balls were moving in opposite directions (Figs. A.8.3. and A.8.4.).

In the next series of the investigations the dynamic friction forces unknown by the controller were switched on (Figs. A.8.5. A.8.6.and A.8.7.).

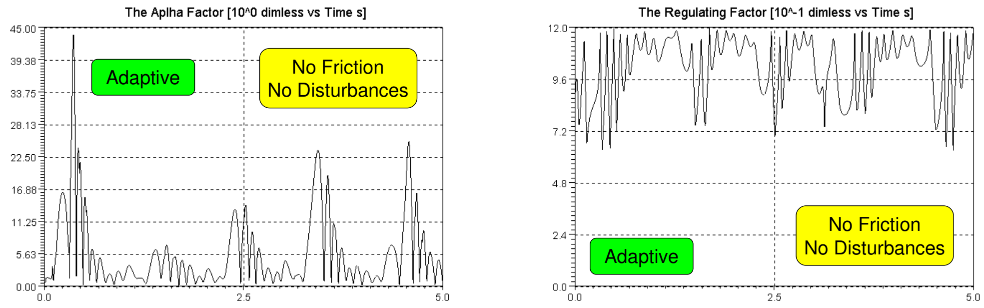


Figure A.8.4. The variation of the adaptive factors α and λ versus time for balls moving in the opposite directions [counterpart of Fig. A.8.2.]

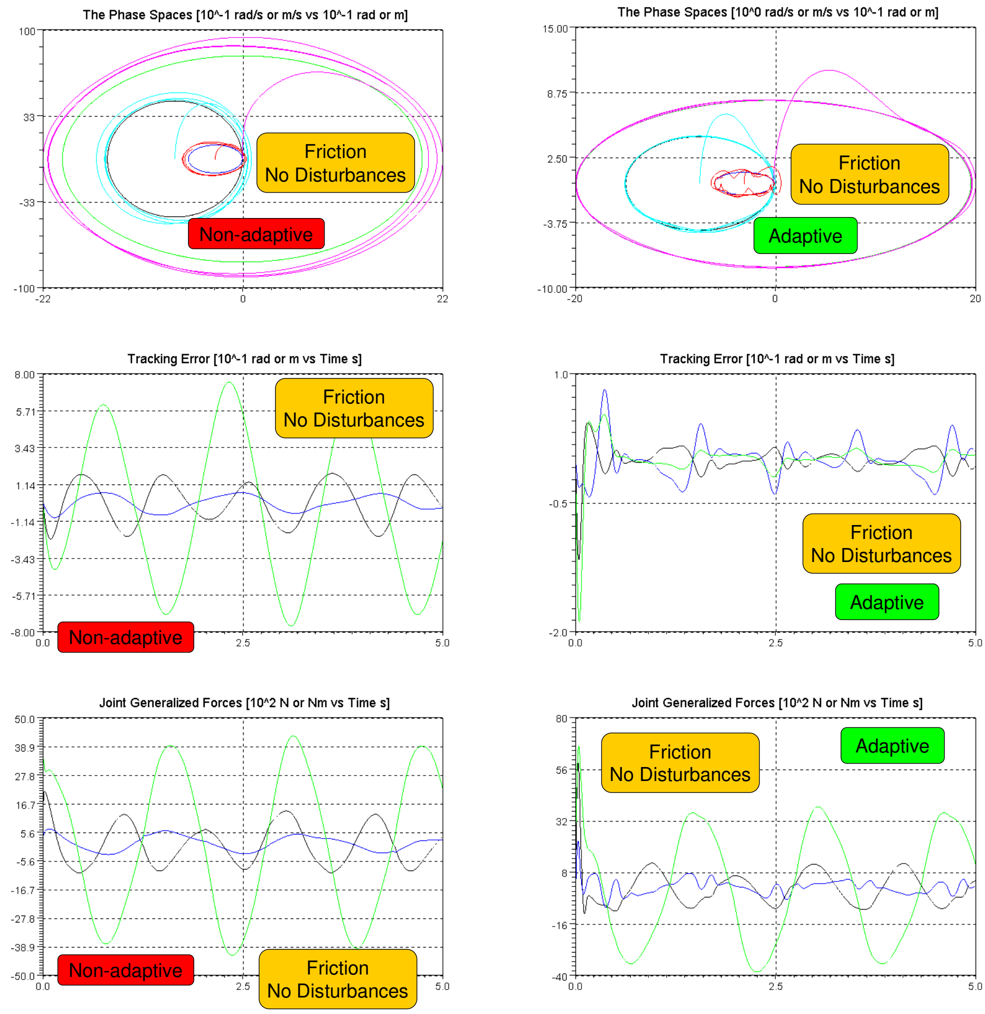


Figure A.8.5. The phase trajectories (1st row), the tracking error (2nd row), and the exerted generalized forces (3rd row) for the non-adaptive (LHS) and the adaptive (RHS) control with dynamic friction in the controlled system (for the nominal motion: q_1: black, q_2: blue, q_3: green, for the simulated motion: q_1: light blue, q_2: red, q_3: magenta line in the phase trajectories, and q_1: black, q_2: blue, q_3: green for the rest)

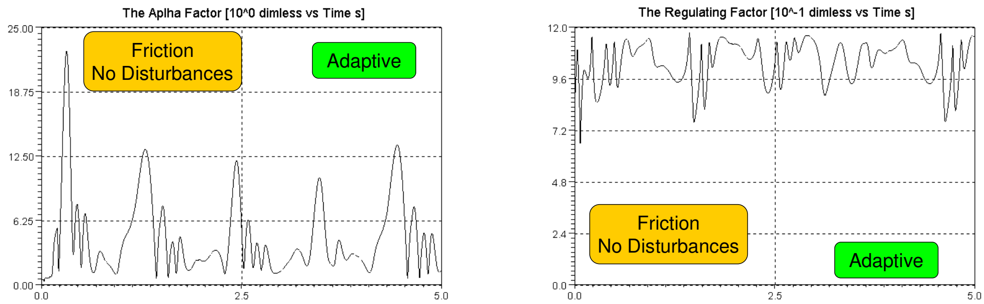


Figure A.8.6. The variation of the adaptive factors α and λ versus time in the case of dynamic friction in the controlled system

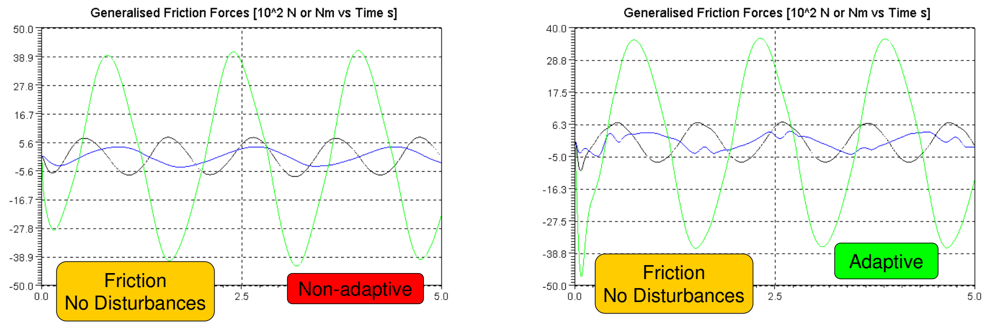


Figure A.8.7. The variation of the friction forces versus time in the case of dynamic friction in the controlled system (for q_1: black, q_2: blue, q_3: green line)

Figures A.8.5. and A.8.6. again reveal the superiority of the proposed adaptive control. In Fig. A.8.7. the friction forces are described that are quite significant and they considerably destroy the tracking quality of the non-adaptive controller.

To make the control task even more difficult, besides that of the internal friction, the effects of additional external disturbance forces were studied in the last series of simulations. The appropriate results are described by Fig. A.8.8. that reveals that while the non-adaptive controller is very considerably disturbed, the adaptive version quite efficiently resists.

As a summary of the simulation investigations it can be stated that in this section the generalization of certain parametric fixed point transformations was presented from SISO to MIMO systems for control technical purposes. The theoretically expected adaptive behavior was also illustrated by simulation results for a very wide range of motion velocities. The method is based on the properties of the *SVD* of an approximation of the Jacobian of the system's response.

In the presented example the matrices of the decomposed models were stored within certain typical regions of the generalized coordinates **q** (in the case numerically investigated the rigid translation in the direction of q_3 is internal symmetry of the system, therefore it is satisfactory to consider the part of the **q** space determined by the coordinates q_1 and q_2). In combination with the adaptive approach this idea is the counterpart of storing fuzzy rules over the whole domain of interest.

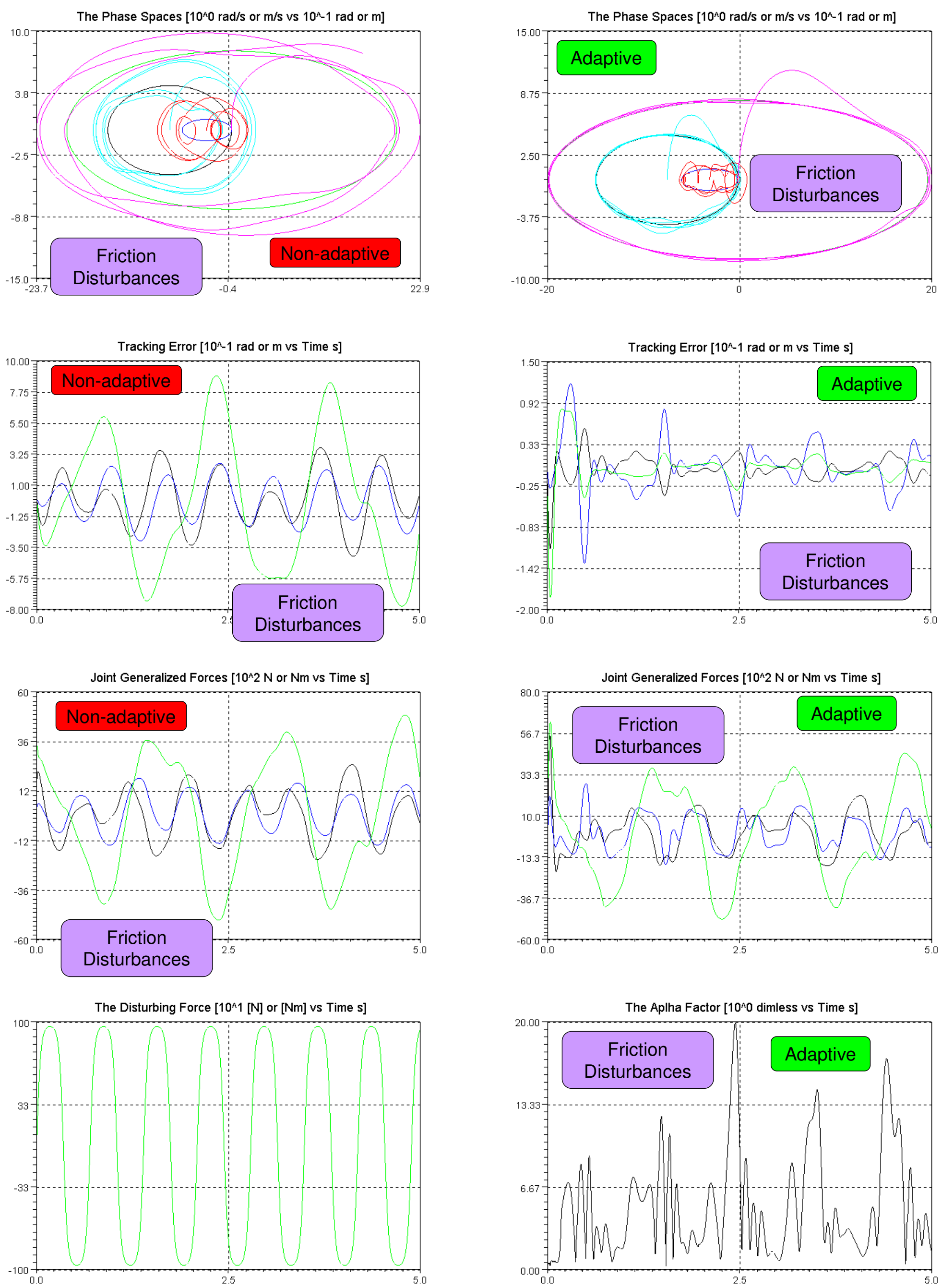


Figure A.8.8. The phase trajectories (1st row), the tracking error (2nd row), and the exerted generalized forces (3rd row) for the non-adaptive (LHS) and the adaptive (RHS) control with dynamic friction in the controlled system and the presence of external disturbances (for the nominal motion: q_1: black, q_2: blue, q_3: green, for the simulated motion: q_1: light blue, q_2: red, q_3: magenta line in the phase trajectories, and q_1: black, q_2: blue, q_3: green for the rest); In the 4th row the components of the disturbance forces and the control variable α are described vs. time

A.9. Simulation Results for Section "11.3. Application Example: the Use of Fractional Order Terms in the Control of Integer Order Systems"

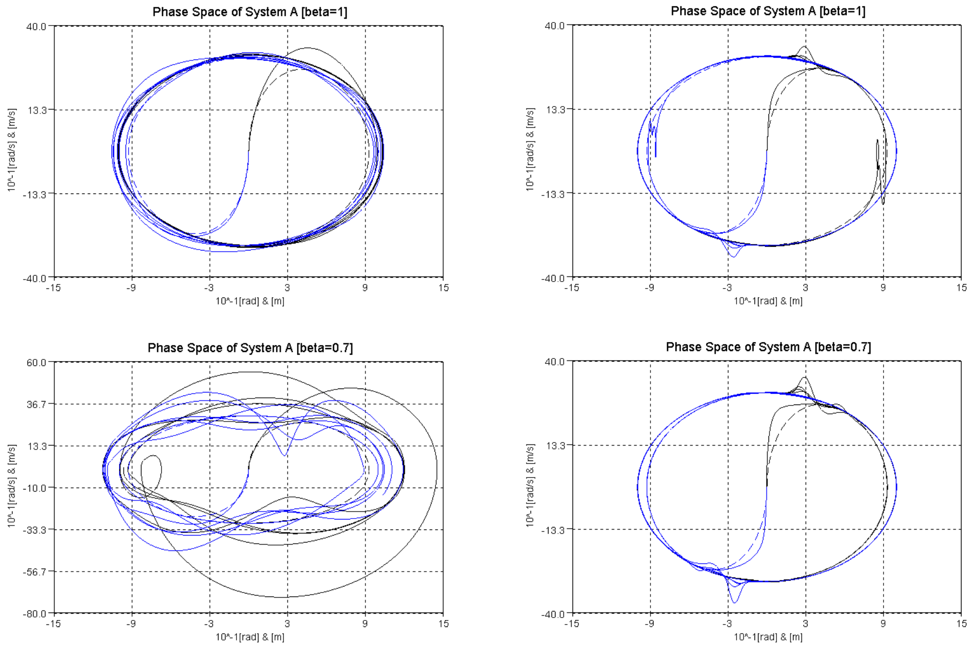


Figure A.9.1. The phase space of subsystem A: non-adaptive integer order (upper left corner), adaptive integer order (upper right corner), non-adaptive fractional order (lower left corner), and adaptive fractional order (lower right corner)

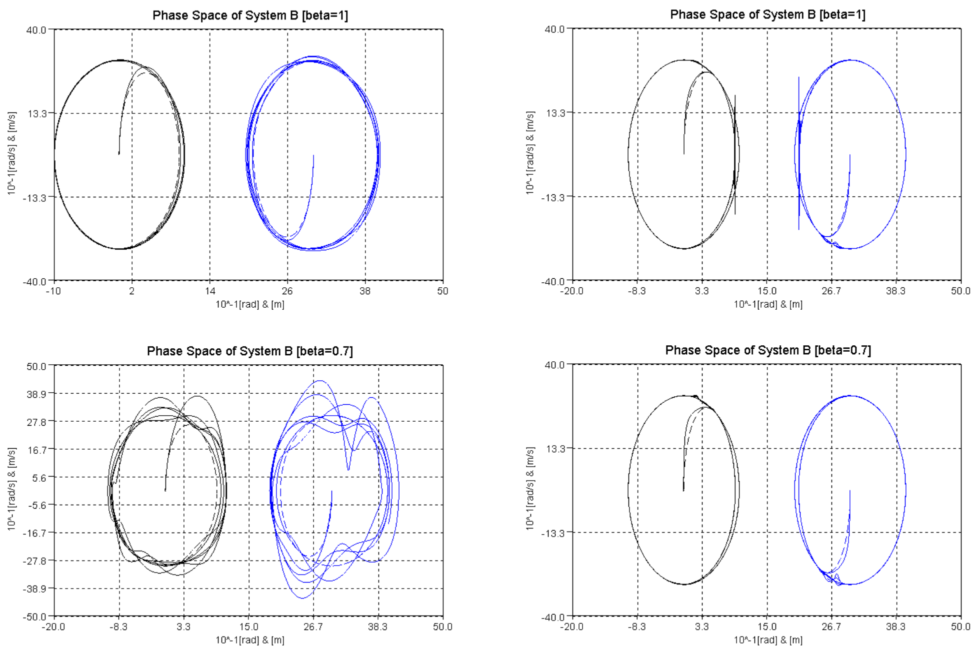


Figure A.9.2. The phase space of subsystem B: non-adaptive integer order (upper left corner), adaptive integer order (upper right corner), non-adaptive fractional order (lower left corner), and adaptive fractional order (lower right corner)

The phase trajectories obtained for adaptive and non-adaptive, integer and fractional order derivatives order with β=0.7, T=10 time-step memory in *ms* units are given in Figs. A.9.1. and A.9.2. that well reveal the small degradation in the tracking accuracy and the smoothing effects in the adaptive control, too. The tracking errors are detailed in Figs. A.9.3. and A.9.4.

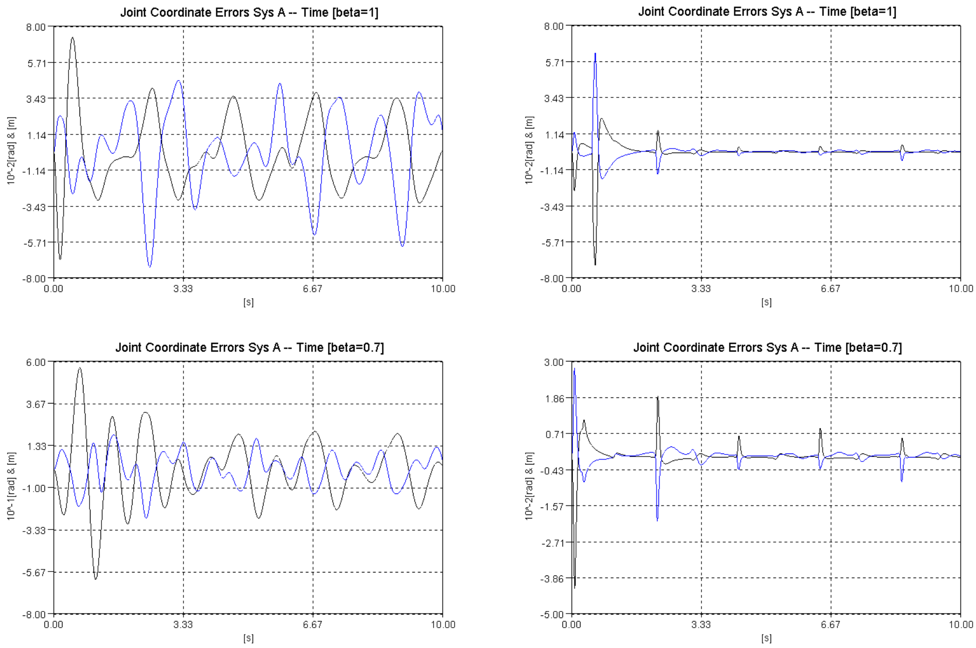


Figure A.9.3. The tracking error of subsystem A: non-adaptive integer order (upper left corner), adaptive integer order (upper right corner), non-adaptive fractional order (lower left corner), and adaptive fractional order (lower right corner)

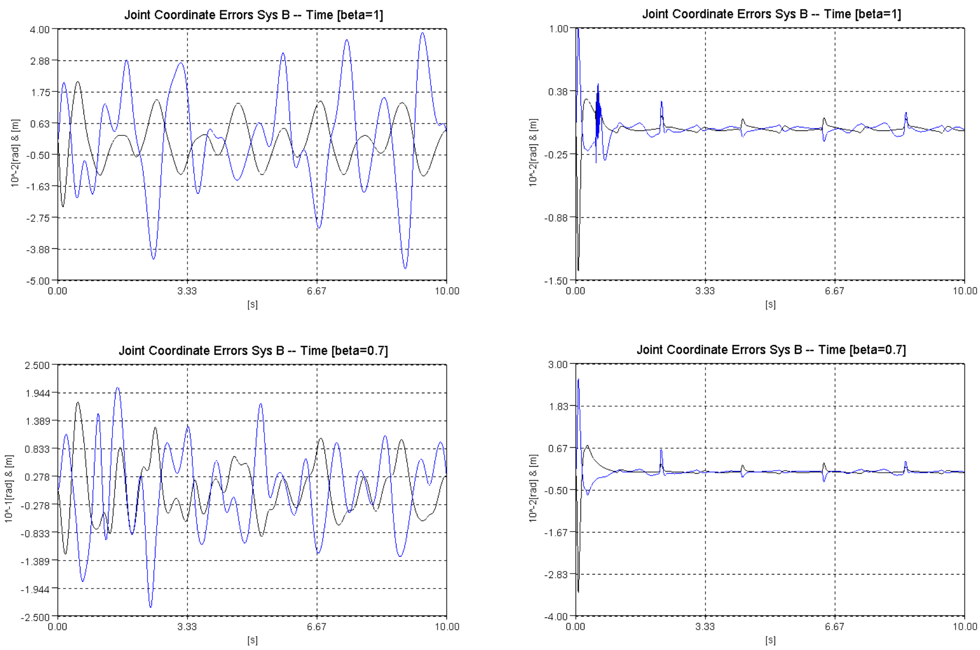


Figure A.9.4. The tracking error of subsystem B: non-adaptive integer order (upper left corner), adaptive integer order (upper right corner), non-adaptive fractional order (lower left corner), and adaptive fractional (lower right corner)

A.10. Geometric Analogies by Fundamental Quadratic Forms

In this paragraph strict analogies between three different geometries frequently occurring in natural and technical sciences are considered. These are the *Euclidean Geometry*, the *Minkowski Geometry*, and the *Symplectic Geometry*. Each of them is defined by a *fundamental quadratic expression* having different physical interpretation. The strict analogies are revealed by considering them as different representatives of the concept of *Lie Groups*.

A.10.1. The Euclidean Geometry:

The fundamental quadratic expression is the *Scalar Product* of the vectors **a** and **b**:

$$\mathbf{a}^T\mathbf{I}\mathbf{b} \tag{A.10.1}$$

that is interpreted by the absolute values of the vectors (more precisely by their norms as introduced by Frobenius), and the angle φ between these vectors as $|\mathbf{a}|\cdot|\mathbf{b}|\cdot\cos\varphi(\mathbf{a},\mathbf{b})$. The quadratic matrix defining this quadratic expression in (A.10.1) is the unit matrix **I**. *The linear transformations of the vectors as* **a'=Oa**, and **b'=Ob** that *leave the form as well as the numerical value of the scalar product for arbitrary* **a**, **b** *vectors invariant, that is for which*

$$\mathbf{a'}^T\mathbf{I}\mathbf{b'}=\mathbf{a}\mathbf{O}^T\mathbf{I}\mathbf{O}\mathbf{b} \Rightarrow \mathbf{I}=\mathbf{O}^T\mathbf{I}\mathbf{O} \tag{A.10.2}$$

are referred to as the ***Orthogonal Transformations****. These transformations describe one of the fundamental symmetries of Euclidean Geometry.*

A.10.2. The Minkowski Geometry:

A fundamental experimental observation in Electrodynamics (the Michelson-Morley Experiment) postulated that it is possible to so set the clocks and distance measures in inertial frames (i.e. bringing about systems of coordinates) in the measures of which the velocity of the light signals in each direction is c. By introducing the *four component vectors describing the separation of two events in space and time* as $\mathbf{x}=[\Delta\mathbf{r},\Delta t]^T$, *the fundamental quadratic expression of Electrodynamics* can be introduced by the *diagonal matrix* $\mathbf{g}:=<1,1,1,-c^2>$

$$\mathbf{x}^T\mathbf{g}\mathbf{x} \tag{A.10.3}$$

that is positive number for events that can be connected by signals having lower speed of propagation than that of the light signals in vacuum, exactly zero if light signals can connect the two events, and are negative number if signal of higher speed than c is needed for connecting these events. The above form can be extended for different **x** and **y** vectors as $\mathbf{x}^T\mathbf{g}\mathbf{y}$ that is called as the *scalar product of four dimensional vectors in the Minkowski Geometry. The linear transformations of the vectors as* **x'=Λa**, and **y'=Λy** that *leave the form as well as the numerical value of the scalar product for arbitrary* **x**, **y** *vectors invariant, that is for which*

$$\mathbf{x'}^T\mathbf{g}\mathbf{y'}=\mathbf{x}\Lambda^T\mathbf{g}\Lambda\mathbf{y} \Rightarrow \mathbf{g}=\Lambda^T\mathbf{g}\Lambda \tag{A.10.4}$$

are referred to as the ***Lorentz Transformations****. These transformations describe one of the fundamental symmetries of Minkowski Geometry.*

A.10.3. The Symplectic Geometry:

In the Canonical Equations of Motion of Classical Mechanics [R25] a quadratic expression occurs in the *Poisson Bracket* that describes the time-derivatives of physical quantities depending exceptionally only on the physical state of the isolated mechanical system. By writing arrays of 2×*DOF* dimensions (*DOF=Degree of Freedom* of the mechanical system) strict analogy of (A.10.1) can be obtained as

$$\mathbf{u}^T\mathfrak{I}\mathbf{v},\ \ \mathfrak{I}:=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{I} & \mathbf{0}\end{bmatrix} \tag{A.10.5}$$

The linear transformations of the vectors as **u'=Su**, and **v'=Sv** that *leave the form as well as the numerical value of* (A.10.5) *for arbitrary* **u**, **v** *"vectors" invariant, that is for which*

$$\mathbf{u'}^T\mathfrak{I}\mathbf{v'}=\mathbf{u}\mathbf{S}^T\mathfrak{I}\mathbf{S}\mathbf{v}\Rightarrow\mathfrak{I}=\mathbf{S}^T\mathfrak{I}\mathbf{S} \tag{A.10.6}$$

are referred to as the ***Symplectic Transformations****. These transformations describe one of the fundamental symmetries of Classical Mechanics.* We note that the definitions in (A.10.6) and the definition $\mathbf{S}\mathfrak{I}\mathbf{S}^T=\mathfrak{I}$ occurring in Classical Mechanics are essentially equivalent to each other because from $\mathfrak{I}^2=-\mathbf{I}$ it follows that $\mathfrak{I}^{-1}=\mathfrak{I}^T$, therefore if $\mathbf{S}\mathfrak{I}\mathbf{S}^T=\mathfrak{I}$, then $\mathfrak{I}^T\mathbf{S}\mathfrak{I}\mathbf{S}^T=\mathbf{I}$ that means that $\mathbf{S}^T=(\mathfrak{I}^T\mathbf{S}\mathfrak{I})^{-1}$ or $\mathbf{S}^T=\mathfrak{I}^T\mathbf{S}^{-1}\mathfrak{I}$, therefore $\mathbf{S}^T\mathfrak{I}\mathbf{S}=\mathfrak{I}^T\mathbf{S}^{-1}\mathfrak{I}\mathfrak{I}\mathbf{S}=-\mathfrak{I}^T\mathbf{S}^{-1}\mathbf{I}\mathbf{S}=\mathfrak{I}$, too.

A.10.4. Analogies on the Basis of Group Theory:

To establish formal analogies at first we note that none of the matrices defining the fundamental quadratic expressions is singular. This statement is trivial for **I** of the Euclidean Geometry, and for the diagonal **g** in (A.10.3). However, it is easy to calculate det$\mathfrak{I}$ for arbitrary size on the basis of its definition:

$$\begin{aligned}\det\mathfrak{I}:=&\sum_{i_1,\dots,i_n,i_{n+1},\dots,i_{2n}}\varepsilon_{i_1,\dots,i_n,i_{n+1},\dots,i_{2n}}\mathfrak{I}_{1,i_1}\dots\mathfrak{I}_{n,i_n}\mathfrak{I}_{n+1,i_{n+1}}\dots\mathfrak{I}_{2n,i_{2n}}=\\ &=\varepsilon_{n+1,\dots,i_n,1,\dots,i_{2n}}\mathfrak{I}_{1,n+1}\dots\mathfrak{I}_{n,2n}\mathfrak{I}_{n+1,1}\dots\mathfrak{I}_{2n,n+1}=\\ &=\varepsilon_{n+1,\dots,i_n,1,\dots,i_{2n}}1^n(-1)^n=\varepsilon_{1,\dots,i_n,n+1,\dots,i_{2n}}1^n(-1)^{n-1}=\dots=1\end{aligned} \tag{A.10.7}$$

In the 2nd line it is taken into account that only n ones ad (-1) matrix elements occur in $\mathfrak{I}$, and the effect of the multiplication factor $(-1)^n$ is just compensated by the by the n number of index swapping in the Levi-Cività symbol to arrive to $\varepsilon_{1,2,\dots,2n}=1$. From the nonsingular value of the defining matrix immediately follows that the transformation matrices cannot be singular, moreover they may have the determinant ±1 [e.g. $\det(\mathbf{S}^T\mathfrak{I}\mathbf{S})=\det\mathfrak{I}\Rightarrow\det\mathbf{S}=\pm1$.

The *associativity of the matrix product* guarantees that the symmetry transformations considered satisfy the group properties, e.g. $(\mathbf{O}^{(1)}\mathbf{O}^{(2)})^T\mathbf{I}(\mathbf{O}^{(1)}\mathbf{O}^{(2)})=\mathbf{O}^{(2)\mathrm{T}}\mathbf{O}^{(1)\mathrm{T}}\mathbf{I}\mathbf{O}^{(1)}\mathbf{O}^{(2)}=\mathbf{O}^{(2)T}(\mathbf{O}^{(1)\mathrm{T}}\mathbf{I}\mathbf{O}^{(1)})\mathbf{O}^{(2)}=\mathbf{O}^{(2)\mathrm{T}}\mathbf{I}\mathbf{O}^{(2)}=\mathbf{I}$, the unit matrix **I** is evidently included in the set of each symmetry transformation, the existence of the inverse matrices and that the left and right hand side inverses are identical to each other as well as the membership of the inverses in the group elements in the group follow from the properties of the matrix product.

Taking into account, that the determinant is continuous function of the matrix elements and det**I**=1 only the matrices with the determinant +1 can be continuously connected with the unit matrix, therefore only the *unimodular symmetry* *transformations form a Lie Group*. The generators and the appropriate exponentials can be calculated as in the case of the Orthogonal Group.

A particularly interesting but not very strict "analogy" between Euclidean and Symplectic Geometries are the concepts of *orthogonal vectors* ($\mathbf{a}$ is orthogonal to $\mathbf{b}$ in the Euclidean Geometry if $\mathbf{a}^T\mathbf{I}\mathbf{b}=0$) *and antiorthogonal vectors* ($\mathbf{u}$ is antiorthogonal to $\mathbf{v}$ in the Symplectic Geometry if $\mathbf{u}^T\mathfrak{J}\mathbf{v}=0$), the notion of *orthogonal* and *antiorthogonal linear subspaces* [for arbitrary $\alpha, \beta\in\Re$ if $\mathbf{a}$ and $\mathbf{b}$ is orthogonal to $\mathbf{c}$ then $\alpha\mathbf{a}+\beta\mathbf{b}$ is also orthogonal to $\mathbf{c}$ since $(\alpha\mathbf{a}+\beta\mathbf{b})^T\mathbf{c}==\alpha\mathbf{a}^T\mathbf{c}+\beta\mathbf{b}^T\mathbf{c}=0+0=0$; if $\mathbf{a}$ and $\mathbf{b}$ is antiorthogonal to $\mathbf{c}$ then $\alpha\mathbf{a}+\beta\mathbf{b}$ is also antiorthogonal to $\mathbf{c}$ since $(\alpha\mathbf{a}+\beta\mathbf{b})^T\mathfrak{J}\mathbf{c}=\alpha\mathbf{a}^T\mathfrak{J}\mathbf{c}+\beta\mathbf{b}^T\mathfrak{J}\mathbf{c}=0+0=0$]. As in the case of the *Euclidean Geometry* it is the simplest and most convenient way to use *orthonormal basis vectors* (by definition $\mathbf{e}^{(i)T}\mathbf{I}\mathbf{e}^{(j)}=\delta_{ij}$) for representing various vectors, in the case of the *Symplectic Geometry* it is the most expedient choice is the use of *symplectic basis vectors* (by definition $\mathbf{f}^{(i)T}\mathfrak{J}\mathbf{f}^{(j)}=\mathfrak{J}_{ij}$) since in the first case we normally have to work with scalar products, while in the second one normally evaluation the Poisson Brackets is needed, and these expressions can very conveniently be evaluated by using orthonormal/symplectic basis vectors.

As in the case of the Euclidean Geometry by the use of the Gram-Schmidt Algorithm it is very easy to create orthonormal basis vectors from arbitrary but sufficient set of linearly independent vectors, using the concept of antiorthogonal subspaces it is very easy to create symplectic set, too [for details see Table A.10.1. below].

The Gram-Schmidt Algorithm	**The Symplectizing Algorithm**
Let $\{\mathbf{a}^{(i)}\vert i=1,...,n\}$ a linearly independent set of basis vectors.	Let $\{\mathbf{b}^{(i)}\vert i=1,...,2n\}$ a linearly independent set of basis vectors.
Since $\mathbf{a}^{(1)}\neq\mathbf{0}$ it can be *normed* for forming the *first element* of the orthonormal set $\mathbf{e}^{(1)}:=\mathbf{a}^{(1)}/\Vert\mathbf{a}^{(1)}\Vert$.	Since $\mathfrak{J}$ is non-singular, none of the $\mathfrak{J}\mathbf{b}^{(j)}$ (j=1,...,2n) vectors can be zero. Due to its skew-symmetry $\mathbf{b}^{(1)T}\mathfrak{J}\mathbf{b}^{(j)}=0$, therefore the remaining set must contain at least one vector, $\mathbf{c}$, for which $\mathbf{b}^{(1)T}\mathfrak{J}\mathbf{c}\neq0$. Via permutation of the remaining vectors let the index "n+1" assigned to it. Via the normalization $\mathbf{b}'^{(n+1)}:=\mathbf{b}^{(n+1)}/[\mathbf{b}^{(1)T}\mathfrak{J}\mathbf{b}^{(n+1)}]$, the *symplectic "mate"* of $\mathbf{b}^{(1)}$ is obtained.
Those $\mathbf{a}^{(j)}$ vectors of the remaining set which are not orthogonal to $\mathbf{e}^{(1)}$ can be made orthogonal to it by the transformation $\mathbf{a}'^{(j)}:=\mathbf{a}^{(j)}-\mathbf{e}^{(1)}[\mathbf{e}^{(1)T}\mathbf{a}^{(j)}]\neq0$.	Those $\mathbf{b}^{(j)}$ vectors of the remaining set which are not anti-orthogonal to the pair $\mathbf{b}^{(1)}$ and $\mathbf{b}'^{(n+1)}$ can be made anti-orthogonal to them by the transformation $\mathbf{b}'^{(j)}=\mathbf{b}^{(j)}+\mathbf{b}^{(1)}[\mathbf{b}^{(n+1)T}\mathfrak{J}\mathbf{b}^{(j)}]-\mathbf{b}^{(n+1)}[\mathbf{b}^{(1)T}\mathfrak{J}\mathbf{b}^{(j)}]$.
Due to the completeness and linear independence of the original set of vectors the transformed remaining set must consist of (n-1) linearly independent non-zero vectors each of which is orthogonal to $\mathbf{e}^{(1)}$.	Due to the completeness and linear independence of the original set the remaining set must consist of (2n-2) non-zero, linearly independent vectors each of which is anti-orthogonal to the pair $\mathbf{b}^{(1)}$ and $\mathbf{b}'^{(n+1)}$.
The above steps can be repeated within the linear sub-space orthogonal to $\mathbf{e}^{(1)}$.	The above steps can be repeated within the linear sub-space anti-orthogonal to the pair $\mathbf{b}^{(1)}$ and $\mathbf{b}'^{(n+1)}$.

The final result is an ***orthonormal*** *set of basis vectors.*	*The final result is a* ***symplectic*** *set of basis vectors.*

Table A.10.1. The formal analogy between the Gram-Schmidt Algorithm in Hilbert Spaces and the Symplectizing Algorithm

A.11. Geometric Interpretation of Real SVD

Though the general theory of *Singular Value Decomposition (SVD)* has been elaborated for *complex matrices* by the use of *unitary transformations*, for the purposes of the present work it is satisfactory to restrict ourselves to *real matrices* that can be tackled by *orthogonal transformations.* The geometric interpretation of *SVD* is strictly related to spanning the *input* and the *output* space of *abstract linear transformations* by *orthonormal basis vectors.* Consider the $\boldsymbol{\mathcal{b}}=\boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{a}}$ linear transformation in which the dimensions of the input and the output spaces may differ from each other. Let the orthonormal set $\{\boldsymbol{e}^{(i)}\}$ be defined in the input, and $\{\boldsymbol{\mathcal{f}}^{(i)}\}$ be defined in the output spaces. Necessarily

$$\boldsymbol{a}=\sum_i a_i\boldsymbol{e}^{(i)},\ \boldsymbol{\mathcal{L}}\boldsymbol{a}=\sum_i a_i\boldsymbol{\mathcal{L}}\boldsymbol{e}^{(i)} \tag{A.11.1}$$

The $\{a_i\}$ coefficients can be obtained by considering scalar products utilizing the orthonormality of the set $\{\boldsymbol{e}^{(i)}\}$:

$$\left(\boldsymbol{e}^{(k)},\boldsymbol{a}\right)=\left(\boldsymbol{e}^{(k)},\sum_i a_i\boldsymbol{e}^{(i)}\right)=\sum_i\underbrace{\left(\boldsymbol{e}^{(k)},\boldsymbol{e}^{(i)}\right)}_{\delta_{ki}}a_i=a_k \tag{A.11.2}$$

The transformed vector can similarly be computed by using the basis $\{\boldsymbol{\mathcal{f}}^{(i)}\}$:

$$\boldsymbol{\mathcal{b}}=\sum_s b_s\boldsymbol{\mathcal{f}}^{(s)}=\boldsymbol{\mathcal{L}}\boldsymbol{a}=\sum_i a_i\boldsymbol{\mathcal{L}}\boldsymbol{e}^{(i)},\quad b_k=\sum_i a_i\underbrace{\left(\boldsymbol{\mathcal{f}}^{(k)},\boldsymbol{\mathcal{L}}\boldsymbol{e}^{(i)}\right)}_{L_{ki}}=\sum_i L_{ki}a_i \tag{A.11.3}$$

Instead of the original basis $\{\boldsymbol{e}^{(i)}\}$ a new basis can also be used the rigid rotation $\boldsymbol{\mathcal{U}}$ of which corresponds to the old one, i.e. $\{\boldsymbol{e}^{(i)}\}=\{\boldsymbol{\mathcal{U}}\boldsymbol{e}'^{(i)}\}$. Since the rotation around the origin is a linear transformation [$\boldsymbol{\mathcal{U}}(\alpha\boldsymbol{a})=\alpha\boldsymbol{\mathcal{U}}\boldsymbol{a}$ and $\boldsymbol{\mathcal{U}}(\boldsymbol{a}+\boldsymbol{\mathcal{b}})=\boldsymbol{\mathcal{U}}\boldsymbol{a}+\boldsymbol{\mathcal{U}}\boldsymbol{\mathcal{b}}$] the elements of the rotated basis can be computed by the linear combination of the elements of the unrotated basis, that is $\boldsymbol{e}^{(i)}=\sum_z U_{iz}\boldsymbol{e}'^{(z)}$, therefore

$$b_k=\sum_{i,z}a_iU_{iz}\left(\boldsymbol{\mathcal{f}}^{(k)},\boldsymbol{\mathcal{L}}\boldsymbol{e}'^{(z)}\right) \tag{A.11.4}$$

In similar manner a new orthonormal basis can be also introduced in the output space as $\boldsymbol{\mathcal{f}}^{(k)}=\sum_v V_{kv}\boldsymbol{\mathcal{f}}'^{(v)}$ leading to

$$b_k=\sum_{i,z,v}a_iU_{iz}V_{kv}\underbrace{\left(\boldsymbol{\mathcal{f}}'^{(v)},\boldsymbol{\mathcal{L}}\boldsymbol{e}'^{(z)}\right)}_{L'_{vz}}=\sum_{i,z,v}V_{kv}L'_{vz}\left(U^T\right)_{zi}a_i \tag{A.11.5}$$

that simply sets the transformation rule of the matrix elements of the linear operators when choosing various orthonormal basis sets in the input and the output spaces. Rotations may not have arbitrary matrix structure. Since the scalar products must be left invariant $(\boldsymbol{\mathcal{U}}\boldsymbol{e}'^{(i)},\boldsymbol{\mathcal{U}}\boldsymbol{e}'^{(j)})=(\boldsymbol{e}'^{(i)},\boldsymbol{e}'^{(j)})$

$$\underbrace{\left(\boldsymbol{e}^{(i)},\boldsymbol{e}^{(k)}\right)}_{\delta_{i,k}}=\sum_{s,t}U_{is}U_{kt}\underbrace{\left(\boldsymbol{e}'^{(s)},\boldsymbol{e}'^{(t)}\right)}_{\delta_{s,t}}\Rightarrow \mathbf{U}\mathbf{U}^T=\mathbf{I} \qquad \text{(A.11.6)}$$

that means that the matrices **U** and **V** must be *orthogonal matrices*. So the transformation rule of the linear operators in (A.11.5) means that $\mathbf{L}=\mathbf{V}\mathbf{L}'\mathbf{U}^T$ in which **V** and **U** are orthogonal matrices that describe the effect of changing the orthonormal basis vector sets in the matrix elements.

For a given $\{\boldsymbol{e}'^{(i)}\}$ and $\{\boldsymbol{f}'^{(i)}\}$ set **L'** may be "complicated". For better understanding the properties of the abstract linear operator $\boldsymbol{\mathcal{L}}$ it would be expedient to choose special basis vectors in which **L** has the simplest possible form. It is easy to see that it is possible to find diagonal **L** if we consider $\mathbf{L}^T\mathbf{L}=\mathbf{U}\mathbf{L}'^T\mathbf{L}'\mathbf{U}^T$ and $\mathbf{L}\mathbf{L}^T=\mathbf{V}\mathbf{L}'\mathbf{L}'^T\mathbf{V}^T$ that means that the symmetric, generally positive semidefinite real matrices as $\mathbf{L}'^T\mathbf{L}'$ and $\mathbf{L}'\mathbf{L}'^T$ have to be diagonalized by the appropriate orthogonal matrices. This certainly can be done if we choose the normalized eigenvectors of these matrices to serve as the columns of these orthogonal matrices (the eigenvectors belonging to different eigenvalues must be orthogonal to each other, while in the linear subspace of the eigenvectors belonging to the same eigenvalue orthogonal ones can be found and chosen). In the diagonal form of **L** *in the main diagonals the square roots of the appropriate eigenvalues have to stand.*

According to [R48] the standard procedure of diagonalizing *real, symmetric, positive semidefinite matrices* can be solved by efficient *numerical techniques.* Taking into account that any **O** orthogonal matrix of appropriate size leaves the eigenvalues of the real matrices invariant $\det(\mathbf{A}-\lambda\mathbf{I})=0$ and $\det(\mathbf{O})=\det(\mathbf{O}^T)=\pm1 \Rightarrow \det(\mathbf{O})\det(\mathbf{A}-\lambda\mathbf{I})\det(\mathbf{O}^T)=\det(\mathbf{O}\mathbf{A}\mathbf{O}^T-\lambda\mathbf{O}\mathbf{I}\mathbf{O}^T)=\det(\mathbf{O}\mathbf{A}\mathbf{O}^T-\lambda\mathbf{I})=0$ at first *orthogonal transformations* constructed of properly chosen diadic terms as $(\mathbf{I}-2\mathbf{u}\mathbf{u}^T)$ are chosen to convert the original symmetric real, positive semidefinite matrix into continuant matrix. Then the eigenvalue and eigenvector problem of such special matrices can efficiently be solved *numerically*. Following that the eigenvectors can be properly transformed since if $\mathbf{A}\mathbf{a}=\lambda\mathbf{a}$, then $\mathbf{O}\mathbf{A}\mathbf{a}=\mathbf{O}\mathbf{A}\mathbf{O}^T\mathbf{O}\mathbf{a}=\lambda\mathbf{O}\mathbf{a}$.

In our days *SVD* is a standard service (function) of software designed for the use in research, as e.g. INRIA's SCILAB. By using the coordinate representations of the abstract transformation $\boldsymbol{b}=\boldsymbol{\mathcal{L}a}$ the diagonalized version takes the form of

$$\begin{aligned}
\mathbf{b}=\mathbf{V}\mathbf{D}\mathbf{U}^T\mathbf{a} &= \left[\mathbf{v}^{(1)}\mid\cdots\mid\mathbf{v}^{(n)}\right]\begin{bmatrix} D_{11} & 0 & 0\\ \vdots & \ddots & \vdots\\ 0 & 0 & D_{nm}\end{bmatrix}\begin{bmatrix}\mathbf{u}^{(1)T}\\ \hline \cdots\\ \hline \mathbf{u}^{(m)T}\end{bmatrix}\mathbf{a}=\\
&=\left[D_{11}\mathbf{v}^{(1)}\mid\cdots\mid D_{kk}\mathbf{v}^{(k)}\mid\cdots\right]\begin{bmatrix}\left(\mathbf{u}^{(1)T},\mathbf{a}\right)\\ \hline \cdots\\ \hline \left(\mathbf{u}^{(m)T},\mathbf{a}\right)\end{bmatrix}= \qquad \text{(A.11.7)}\\
&=\left(\mathbf{u}^{(1)T},\mathbf{a}\right)D_{11}\mathbf{v}^{(1)}+\ldots+\left(\mathbf{u}^{(k)T},\mathbf{a}\right)D_{kk}\mathbf{v}^{(k)}
\end{aligned}$$

in which $k=\min(n,m)$, and in the central line following D_{kk} in "$[\ldots|D_{kk}|\ldots]$" either nothing stands or zeros are located. The geometric interpretation of (A.11.7) is straightforward: characteristic pairs of orthogonal directions are found in the *input* and the *output spaces* to which characteristic stretch/shrink denoted by the *singular values* $D_{ii}\geq0$ belong.

Publications Related to the Dissertation

Book Excerpts

[B1] J.K. Tar, O. Kaynak, I.J. Rudas, J.F. Bitó: "The Use of Partially Decoupled Uniform Structures and Procedures for the Robust and Adaptive Control of Mechanical Devices", Recent Advances in Mechatronics, (O. Kaynak, S. Tosunoglu, M. Ang editors) Springer-Verlag Singapore Pte. Ltd. 1999, pp. 138-151.

[B2] J.F. Bitó, J.K. Tar, I.J. Rudas: "Novel Adaptive Control of Mechanical Systems Driven by Electromechanical Hydraulic Drives", Knowledge and Technology Integration in production and Series Balancing Knowledge and Technology in Product and Service Life Cycle, Ed. V. Marik, Kluwer Academic Publishers, USA, 2002, pp. 518-532.

[B3] József K. Tar, Imre J. Rudas, János F. Bitó, K. Kozlowski: "A New Approach in Computational Cybernetics Based on the Modified Renormalization Algorithm Guaranteeing Complete Stability in the Control of a Wide Class of Physical Systems", Intelligent Systems at the Service of Mankind Vol. I, pp. 3-14 (Eds.: Wilfried Elmenreich, J. Tenreiro Machado, and Imre J. Rudas), Ubooks 2004, Printed in Germany, ISBN: 3-935798-25-3.

[B4] József K. Tar, Imre J. Rudas, János F. Bitó, J. Tenreiro Machado: "Improved Numerical Simulation for a Novel Adaptive Control Using Fractional Order Derivatives", Intelligent Systems at the Service of Mankind-vol. 2, Ubooks, Germany, 2005, W. Elmenreich, J. Tenreiro Machado, I.J. Rudas (Eds), pp. 283-294, ISBN 3-86608-052-2.

[B5] József K. Tar, Imre J. Rudas, János F. Bitó, J. Tenreiro Machado: "Simple Kinematic Design for Evading the Forced Oscillation of a Car-Wheel Suspensionn System", Intelligent Systems at the Service of Mankind-vol. 2, Ubooks, Germany, 2005, W. Elmenreich, J. Tenreiro Machado, I.J. Rudas (Eds), pp. 161-171, ISBN 3-86608-052-2.

[B6] József K. Tar, Imre J. Rudas, Ágnes Szeghegyi and Krzysztof Kozłowski: "Novel Adaptive Control of Partially Modeled Dynamic Systems", in Lecture Notes in Control and Information Sciences, Springer Berlin/Heidelberg, Robot Motion and Control: Recent Development, Part II - Control and Mechanical Systems, Ed. Krzysztof Kozlowski, Volume 335 / 2006, pp. 99 – 111, ISBN: 1-84628-404-X.

[B7] József K. Tar, Imre J. Rudas and Krzysztof R. Kozłowski: "Fixed Point Transformations-Based Approach in Adaptive Control of Smooth Systems", (delivered at the 6th International Workshop on Robot Motion and Control, June 11-13, 2007, Bukowy Dworek, Poland);in Lecture Notes in Control and Information Sciences 360 (Eds.: M. Thoma and M. Morari), Robot Motion and Control 2007 (Ed.: Krzysztof R. Kozłowski), pp. 157-166, Springer Verlag London Ltd. 2007, ISBN-13:978-1-84628-973-6.

[B8] József K. Tar and Imre J. Rudas: "Fixed Point Transformations Based Iterative Control of a Polymerization Reaction", in Intelligent Engineering Systems and Computational Cybernetics (Eds. J.A. Tenreiro Machado, Imre J. Rudas, Béla Pátkai), Springer Science+Business Media B.V., 2008, pp. 293-303 [ISBN 978-1-4020-8677-9, e-ISBN 978-1-4020-8678-6, Library of Congress Control Number: 2008934137].

Journal Publications

[J1] I.J. Rudas, J.F. Bitó, J.K. Tar: “An Advanced Robot Control Scheme Using ANN and Fuzzy Theory Based Solutions”, Robotica (1996) Vol. 14, pp. 189-198.

[J2] J.K. Tar, I.J. Rudas, J.F. Bitó: “Group Theoretical Approach in Using Canonical Transformations and Symplectic Geometry in the Control of Approximately Modeled Mechanical Systems Interacting with Unmodelled Environment”, Robotica, Vol. 15, pp. 163-179, 1997.

[J3] J.K. Tar, I.J. Rudas, L. Madarász, J.F. Bitó: “Simultaneous Optimization of the External Loop Parameters in an Adaptive Control Based on the Co-operation of Uniform Procedures”, Journal of Advanced Computational Intelligence, Vol. 4, No. 4, 2000, pp. 279-285.

[J4] J.K. Tar, I.J. Rudas, J.F. Bitó, O.M. Kaynak: ”A New Utilization of the Hamiltonian Formalism in the Adaptive Control of Mechanical Systems under External Perturbation” Intelligent Automation and Soft Computing, Vol.5, Issue:4, 1999, pp. 303-312.

[J5] J.K. Tar, M. Rontó: “Adaptive Control Based on the Application of Simplified Uniform Structures and Learning Procedures”, Zbornik Radova, Vol. 24 No. 2, 2000, pp. 174-194. (ISSN: 0351-1804)

[J6] József K. Tar, Imre J. Rudas, János F. Bitó, Ladislav Madarász: “An Emerging Branch of Computational Cybernetics Dedicated to the Solution of Reasonably Limited Problem Classes”, AT & P Journal Plus2 2001, pp 19-25.

[J7] J.K. Tar, I.J. Rudas, J.F. Bitó, S.J. Torvinen: “Symplectic Geometry Based Simple Algebraic Possibilities for Developing Adaptive Control for Mechanical Systems”, Journal of Advanced Computational Intelligence (JACI), Vol. 5, No. 4, 2001, pp. 254-262.

[J8] József K. Tar, Imre J. Rudas, J.F. Bitó, Krzysztof Kozlowski: “Special Symplectic Transformations Used in Nonlinear System’s Control”, International Journal of Mechanics and Control, Vol. 04 No 02 – 2003, ISSN 1590-8844, pp. 3-8.

[J9] József K. Tar, Imre J. Rudas, János F. Bitó: “Simulation Based Verification of the Applicability of a Novel Branch of Computational Cybernetics in the Adaptive Control of Imperfectly Modeled Physical Systems of Asymmetric Delay Time and Strong Non-linearities”, Acta Polytechnica Hungarica, issued by Budapest Polytechnic, 2004 May, No. 1, pp. 26-43., ISSN 1785-8860.

[J10] József K. Tar, Imre J. Rudas, Ágnes Szeghegyi, Krzysztof Kozłowski: “Non-Conventional Processing of Noisy Signal in the Adaptive Control of Hydraulic Differential Servo Cylinders”, in the Special Issue on Signal Processing, IEEE Transactions on Instrumentation and Measurement, Vol. 54, No. 6, pp. 2176-2179, December 2005, ISSN 0018-9456.

[J11] József K. Tar, Imre J. Rudas, Stefan Preitl, Radu-Emil Precup: “Robust, Potential Limited Control for an Indirectly Driven Saturated System”, Buletinul Stiintific al Universitatii “Politehnica” din Timişoara, Romania, Seria Automatica si Calculatore, Scientific Bulletin of “Politehnica” University of Timişoara, Romania, Transactions on Automatic Control and Computer Science, Vol. 51 (65), No. 1, 2006, ISSN 1224-600X.

[J12] József K. Tar, Imre J. Rudas, Miklós Rontó: “Geometric Identification and Control of Nonlinear Dynamics Based on Floating Basis Vector

Representation", Journal of Advanced Computational Intelligence and Intelligent Informatics, Vol. 10 No.4, 2006, pp. 542-548.

[J13] József K. Tar, Imre J. Rudas and Béla Pátkai: "Comparison of Fractional Robust and Fixed Point Transformations Based Adaptive Compensation of Dynamic Friction", Journal of Advanced Computational Intelligence and Intelligent Informatics, Vol. 11 No. 9, 2007.

[J14] J.K. Tar, I.J. Rudas, Gy. Hermann, J.F. Bitó, J.A. Tenreiro Machado: "On the Robustness of the Slotine-Li and the FPT/SVD-based Adaptive Controllers", WSEAS Transactions on Systems and Control, Issue 9, Volume 3, September 2008, pp. 686-700, ISSN: 1991-8763.

[J15] József K. Tar, János F. Bitó, László Nádai, José A. Tenreiro Machado: "Robust Fixed Point Transformations in Adaptive Control Using Local Basin of Attraction", Acta Polytechnica Hungarica, Vol. 6 Issue No. 1 2009, pp. 21-37, ISSN:1785-8860.

[J16] József K. Tar, János F. Bitó, Csaba Ráti: "Avoiding the saturation and resonance effects via simple adaptive control of an electrically driven vehicle using omnidirectional wheels", Acta Technica Jaurinensis Vol. 2 No. 2, 2009, pp. 217-231, ISSN 1789-6932.

[J17] J.A. Tenreiro Machado, Alexandra M. Galhano, Anabela M. Oliveira, József K. Tar: "Approximating fractional derivatives through the generalized mean", Commun Nonlinear Sci Numer Simulat (2009), doi:10.1016/j.cnsns.2009.03.004

[J18] József K. Tar, Imre J. Rudas: "Adaptive Optimal Dynamic Control for Nonholonomic Systems", Computing and Informatics, Vol. 28, 2009, 1001–1013, V 2009-Apr-6

[J19] J.K. Tar: Robust Fixed Point Transformations Based Adaptive Control of an Electrostatic Microactuator, Acta Electrotechnica et Informatica, Vol. 10, No. 1, 2010, pp. 18-23

[J20] J.K. Tar: "Replacement of Lyapunov Function by Locally Convergent Robust Fixed Point Transformations in Model-based Control A Brief Summary", Journal of Advanced Computational Intelligence and Intelligent Informatics, Vol. 14 No. 2, pp. 224-236

Publications in Conference Proceedings and Lectures

[C1] J.K. Tar, J.F. Bitó, I.J. Rudas, O. Kaynak: "Robust Robot Controller Improved by Adaptive Loop Based on Abstract Geometric Principles of Partial Identification", Proc. of 11th ISPE/IEE/IFAC International Conference on CAD/CAM, Robotics and Factories of the Future, Pereira, Colombia South America, 1995, pp. 283-287.

[C2] J.K. Tar, O.M. Kaynak, J.F. Bitó, I.J. Rudas: "Generalised Canonical Equations of Classical Mechanics: A New Approach towards the Adaptive Treatment of Unmodelled Dynamics", Proceedings of the International Conference on Recent Advances in Mechatronics (ICRAM'95), 14-16 August, 1995, Istanbul, Turkey, pp. 1092-1097.

[C3] J.K. Tar, O.M. Kaynak, J.F. Bitó, I.J. Rudas, D. Mester: "A New Method for Modelling the Dynamic Robot-Environment Interaction Based on the Generalization of the Canonical Formalism of Classical Mechanics", Proceedings of the First International ECPD Conference, Athens, Greece, 1995, pp. 687-692.

[C4] J.K. Tar, I.J.Rudas, J.F.Bitó, M.O. Kaynak: "Adaptive Robot Control Gained by Partial Identification Using the Advantages of Sympletic Geometry", Proceedings of 1995 IEEE 21st International Conference on Industrial Electronics, Control, and Instrumentation. November, 1995, Orlando, USA, pp 75-80.

[C5] J.K. Tar, J.F. Bitó, I.J. Rudas, M. Dinev: "Towards the Integration of Analytical Modeling and Soft Computing in Adaptive Robot Control: the Symplectic-Fuzzy Approach". In the Proc. of the 5th International Workshop on Robotics in Alpe-Adria-Danube Region, 10-13 June, 1996, Budapest, Hungary, pp. 163-169.

[C6] J.K. Tar, J.F. Bitó, I.J. Rudas, B. Pátkai: "On the Effect of Free Parameter Selection in an Adaptive Robot Control Based on Abstract Geometric Principles". In the Proc. of the 5th International Workshop on Robotics in Alpe-Adria-Danube Region, 10-13 June, 1996, Budapest, Hungary, pp. 171-176.

[C7] J.K. Tar, I.J. Rudas, J.F. Bitó: "Free Parameter Selection Methods in Geometric Adaptive Control of Robots Having Dynamic Interaction with Unmodeled Environment", 7th International Power Electronics & Motion Control Conference, September, 1996, Budapest, Hungary.

[C8] I.J. Rudas, J.K. Tar, J.F. Bitó, M.O. Kaynak: "A Multiple-Purpose Uniform Structure for Use in Dynamic Control of Mechanical Devices", Proceedings of the 1996 Engineering Systems Design and Analysis Conference ESDA, July 1-4, 1996, Montpellier, France. Vol 2. pp. 215-220.

[C9] J.K. Tar, I.J. Rudas, J.F. Bitó, O.M. Kaynak: "Constrained Canonical Transformations Used for Representing System Parameters in an Adaptive Control of Manipulators", in the Proc. of the 8th International Conference on Advanced Robotics (ICAR97), Monterey, California, USA, 1997. July 7-9, pp. 873-878.

[C10] J.K. Tar, I.J. Rudas, J.F. Bitó, Á. Szeghegyi: "Self-Consistent Approach in an Adaptive Robot Control based on Constrained Canonical Transformations", in the proc. of the 6th International Symposium on Robotics in Alpe-Adria-Danube Region (RAAD97), 1997, June 26-28, Cassino, Italy, pp. 230-234, ISBN88-87054-00-2.

[C11] J.K: Tar, I.J. Rudas, J.F. Bitó, O.M. Kaynak "On the Tuning of the Free Parameters of a New Adaptive Robot Control Based on Geometric Principles of Hamiltonian Mechanics", in the Proc. of 1997 IEEE International Symposium on Computational Intelligence in Robotics (CIRA 97), Monterey, California, USA, 10-11 July, 1977, pp. 326-331.

[C12] J.K. Tar, I.J. Rudas, J.F. Bitó, O.M. Kaynak: "A Simple Method for Modeling Robots in Adaptive Control", Proc. of the 4th International Symposium on Methods and Models in Automation and Robotics (MMAR'97), Miedzyzdroje, Poland, 26-29 August, 1997, Vol. 3, pp. 907-912, ISBN 83-87423-30-0

[C13] J.K. Tar, J.F. Bitó, I.J. Rudas, M. Dinev: "Adaptive Tuning in an Extended Parameter Space in a Robot Control Based on the Hamiltonian Mechanics", in the Proc. of the 1997 IEEE Int. Conf. on Intelligent Engineering Systems (INES'97), 15-17 September, 1997, Budapest, Hungary, p. 183 (ISBN 0-7803-3627-5).

[C14] J.K. Tar, I.J. Rudas, J.F. Bitó, B. Pátkai: "New Prospects in the Adaptive Control of Robots Under Unmodeled Environmental Interactions", in the

Proc. of the 23rd International Conference of the Industrial Electronics Society (IECON'97), New Orleans, Lousiana, USA, 1997, p. 1349 (ISBN 0-7803-3932-0).
[C15] J.K. Tar, I.J. Rudas, J.F. Bitó, O.M. Kaynak: "On the Effect of Tuning in an Extended Parameter Space in a Lagrangian Model-Based Adaptive Control", in the Proc. of the 7th International Workshop Robotics in Alpe-Adria-Danube Region (RAAD'98), Smolenice, Slovakia, June 26-28, 1998), pp. 315-320.
[C16] J.K. Tar, O.M. Kaynak, J.F. Bitó, I.J. Rudas, T. Kégl: "The Use of Uniform Structures in Adaptive Dynamic Control of Robots", in the Proc. of IEEE International Symposium on Industrial Electronics (ISIE'98), Pretoria, South Africa, 7-10 July, 1998, Vol. 1, pp. 137-142.
[C17] J.K. Tar, I.J. Rudas, José A. Tenreiro Machado, Okyay M. Kaynak: "On the Tuning of the Lagrangian Parameters in a Uniform Structure in Adaptive Robot Control", in the Proc. of the 2nd IEEE International Conference on Intelligent Processing Systems (ICIPS'98), Gold Coast, Australia, August 4-7, 1998., pp. 599-602.
[C18] J.K. Tar, K. Kozlowski, I.J. Rudas, J.F. Bitó: "Combined Application of Regression Analysis and Approximate Modeling in an Adaptive Control for Mechanical Devices", in the Proc. of the 5th International Symposium on Methods and Models in Automation and Robotics -- MMAR'98, 25-29 August 1998, Miedzyzdroje, Poland, pp. 807-812, ISBN 83-87423-81-5.
[C19] J.K. Tar, J.F. Bitó, O.M. Kaynak, I.J. Rudas: "Application of ANN-like Uniform Structures in Robot Control", in the Proc. of the 1998 IEEE International Conference on Intelligent Engineering Systems (INES'98), September 17-19, 1998, Vienna, Austria, pp. 397-402.
[C20] J.K. Tar, J.F. Bitó, I.J. Rudas, M. Dinev: "Application of Regression Analysis, PID/ST and Uniform Structures in an Adaptive Control for Mechanical Systems" (in Hungarian), in the Proc. of the Measurement and Automation'98 Conference" in the Terms of Applied Information Technology", 3-5 November 1998, Budapest, Hungary, pp. 153-163, (ISBN: 963 8231 90 4).
[C21] J.K. Tar, O.M. Kaynak, J.F. Bitó, I.J. Rudas: "Formally Non-Exact Analytical Modeling of Mechanical Systems and Environmental Interactions in an Adaptive Control", in the Proc. of the 1999 IEEE International Conference on Robotics and Automation (ICRA'99), 10-15 May 1999, Detroit, Michigan, USA, pp. 831-838.
[C22] J.K. Tar, I.J. Rudas, K. Kozlowski, J.F. Bitó: "Global Parameter Optimization in an Adaptive Control Built up of Uniform Structures and Procedures", in the Proc of the "Ninth International Conference on Advanced Robotics ('99 ICAR)", 25-27 October 1999, Tokyo, Japan (accepted for publication)
[C23] J.K. Tar, José A. Tenreiro Machado, I.J. Rudas, János F. Bitó: "An Adaptive Robot Control for Technological Operations Based on Uniform Structures and Reduced Number of Free Parameters", in the Proc. of the 8th International Workshop on Robotics in Alpe-Adria-Danube Region (RAAD'99), 17 - 19 June, 1999, Munich, Germany, pp. 106-111.
[C24] J.K. Tar, L. Horváth, J.A. Tenreiro Machado, I.J. Rudas: "Advanced Control of Robot in Technological Operation", in Proc. of the 3rd IEEE International Conference on Intelligent Engineering Systems 1999 (INES'99), November 1-3 1999, Stará Lesná, Slovakia, pp. 139-144. (ISBN 80-88964-25-3).

[C25] J.K. Tar, I.J. Rudas, L. Madarász, J.F. Bitó: "Simultaneous Optimization of the External Loop Parameters in an Adaptive Control Based on the Co-operation of Uniform Procedures", in Proc. of the 3rd IEEE International Conference on Intelligent Engineering Systems 1999 (INES'99), November 1-3 1999, Stará Lesná, Slovakia, pp. 449-454. (ISBN 80-88964-25-3).
[C26] J.K. Tar, O.M. Kaynak, I.J. Rudas, J.F. Bitó: "Application of Uniform Structures and Passive Compliant Components in an Adaptive Robot Control for Technological Operation", in the Proc. of the 1999 IEEE International Symposium on Industrial Electronics (ISIE'99), 12-16 July, 1999, Bled, Slovenia, Vol. 2, pp.867-871.
[C27] J.K. Tar, I.J. Rudas, J.F. Bitó, K. Kozlowski: "Certain Aspects of Parameter Tuning in the Adaptive Control of Mechanical Systems Based on Uniform Structures and Procedures" in Proc. of the "Jubilee Scientific Conference at Bánki Donát Polytechnic", 1-2 September 1999, Budapest, Hungary, pp. 69-74 (ISBN 963 7154 03 5),
[C28] J.K. Tar, I.J. Rudas, K. Kozlowski, J.F. Bitó "Dynamic Tuning of the Parameters of an Adaptive Control for Mechanical Processes", in the Proceedings of the "25th Annual Conference of the IEEE Industrial Electronics Society (IECON'99)", 29 November-3 December, 1999, San Jose, CA, USA, CD ROM Issue, ISBN 0-7803-5735-3.
[C29] J.K. Tar, K. Kozlowski, I.J. Rudas, L. Horváth: "The Use of Truncated Joint Velocities and Simple Uniformized Procedures in an Adaptive Control of Mechanical Devices", in the Proc. of the First Workshop on Robot Motion and Control (ROMOCO'99), 28-29 June, 1999, Kiekrz, Poland, pp. 129-134., (ISBN 0-7803-5655-1, IEEE Catalog Number: 99EX353).
[C30] J.K. Tar, L. Horváth, J.A. Tenreiro Machado, I.J. Rudas: "Selection of Appropriate Uniform Structures for an Adaptive Control of Robot Under Environmental Interaction", in Proc. of the "Jubilee Scientific Conference at Bánki Donát Polytechnic", 1-2 September 1999, Budapest, Hungary, pp. 147-152.
[C31] J.K. Tar: "Integrated Use of Fuzzy Concepts and Uniform Structures of the Fundamental Groups in Classical Mechanics: an Overview Regarding Adaptive Control" invited paper, in Proc. of the 3rd IEEE International Conference on Intelligent Engineering Systems 1999 (INES'99), November 1-3 1999, Stará Lesná, Slovakia, pp. 27-32. (ISBN 80-88964-25-3).
[C32] J.K. Tar, I.J. Rudas, Touré Balladji: "Minimum Operation Algebraic Adaptive Control for Robot Based on a Novel Branch of Soft Computing", in the Proc. of EUREL '2000, Vol. 2, Session 3, (European Advancded Robotics Systems - Masterclass and Conference - Robotics 2000), 12-14 April 2000, The University of Salford, Manchester, UK.
[C33] J.K. Tar, J.F. Bitó, I.J. Rudas, B. Pátkai: "A Minimum Operation Adaptive Robot Control Using Symplectic Geometry-Based Prediction", in the Proc. of the "9th International Workshop on Robotics in Alpe-Adria-Danube region (RAAD'2000), June 1-3 2000, Maribor, Slovenia, pp. 187-192.
[C34] J.K. Tar, I.J. Rudas, J.F. Bitó: "Application of 'Minimum Symplectic Transformations' in an Adaptive Control for Polishing Operations", In the Proc. of the 4th IEEE / IFIP International Conference on Information Technology for BALANCED AUTOMATION SYSTEMS in Production and Transportation, September 27-29, 2000 Berlin, Germany (BASYS 2000).

[C35] J.K. Tar, I.J. Rudas, K. Kozlowski, G. Molnár: “Improved Adaptive Robot Control Based on Minimum Symplectic Transformations”, (in the proc. of the 6th International Conference on Methods annd Models in Automation and Robotics (MMAR 28-31 Aug. 2000, Miedzyzdroje, Poland), amongst the Invited Sessions.

[C36] J.K. Tar, J.F. Bitó, I.J. Rudas, H.M. Amin Shamshudin: “A New Approach in The Use of Symplectic Geometry as an AI Tool in Robot Control: The Idea of "Minimum Operation Transformations"”, IFAC Symposium on Artificial Intelligence in Real Time Control (AIRTC' 2000), October 2-4, 2000, Budapest, Hungary, preprints, pp. 335-340.

[C37] J.K. Tar, M. Rontó: “Adaptive Control Based on the Application of Simplified Uniform Structures and Learning Procedures”, invited lecture, in the Proc. of the “11th International Conference on Information and Intelligent Systems”, 20-22 September, 2000, University of Zagreb, Faculty of Organization and Informatics, Varazdin, Croatia (a CD issue).

[C38] J.K. Tar, I.J. Rudas, J.F. Bitó, S.J. Torvinen: “Symplectic Geometry Based Simple Algebraic Possibilities for Developing Adaptive Control for Mechanical Systems”, in the Proc. of the 4thd IEEE International Conference on Intelligent Engineering Systems 2000 (INES’2000), Sept. 17-19 2000, Portoroz, Slovenia, pp. 67-70 (ISBN 961-6303-23-6).

[C39] J.K. Tar, J.F. Bitó, I.J. Rudas, S.J. Torvinen: “A New Possible Branch of Computational Intelligence for Developing Real Time Control in Mechatronics”, in the proceedings of the 2000 IREEE International Conference on Industrial Electronics, Control and Instrumentation (IECON 2000) “21st Century Technologies and Industrial Opportunities”, October 22 - 28, 2000 Nagoya Congress Center, Nagoya, Aichi, JAPAN, paper SS16 in CD issue, pp. 912-917, ISBN 0-7803-6459-7.

[C40] J.K. Tar, I.J. Rudas, J.F. Bitó, K. Kozlowski: “Non-Conventional Integration of the Fundamental Elements of Soft Computing and Traditional Methods in Adaptive Robot Control”, in the Proceedings of the 2000 IEEE International Conference on Robotics & Automation (ICRA’2000), San Francisco, CA April 2000, pp. 3531-3536, (CD Issue, ISBN: 0-7803-5889-9).

[C41] József K. Tar, Anikó Szakál, Imre J. Rudas, János F. Bitó: “Selection of Different Abstract Groups for Developing Uniform Structures to be Used in Adaptive Control of Robots”, Proceedings of ISIE, December 4-8, 2000, Puebla, Mexico, pp. 559-564.

[C42] József K. Tar, Imre J. Rudas, János F. Bitó, Karel Jezernik: “A Generalized Lorentz Group-Based Adaptive Control for DC Drives Driving Mechanical Components”, in the Proc. of The 10th International Conference on Advanced Robotics 2001 (ICAR 2001), August 22-25, 2001, Hotel Mercure Buda, Budapest, Hungary, pp. 299-305 (ISBN: 963 7154 05 1).

[C43] J.K. Tar, I.J. Rudas, J.F. Bitó, Á. Szeghegyi: “Integrated Modeling of Mechanical and Electrical Components Using a New Branch of Soft Computing”, in the Proc. of the 10th International Workshop on Robotics in Alpe-Adria-Danube Region (RAAD'2001), Vienna, May 16-18, 2001 (Paper RD-03.pdf, CD issue).

[C44] J.K. Tar, I.J. Rudas, K. Kozlowski, T. Ilkei: “Application of the Symplectic Group in a Novel Branch of Soft Computing-for Controlling of Electro-Mechanical Devices”, in the Proc. of the Second International Workshop On Robot Motion And Control, RoMoCo'001, October 18-20, 2001, Bukowy

Dworek, Poland, pp. 71-77, ISBN: 83-7143-515-0, IEEE catalog Number: 01EX535., F)

[C45] J.K. Tar, I.J. Rudas, J.F. Bitó, P.H. Andersson, S.J. Torvinen: "Structurally and Procedurally Simplified Soft Computing for Real Time Control", The 2001 IEEE International Conference on Robotics and Automation, 21-26 May, 2001 Seoul, Korea.

[C46] J.K. Tar, J.F.Bitó, I.J. Rudas, Á. Szeghegyi: "Formal Application of the Symplectic Group for Soft Computing-Based Control of Mechanical Devices Free of Direct Physical Interpretation", In the Proc. of the 2001 IEEE International Conference on Intelligent Engineering Systems (INES 2001), Helsinki, Finland, September 16-18, 2001, pp.335-340. (ISBN 952 15-0689-X).

[C47] J.K. Tar, J.F. Bitó, I.J. Rudas, K. Jezernik, Seppo J. Torvinen: "A Lorentz-Group Based Adaptive Control for Electro-Mechanical Systems", Proc. of the 2001 IEEE/RSJ International Conference on Intelligent Robots and Systems, October 29-November 3 2001, Maui, Hawaii, USA., pp. 2045-2050, ISBN 0-7803-6614-X, IEEE Catalog No.: 01CH37180C.

[C48] J.K. Tar, J.F.Bitó, I.J. Rudas, K. Jezernik: "Formal Application of Dummy Parameters in a Soft Computing-Based Control of Mechanical Devices", in the Proc. of the The 27th Annual Conf. of the IEEE Ind. El. Soc., Denver, Colorado, USA, Nov 29 - Dec 2, 2001.pp. 220-225.

[C49] J.K. Tar, J.F.Bitó, I.J. Rudas, Seppo J. Torvinen: "Application of a Novel Branch of Soft Computing Aiming at Platform-Independence and Uniformity in the Adaptive Control of Electromechanical Devices", in the Proc. of the 1st International Conference on Information Technology in Mechatronics, Istanbul, Turkey, October 1-3, 2001, pp. 369-374, ISBN: 975-518-171-7.

[C50] József K. Tar, Imre J. Rudas, János F. Bitó, K. Kozlowski: "A New Approach in Computational Cybernetics Based on the Modified Renormalization Algorithm Guaranteeing Complete Stability in the Control of a Wide Class of Physical Systems", in the Proceedings of the 6th IEEE International Conference on Intelligent Engineering Systems (INES 2002), May 26-28, 2002, Opatija, Croatia, pp. 19-24, ISBN 953-6071-17-7.

[C51] József K. Tar, János F. Bitó, Krzysztof Kozłowski, Béla Pátkai, D. Tikk: "Convergence Properties of the Modified Renormalization Algorithm Based Adaptive Control Supported by Ancillary Methods", in the Proc of the 3rd International Workshop on Robot Motion and Control (ROMOCO '02), Bukowy Dworek, Poland, 9-11 November, 2002, pp. 51-56, ISBN 83-7143-429-4, IEEE Catalog Number: 02EX616.

[C52] József K. Tar, Marcus Bröcker, Krzysztof Kozlowski: "A Novel Adaptive Control for Hydraulic Differential Cylinders", in the Proc. of the 11th Workshop on Robotcs in Alpe-Adria-Danube Region, June 30 - July 2, 2002, Balatonfüred, Hungary, ISBN: 963 7154 10 8 (for the issue on CD), pp. 7-12.

[C53] J.K. Tar, A. Bencsik, J.F. Bitó, K. Jezernik: "Application of a New Family of Symplectic Transformations in the Adaptive Control of Mechanical Systems", in the Proc. of the 2002 28th Annual Conference of the IEEE Industrial Electronics Society, Nov. 5-8 2002 Sevilla, Spain, Paper SF-001810, CD issue, ISBN 0-7803-7475-4, IEEE Catalog Number: 02CH37363C.[Printed issue: IEEE Catalog number: 02CH37363, ISBN: 0-7803-7474-6, Library of Congress: 2002103754, pp. 1499-1504]

[C54] I.J. Rudas, J.K. Tar, K. Kozłowski, K. Jezernik: "Novel Approach in the Adaptive Control of Systems Having Strongly Nonlinear Coupling Between Their Unmodeled Internal Degrees of Freedom", submitted for publication to the 2003 IEEE International Conference on Robotics and Automation, May 12-17, 2003, The Grand Hotel, Taipei, Taiwan.

[C55] J.K. Tar: "Novel Adaptive Control of Roughly Modelled Non-linear Systems Exemplified in Robotics and Chemical Reactions", lecture delivered at the 2nd International Symposiuum on Mechatronics Bridging Over Some Fields of Science, Budapest, November 15, 2002, Budapest Polytechnic, a CD issue, file :\pdf\Tar Jozsef.pdf, ISBN 963 7154 11 6.

[C56] J.F. Bitó, J.K. Tar, I.J. Rudas: "Novel Adaptive Control of Mechanical Systems Driven by Electromechanical Hydraulic Drives", in the Proc. of BASYS 2002, the 5th IFIP International Conference on Information Technology for Balanced Automation Systems in Manufacturing and Services, Cancún; Mexico, Sept. 25-27., 2002, pp. 517-524, ISBN 1-4020-7211-2.

[C57] J.K. Tar, I.J. Rudas, K. Kozłowski, B. Pátkai: "Analysis of the Effect of Joint Acceleration Measurement Noise in a Novel Adaptive Control of Mechanical Systems", Proc. of the 7th International Conference on Intelligent Engineering Systems 2003 (INES'03), March 4-6, Assiut - Luxor, Egypt, pp.376-381, ISBN 977-246-048-3.

[C58] József K. Tar, Imre J. Rudas, J.F.Bitó, Krzysztof Kozłowski: „Special Symplectic Transformations Used in Nonlinear System's Control", Proceedings of RAAD'03, 12th International Workshop on Robotics in Alpe-Adria-Danube Region, Cassino May 7-10, 2003, a CD issue, File: 012RAAD03.pdf, Edited by. M. Ceccarelli.

[C59] J.K. Tar, I.J. Rudas, J.F. Bitó, L. Horváth, K. Kozłowski: "Analysis of the Effect of Backlash and Joint Acceleration Measurement Noise in the Adaptive Control of Electro-mechanical Systems", Proc. of the 2003 IEEE International Symposium on Industrial Electronics (ISIE 2003), June 9-12, 2003, Rio de Janeiro, Brasil, CD issue, file BF-000965.pdf, ISBN 0-7803-7912-8, IEEE Catalog Number: 03th8692.

[C60] I.J. Rudas, J.K. Tar, J.F. Bitó, K. Kozłowski: "Improvement of a Non-linear Adaptive Controller Designed for Strongly Non-linear Plants", Proc. of the The 11th International Conference on Advanced Robotics (ICAR 2003), June 30 - July 3, 2003, University of Coimbra, Portugal, pp. 1381-1386, ISBN 972-96889-9-0.

[C61] József K. Tar, Imre J. Rudas, Ágnes Szeghegyi, Krzysztof Kozłowski: "Non-Conventional Processing of Noisy Signal in the Adaptive Control of Hydraulic Differential Servo Cylinders", in the Proceedings of the 2003 IEEE International Symposium on Intelligent Signal Processing "From classical measurement to computing with perceptions", 4-6 September 2003, Hotel Gellért, Budapest, Hungary, pp. 237-242, ISBN 0-7803-7864-4.

[C62] József K. Tar, Imre J. Rudas, Domonkos Tikk, Krzysztof Kozłowski: "The Effect of Asymmetric Delay Time in a Simple PID and a Novel Adaptive Control of a Strongly Nonlinear System", in the Proc. of the 1st Serbian-Hungarian Joint Symposium on Intelligent Systems September 19-20, 2003, Subotica, Serbia-Montenegro (SiSy 03), pp. 175-187, ISBN 963-7154-19-1.

[C63] J.K. Tar, I.J. Rudas, L. Horváth, Spyros G. Tzafestas: "Adaptive Control of the Double Inverted Pendulum Based on Novel Principles of Soft

Computing", Proc. of the International Conference in Memoriam John von Neumann, Budapest, Hungary, 12th December 2003, pp. 257-268, ISBN 963 7154213.

[C64] Tar J.K., Rudas I.J, Szeghegyi Á., Kozłowski K.: "Adaptive Control of a Dynamic System Having Unmodeled and Unconstrained Internal Degree of Freedom", Proc. of the 4th International workshop on Robot Motion and Control (RoMoCo'2004), June 17-20, 2004, Puszczykowo, Poland, pp. 41-46, ISBN 83-7143-272-0.

[C65] Tar J.K., Rudas I.J, Bitó, J.F., Kozłowski K.: "Application of a Novel Branch of Soft Computing in the Adaptive Control of Non-linear Systems of Strictly and Loosely Bounded Unmodeled and Uncontrolled Internal Degrees of Freedom", Proc. Of the 13th International Workshop on Robotics in Alpe-Adria-Danube Region, June 2-5, 2004, Brno, the Czech Republic, pp. 52-57, ISBN 80-7204-341-2.

[C66] József K. Tar, Imre J. Rudas, János F. Bitó, José A. Tenreiro Machado: "Adaptive Nonlinear Vibration Damping Inspired by the Concept of Fractional Derivatives", Proc. of the 2nd Workshop on Intelligent Solutions in Embedded Systems (WISES 2004), June 25, 2004, Graz, Austria, pp. 183-192, ISBN 3-902463-00-7.

[C67] J.K. Tar, I.J. Rudas, Á. Szeghegyi, K. Kozlowski: "Adaptive Control of a Wheel of Unmodeled Internal Degree of Freedom", In the Proc. of the Symposium on Applied Machine Intelligence (SAMI 2004), Herlany, Slovakia, January 16-17 2004, pp. 289-300, ISBN 963 7154 23 X.

[C68] József K. Tar, Imre J. Rudas, János F. Bitó: "Comparison of the Operation of the Centralized and the Decentralized Variants of a Soft Computing Based Adaptive Control", Proc. of the Budapest Tech Polytechnical Institution's Jubilee Conference 1879-2004, September 4, 2004, Budapest, Hungary, pp. 331-342, ISBN 963-7154-31-0.

[C69] József K. Tar, Imre J. Rudas, János F. Bitó, José A. Tenreiro Machado: "Adaptive Nonlinear Vibration Control Based on Causal Time-invariant Green Functions and on a Novel Branch of Soft Computing", Proc. of the 1st Romanian-Hungarian Joint Symposium on Applied Computational Intelligence (SACI'04), Timişoara, Romania, May 25-26, 2004, pp. 271-282, ISBN 963-7154-26-4.

[C70] József K. Tar, Imre J. Rudas, János F. Bitó, J. A. Tenreiro Machado: "Fractional Order Adaptive Active Vibration Damping Designed on the Basis of Simple Kinematic Considerations", Proc. of the 2nd 2nd IEEE International Conference on Computational Cybernetics (ICCC'04), August 30 - September 1, 2004, Vienna University of Technology, Austria, pp. 353-357, ISBN 3-902463-01-5.

[C71] József K. Tar, Imre J. Rudas, Béla Pátkai, J. A. Tenreiro Machado: "Adaptive Vibration Damping Based on Causal Time-invariant Green-Functions and Fractional Order Derivatives", Proc. of the Fourth International Conference on Intelligent Systems Design and Applications (ISDA'04), 26-28 August 2004, Budapest, Hungary, paper No. 114, pp. 675-679, CD issue, ISBN 963-7154-30-2.

[C72] József K. Tar, János F. Bitó, José A. Tenreiro Machado, Domonkos Tikk: "Application of Extended Numerical Approximation of Fractional Order Derivatives in Adaptive Control", Proc. of the 8th International Conference on Intelligent Engineering Systems 2004 (INES'04), September 19-21, 2004, Technical University of Cluj-Napoca Romania, pp. 476-481, ISBN 973-662-120-0.

[C73] József K. Tar, Imre J. Rudas, János F. Bitó, J. A. Tenreiro Machado, Ágnes Szeghegyi: "Investigation of the Effect of Time Scaling in a Soft Computing Based Control Using Fractional Order Derivatives", Proc. of the 2nd Serbian-Hungarian Joint Symposium on Intelligent Systems (SISY 2004), October 1-2, 2004, Subotica, Serbia and Montenegro, paper No. SZB-022.

[C74] József K. Tar, Gy. Kártyás, János F. Bitó, Ágnes Szeghegyi: "Application of the Scicos Simulator in the Investigation of an Adaptive Control of a Mechanical System of Free Degree of Freedom", Proc. of the 3rd Slovakian-Hungarian Joint Symposium on Applied Machine Intelligence, January 21-22, 2005, Herl'any, Slovakia, pp. 359-370, ISBN 963 7154 35 3

[C75] József K. Tar, Imre J. Rudas, János F. Bitó, J. A. Tenreiro Machado: "Scicos Based Investigation of an Adaptive Vibration Damping Technique Using Fractional Order Derivatives", Proceedings of the 3rd International Conference on Computational Cybernetics, ICCC 2005, April 13-16, 2005, Hotel Le Victoria, Mauritius, pp. 213-218, ISBN 963 7154 37 X.

[C76] József K. Tar, Attila L. Bencsik: "Fractional Order Adaptive Control for Hydraulic Differential Cylinders", Proceedings of the 3rd International Conference on Computational Cybernetics, ICCC 2005, April 13-16, 2005, Hotel Le Victoria, Mauritius, pp. 225-229, ISBN 963 7154 37 X.

[C77] József K. Tar: "Extension of the Modified Renormalization Transformation for the Adaptive Control of Negative Definite SISO Systems", Proc. of the 2nd Romanian-Hungarian Joint Symposium on Applied Computational Intelligence (SACI 2005), May 12-14, 2005, Timisoara, Romania, pp. 447-457, ISBN: 963 7154 39 6.

[C78] József K. Tar, János F. Bitó, Imre J. Rudas, José. A. Tenreiro Machado: "Adaptive Reduction of the Order of Derivation in the Control of a Hydraulic Differential Cylinder", Proceedings of RAAD'05, 14th International Workshop on Robotics in Alpe-Adria-Danube Region, Bucharest, Romania, May 26-28, 2005, pp. 513-518, ISBN: 973-718-241-3.

[C79] József K. Tar, Attila L. Bencsik, K. Kozłowski: "Fractional Order Adaptive Control for Systems of Locally Nonlinearizable Nonlinearities", Proc. of the 5th International Workshop on Robot Motion and Control (RoMoCo'05), June 23-25, 2005, Dymaczewo, Poland, pp. 355-360, ISBN: 83-7143-266-6.

[C80] József K. Tar, Imre J. Rudas, Miklós Rontó: "Geometric Identification and Control of Nonlinear Dynamic Systems Based on Floating Basis Vector Representation", Proc. of the 3rd Serbian-Hungarian Joint Symposium on Intelligent Systems (SISY 2005), August 31-September 1, 2005, Subotica, Serbia and Montenegro, ISBN: 963 7154 41 8, pp. 35-46.

[C81] József K. Tar, Imre J. Rudas, János F. Bitó, J. A. Tenreiro Machado: "Centralized and Decentralized Applications of a Novel Adaptive Control", Proc. 9th International Conference on Intelligent Engineering Systems 2005, September 16-19, 2005, Cruising on Mediterranean Sea, IEEE Catalog Number: 05EX1202C, ISBN: 0-7803-9474-7, file: Tar.pdf (CD issue).

[C82] József K. Tar, Imre J. Rudas, Péter Kerepesi: "Vibration Control of a System having Non-linearly Coupled Unmodeled Internal Degree of Freedom", in the Proc. Of the Eleventh IASTED International Conference on Robotics and Applications (RA 2005) October 31 – November 2, 2005, Cambridge, MA, USA, pp. 76-81.

[C83] József K. Tar, Imre J. Rudas, Miklós Rontó, José Antonio Tenreiro Machado: "Simple Geometric Approach of Identification and Control Using Floating Basis Vectors for Representation", in the Proc. of the 6th International Symposium of Hungarian Researchers on Computational Intelligence, Nov. 18-19 2005 Budapest, Hungary, pp. 437-449, ISBN: 963 7154 43 4.

[C84] J.K. Tar: "Dynamic Nonlinear Control of Mechanical and Analogous Devices/Processes", Plenary Talk at the 4th IEEE International Conference on Computational Cybernetics, 20-22 August, 2006, Tallinn, Estonia, ISBN 1-4244-0071-6.

[C85] József K. Tar, Imre J. Rudas, Kazuhiro Kosuge: "Adaptive Control of a Polymerization Process", in the Proc. of the 4th Slovakian-Hungarian Joint Symposium on Applied Machine Intelligence (SAMI 2006), Herlany, Slovakia, January 20-21, 2006, pp. 414-425, ISBN: 963 7154 44 2.

[C86] József K. Tar, Imre J. Rudas, Kazuhiro Kosuge: "Comparative Analysis of Two Adaptive Controls based on Geometric Principles of Identification and the Extension of the Modified Renormalization Transformation", Proc. of the AMC'06 Workshop, Istanbul, Turkey, March 27-29, 2006, pp. 39-43, CD Proceedings, Paper: RS-2 file f197.pdf, IEEE Catalog No.: 06TH8850C, ISBN 0-7803-9512-3

[C87] József K. Tar, Imre J. Rudas, Kazuhiro Kosuge: "Dynamic Analysis and Control of a Polymerization Reaction", Proc. of the 10th International Conference on Intelligent Engineering Systems 2006, London Metropolitan University, London, UK, June 26-28, 2006, pp. 123-128, CD issue: ISBN 1-4244-9709-6, IEEE Catalog Number 06EX1430C, Printed proceedings: ISBN 1-4244-9708-8, 06EX1430

[C88] József K. Tar, János F. Bitó, Imre J. Rudas, Stefan Preitl, Radu-Emil Precup: "The Effect of the Static Striebeck Friction in the Robust VS/Sliding Mode Control of a Ball-Beam System", in the Proc. Of the 15th International Workshop on Robotics in Alpe-Adria-Danube Region, June 15-17, 2006, Balatonfüred, Hungary, CD issue, ISBN 963 7154 48 5

[C89] József K. Tar, János F. Bitó, Stefan Preitl, Radu-Emil Precup: "Robust, Potential Limited Control for Systems of Unmodeled Internal Degrees of Freedom", 3rd Romanian-Hungarian Joint Symposium on Applied Computational Intelligence, May 25-26, 2006, (SACI 2006), Timişoara, Romania, pp. 278-285

[C90] Imre J. Rudas, József K. Tar, Kazuhiro Kosuge: "Stabilization of the Adaptive Control of a 4th Order System Using Coordinate and Velocity Potentials", 3rd IEEE International Conference on Mechatronics, July 3-5, 2006, Budapest, Hungary (ICM 2006), pp. 513-518, CD issue: ISBN 1-4244-9713-4, IEEE Catalog Number: 06EX1432C, Printed Proceedings: 1-4244-9712-6, EEE Catalog Number: 06EX1432

[C91] József K. Tar, Imre J. Rudas, Kazuhiro Kosuge: "Improved Adaptive Dynamic Control of a Polymerization Process", World Automation Congress 2006 (WAC 2006), July 24-27, 2006, Budapest Hilton, Budapest, Hungary, CD issue, file: isiac_134, ISBN 1-889335-26-6, IEEE Catalog Number: 06EX1486

[C92] Imre J. Rudas, Jozsef K. Tar, Kazuhiro Kosuge: "Fractional Robust Control of a Ball-Beam System", The 32nd Annual Conference of the IEEE Industrial Electronics Society (IECON 2006), Conservatoire National des Arts &

Metiers Paris - FRANCE - November 7-10, 2006 292, rue St Martin and 2, rue Conté - Paris District 3, pp. 5408-5413

[C93] József K. Tar, Imre J. Rudas, János F. Bitó, Kazuhiro Kosuge: "Adaptive Control of a Differential Hydraulic Cylinder with Dynamic Friction Model", Proc. of the 4th Serbian-Hungarian Joint Symposium on Intelligent Systems (SISY 2006), September 29-30, 2006, Subotica, Serbia, pp. 361-374, ISBN 963 7154 50 7

[C94] Imre J. Rudas, József K. Tar, Béla Pátkai: "Compensation of Dynamic Friction by a Fractional Order Robust Controller, IEEE International Conference on Computational Cybernetics (ICCC 2006)", Tallinn, Estonia, August 20-22, 2006, pp. 15-20, ISBN 1-4244-0071-6

[C95] József K. Tar, Imre J. Rudas: "Sophisticated Dynamic Adaptive Control of a Polymerization Process", in the Proc. of the 7th International Symposium of Hungarian Researchers on Computational Intelligence (HUCI 2006), November 24-25, 2006, Budapest, Hungary, pp. 107-120

[C96] József K. Tar, Imre J. Rudas: "Geometric Approach to Nonlinear Adaptive Control" -- Tutorial, in the Proc. of the 4th International Symposium on Applied Computational Intelligence and Informatics (SACI 2007), May 17-18, 2007, Timişoara, Romania, pp. 9-23, ISBN 1-4244-1234-X, IEEE Catalog Number:07EX1788

[C97] József K. Tar, János F. Bitó: "Robustness Analysis of a Novel Adaptive Control based on Geometric Approach", in the Proc. of the 4th International Symposium on Applied Computational Intelligence and Informatics (SACI 2007), May 17-18, 2007, Timişoara, Romania, pp. 99-104, ISBN 1-4244-1234-X, IEEE Catalog Number:07EX1788

[C98] József K. Tar, Imre J. Rudas, Stefan Preitl, Radu-Emil Precup: "Adaptive Control of the TORA System based on a Simple Causal Filter", in the Proceedings of 16th Int. Workshop on Robotics in Alpe-Adria-Danube Region - RAAD 2007, Ljubljana, June 7-9, 2007, pp. 363-370, CD issue, ISBN 978-961-243-067-2

[C99] József K. Tar, Katalin Lőrincz, Roland Kovács: "Adaptive Control of an Automatic Convoy of Vehicles", in the Proc of the 11th IEEE International Conference on Intelligent Engineering Systems 2007, June 29-July 2, 2007, Budapest, Hungary, pp. 21-26, ISBN: 1-4244-1148-3

[C100] József K. Tar, Katalin Lőrinc, Krishnan Agbemasu, László Nádai, Roland Kovács: "Adaptive Control of a Semi-Automatic Convoy of Unmodeled Internal Degrees of Freedom", Proc. of the 5th International Symposium on Intelligent Systems and Informatics, August 24-25, 2007, Subotica, Serbia, ISBN: 1-4244-1443-1, IEEE Catalog Number: 07EX1865C, Library of Congress: 2007930059, pp. 129-134.

[C101] József K. Tar, Katalin Lőrinc, Krishnan Agbemasu, László Nádai, Roland Kovács: "Investigation of the Behavior of Adaptively Controlled Platoons with Unmodeled Loads", Proceedings of the International Symposium on Logistics and Industrial Informatics, September 13-15, 2007, Wildau, Germany, ISBN: 1-4244-1441-5, pp. 137-142.

[C102] József K. Tar, József Gáti, Zoltán Puklus: "SVD-Based Multiple Dimensional Generalization of an Adaptive Control of Geometric Interpretation", Proceedings of the 5th IEEE International Conference on Computational Cybernetics, October 19-21, 2007, Gammarth, Tunis, ISBN 1-4244-1146-7, pp. 81-86.

[C103] József K. Tar: „Fixed Point Transformations as Simple Geometric Alternatives in Adaptive Control", invited plenary lecture, in the Proc. of the 5^{th} IEEE International Conference on Computational Cybernetics, October 19-21, 2007, Gammarth, Tunis, ISBN 1-4244-1146-7, pp. 19-34.

[C104] József K. Tar, Imre J. Rudas, János F. Bitó, Stefan Preitl and Radu E. Precup: "Dynamic Friction Compensation in the Slotine--Li and in an SVD--Based Adaptive Control, Proceedings of the 17^{th} International Workshop on Robotics in Alpe--Adria--Danube Region (RAAD 2008), September 15-17, 2008, Ancona, Italy, paper #5 in a CD issue, ISBN: 978-88-903709-0-8, Alexa edizioni.

[C105] József K. Tar, János F. Bitó, Imre J. Rudas, Krzysztof R. Kozłowski, José A. Tenreiro Machado: "Possible Adaptive Control by Tangent Hyperbolic Fixed Point Transformations Used for Controlling the Φ^6-Type Van der Pol Oscillator, Proc. of the 6th IEEE International Conference on Computational Cybernetics (ICCC 2008), November 27-29, 2008, Hotel Academia, Stará Lesná, Slovakia, pp. 15 20, (CD issue), IEEE Catalog Number: CFP08575-CDR, ISBN: 978-1-4244-2875-5, Library of Congress: 2008907697.

[C106] J.K. Tar, J.F. Bito, I.J. Rudas, S. Preitl, and R.-E. Precup: "An SVD Based Modification of the Adaptive Inverse Dynamics Controller", Proceedings of 5th International Symposium on Applied Computational Intelligence and Informatics, Timisoara, Romania, 2009, pp. 193-198, ISBN 978-1-4244-4478-6, IEEE Catalog Number CFC0945C-CDR.

[C107] József K. Tar, Imre J. Rudas, János F. Bitó, Stefan Preitl, and Radu E. Precup: "Adaptive Control of a 3rd Order Electromechanical System Using Robust Sigmoidal Fixed Point Transformation", Proceedings of the RAAD 2009 18th International Workshop on Robotics in Alpe–Adria–Danube Region, May 25-27, 2009, Brasov, Romania, CD issue, file: 87.pdf, ISBN: 978 606 521 315 9.

[C108] József K. Tar, János F. Bitó, Krzysztof R. Kozłowski, José A. Tenreiro Machado: „Application of Robust Fixed Point Transformations for Technological Operation of Robots", Lecture at the Seventh International Workshop on Robot Motion and Control, June 1-3, 2009, Czerniejewo, Poland; issued in Lecture Notes in Control and Information Sciences 396 (Eds: M. Thoma, F. Allgöver, M. Morari) – Robot Motion and Control 2009 (Ed. Krzysztof R. Kozłowski), Springer-Verlag Berlin Heidelberg, ISBN 978-1-84882-984-8, e-ISBN 978-1-84882-985-5, Library of Congress Control Number: 2009937154, Chapter 9, pp. 93-101. DOI: 10.1007/978-1-84882-985-5

[C109] József K. Tar, Imre J. Rudas, László Nádai, Krzysztof R. Kozłowski, José A. Tenreiro Machado: "Fixed Point Transformations in the Adaptive Control of Fractional Order MIMO Systems", Lecture at the Seventh International Workshop on Robot Motion and Control, June 1-3, 2009, Czerniejewo, Poland; issued in Lecture Notes in Control and Information Sciences 396 (Eds: M. Thoma, F. Allgöver, M. Morari) – Robot Motion and Control 2009 (Ed. Krzysztof R. Kozłowski), Springer-Verlag Berlin Heidelberg, ISBN 978-1-84882-984-8, e-ISBN 978-1-84882-985-5, Library of Congress Control Number: 2009937154, Chapter 10, pp. 103-112.

[C110] József K. Tar, Imre J. Rudas, János F. Bitó, José A. Tenreiro Machado, and Krzysztof R. Kozłowski: "Decoupled Fixed Point Transformation Based Adaptive Control of the Generalized 2 DOF Φ^6-Type Van der Pol

Oscillator", in Proc. of the ECC'09 European Control Conference, 23-26 August 2009, Budapest, Hungary, ISBN 978-963-311-369-1, pp. 579-584

[C111] József K. Tar, János F. Bitó, István Gergely, László Nádai: "Possible Improvement of the Operation of Vehicles Driven by Omnidirectional Wheels", in Proc. of the 4th International Symposium on Computational Intelligence and Intelligent Informatics, 21–25 October 2009 Egypt (ISCIII 2009), pp. 63 68, IEEE Catalog Number: CFP0936C-CDR, ISBN:978-1-4244-5382-5, Library of Congress: 2009909581

[C112] J.K. Tar, I.J. Rudas, I. Nagy, K.R. Kozłowski, J.A. Tenreiro Machado: „Simple Adaptive Dynamical Control of Vehicles Driven by Omnidirectional Wheels", Proc. of the 7th IEEE International Conference on Computational Cybernetics (ICCC 2009), Palma de Mallorca, Spain, November 26-29, 2009, ISBN: 978-1-4244-5311-5, IEEE Catalog Number: CFP09575-CDR, Library of Congress: 2009936140, pp. 91-95.

[C113] József K. Tar and János F. Bitó: "Adaptive Control Using Fixed Point Transformations for Nonlinear Integer and Fractional Order Dynamic Systems", in the Proc. of the Budapest Tech Jubilee Conference, September 1-2, 2009, Budapest, in the series "Studies in Computational Intelligence 241 – Aspects of Soft Computing, Intelligent Robotics and Control", Eds. János Fodor Janusz Kacprzyk, ISBN 978-3-642-03632-3, e-ISBN 978-3-642-03633-0, Springer-Verlag Berlin Heidelberg 2009, pp. 253-267

[C114] József K. Tar, Imre J. Rudas, János F. Bitó, José A. Tenreiro Machado, Krzysztof R. Kozłowski: "A Higher Order Adaptive Approach to Tackle the Swinging Problem", Proc of the 10th International Symposium of Hungarian Researchers on Computational Intelligence and Informatics (CINTI 2009), Budapest, November 12-14, 2009, pp. 145-153.

[C115] József K. Tar: "Application of Local Deformations in Adaptive Control - A Comparative Survey", invited plenary lecture, Proc of the 7th IEEE International Conference on Computational Cybernetics (ICCC 2009), Palma de Mallorca, Spain, November 26-29, 2009, ISBN: 978-1-4244-5311-5, IEEE Catalog Number: CFP09575-CDR, Library of Congress: 2009936140, pp. 25-38

[C116] József K. Tar, Imre J. Rudas, József Gáti: „Improvements of the Adaptive Slotine & Li Controller –Comparative Analysis with Solutions Using Local Robust Fixed Point Transformations", invited lecture for the 14th WSEAS International Conference on Applied Mathematics (MATH'09), in the Proc. of "Recent Advances in Applied Mathematics", Puerto De La Cruz, Tenerife, Canary Islands, Spain, December 14-16, 2009, pp. 305-311, ISBN: 978-960-474-138-0.

[C117] J.K. Tar, I.J. Rudas, J.F. Bito, Stefan Preitl, Radu-Emil Precup, Convergence stabilization by parameter tuning in Robust Fixed Point Transformation based adaptive control of underactuated MIMO systems, in the Proc. of the International Joint Conference on Computational Cybernetics and Technical Informatics (ICCC-CONTI), 2010, 27-29 May 2010, Timisoara, Romania, pp. 407 – 412, ISBN: 978-1-4244-7432-5.

[C118] József K. Tar, János F. Bitó, Imre J. Rudas: Replacement of Lyapunov's Direct Method in Model Reference Adaptive Control with Robust Fixed Point Transformations, in Proc. of the 14th IEEE International Conference on Intelligent Engineering Systems 2010, May 5-7, 2010, Las Palmas of Gran

Canaria, Spain, pp. 231-235, ISBN 978-1-4244-7651-0, IEEE Catalog Number: CFP10IES-CDR

[C119] J.K. Tar, I.J. Rudas, J.F.Bitó, K.R.Kozłowski, and C. Pozna: "A Novel Approach to the Model Reference Adaptive Control of MIMO Systems", in the Proc of the 19th International Workshop on Robotics in Alpe-Adria-Danube Region, Budapest, Hungary, June 23-25, 2010, pp. 31-36, (CD issue, file: 4_raad2010.pdf), ISBN: 978-1-4244-6884-3

[C120] József K. Tar, Imre J. Rudas and János F. Bitó: "Fixed Point Stabilization in a Novel MRAC Control for MIMO Systems", accepted for publication at the 8th IEEE International Symposium on Intelligent Systems and Informatics (SISY 2010), September 10-11, 2010 --- Subotica, Serbia

[C121] József K. Tar: "Towards Replacing Lyapunov's 'Direct' Method in Adaptive Control of Nonlinear Systems", invited lecture at the 2010 Mathematical Methods in Engineering International Symposium (MME 2010), October 21-24, 2010, Coimbra, Portugal.

[C122] József K. Tar, Zoltán Siska, Imre J. Rudas, János F. Bitó, A Higher Order Adaptive Approach of the Swinging Problem - Implementation Issues accepted for publication at the 14th International Power Electronics and Motion Control Conference, 6-8 September 2010, Ohrid, Republic of Macedonia.

References

[R1] I. J. Rudas, O. Kaynak, J. F. Bitó, J. K. Tar: "Robustness Analysis of a Paradigm for Non-linear Robot-Controllers Based on Soft Computing Approach", Proceedings of the IEEE International Conference on Industrial Electronics (IECON'94), Bologna, Italy, pp. 1633-1638.

[R2] Thomas Bartz-Beielstein: "Experimental Research in Evolutionary Computation – The New Experimentalism", Springer-Verlag Berlin Heidelberg 2006 ISBN-13 978-3-540-32026-5, pp. 3-4.

[R3] E.D. Dolan & J.J. More: "Benchmarking Optimization Software with Performance Profiles", Mathematical Programming, 91, 201-213.

[R4] A. Eiben & M. Jelasity: "A Critical Note on Experimental Research Methodology in EC", in Proc. of the 2002 Congress on Evolutionary Computation (CEC'2002), pp. 582-587. Piscataway NJ: IEEE.

[R5] B. Lantos: "Identification and Adaptive Control of Robots", International Journal Mechatronics, Vol. 2, No. 3, 1993, pp. 149-166.

[R6] J. Somló, B. Lantos, P.T. Cát: "Advanced Robot Control", Akadémiai Kiadó, Budapest, 1997, Chapter 7 Identification of the Robot Model p. 323.

[R7] B. Armstrong, O, Khatib, J. Burdick: "The Explicit Dynamic Model and Internal Parameters of the PUMA 560 Arm", in Proc. IEEE Conf. On Robotics and Automation, pp. 510-518, 1986.

[R8] P.I. Corke and B. Armstrong-Helouvry: "A Search for Consensus Among Model Parameters Reported for the Puma 560 robot", Proc. IEEE Conf. Robotics and Automation, 1994, pp. 1608-1613.

[R9] Lőrinc Márton, Béla Lantos: "Tracking Control of Mechatronic Systems based on Precise Friction Compensation", in Proceedings of the 3rd Romanian-Hungarian Joint Symposium on Applied Computational Intelligence (SACI 2006), Timişoara, Romania, May 25-26 2006, pp. 136-147.

[R10] Lőrinc Márton: "Robust-Adaptive Control of Nonlinear Singlevariable Mechatronic Systems", PhD Thesis, Budapest University of Technology and Economics, Budapest, Hungary, 2005.
[R11] Jean-Jacques E. Slotine and Weiping Li: Applied Nonlinear Control. Prentice Hall International, Inc., Englewood Cliffs, New Jersey, 1991.
[R12] R. J. P. De Figueiredo: "Implications and Applications of Kolmogorov's Superposition Theorem", IEEE Tr. Autom. Control, pp. 1227-1230, 1980.
[R13] K. Hornik, M. Stinchcombe, and H. White: "Multilayer Feedforward Networks are Universal Approximators", Neural Networks, vol. 2, pp. 359-366, 1989.
[R14] L. X. Wang: "Fuzzy Systems are Universal Approximators", in Proc. of the IEEE Int. Conf. on Fuzzy Systems, San Diego, 1992, pp. 1163-1169.
[R15] J. Bokor, P. Baranyi, P. Michelbereger, and P. Várlaki: "TP Model Transformation in Non-linear System Control", in the 3rd IEEE International Conference on Computational Cybernetics (ICCC), Mauritius, 13-16 April 2005, pp. 111-119.
[R16] A.N. Kolmogorov: "On the Representation of Continuous Functions of Many Variables by Superpositions of Continuous Functions of One Variable and Addition", Doklady Akademii Nauk USSR (in Russian), 114:953-956.
[R17] K. Weierstraß: "Über continuirliche Functionen eines reellen Arguments, die für keinen Werth des letzeren einen bestimmten Differentialquotienten besitzen. A paper presented to the 'Königliche Akademie der Wissenschaften' on 18 of July 1872. English translation available in: On continuous functions of a real argument that do not have a well-defined differential quotient, in: G.A. Edgar, Classics on Fractals, Addison-Wesley Publishing Company, 1993, pp. 3-9.
[R18] http://www-history.mcs.st-and.ac.uk/~history/Biographies/Weierstrass.html
[R19] T. Roska: "Development of Kilo Real-time Frame Rate TeraOPS Computational Capacity Topographic Microprocessors", Plenary Lecture at 10th International Conference on Advanced Robotics (ICAR 2001), Budapest, Hungary, August 22--25, 2001.
[R20] N.H. Getz: "Dynamic Inversion of Nonlinear Maps with Applications to Nonlinear Control and Robotics", PhD Thesis, University of California at Berkeley, 1995.
[R21] N.H. Getz, J.E. Marsden: "Linear Algebra and its Applications", 258:311-343, 1997.
[R22] A.M. Lyapunov: "A General Task About the Stability of Motion", PhD Thesis, 1892 (in Russian)
[R23] A.M. Lyapunov: "Stability of Motion", Academic Press, New--York and London, 1966.
[R24] E.N. Lorenz: "Deterministic Non-periodic Flow", Journal of the Atmospheric Sciences, Vol. 20, p. 130, 1963.
[R25] V.I. Arnold: "Mathematical Methods of Classical Mechanics" (original issue in Russian by "Nauka"), Hungarian translation issued by Műszaki Könyvkiadó Budapest, Hungary 1985. English translation available e.g.: V.I. Arnold, Mathematical Methods of Classical Mechanics, Springer-Verlag (1989), [ISBN 0-387-96890-3]
[R26] http://en.wikipedia.org/
[R27] L.D. Landau, E.M. Liphshits: "Hydrodynamics" (Hungarian translation, issued by "Tankönyvkiadó", Budapest, Hungary, 1980.)

[R28] J. Clerk Maxwell: "A Treatise on Electricity and Magnetism", 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68–73.

[R29] W.R. Hamilton: "On a General Method in Dynamics", Philosophical Transaction of the Royal Society Part I (1834) p.247-308; Part II (1835) p. 95-144. (From the collection Sir William Rowan Hamilton (1805-1865): Mathematical Papers edited by David R. Wilkins, School of Mathematics, Trinity College, Dublin 2, Ireland. (2000); also reviewed as On a General Method in Dynamics)

[R30] H. Grassmann: „Die Lineale Ausdehnungslehre, ein neuer Zweig der Mathematik", Collected Works, 1894-1911.

[R31] Abonyi I., Bitó J.F. Tar J.K.: "A Quaternion Representation of the Lorentz Group for Classical Physical Applications", J. Phys. A: Math. Gen. 24. (1991) pp.: 3245-3254.

[R32] http://en.wikipedia.org/w/index.php?title=Euclides

[R33] Joseph-Louis Lagrange: «Mécanique Analytique» (Analytical Mechanics) (4th ed., 2 vols. Paris: Gauthier-Villars et fils, 1888--1889. (First Edition in 1788)

[R34] http://en.wikipedia.org/wiki/Joseph_Louis_Lagrange

[R35] http://en.wikipedia.org/wiki/Hermann_Grassmann

[R36] http://en.wikipedia.org/wiki/Clifford_algebra

[R37] W.K. Clifford: "Preliminary Sketch of Biquaternions", (1873)

[R38] M.S. Lie: "On a Class of Geometric Transformations", Ph.D. Dissertation, University of Christiania, 1872.

[R39] http://en.wikipedia.org/wiki/Riemann

[R40] http://en.wikipedia.org/wiki/Hilbert

[R41] http://en.wikipedia.org/wiki/Stefan_Banach

[R42] http://en.wikipedia.org/wiki/Vladimir_Arnold

[R43] Máté László: "Funkcionálanalízis műszakiaknak", (in Hungarian) Műszaki Könyvkiadó, Budapest, 1976, ISBN 963 10 1173 9.

[R44] A. Isidori: "Nonlinear Control Systems" (Third Edition), Springer-Verlag London Limited, 1995.

[R45] V. Jurdjevic: "Geometric Control Theory", Cambridge University Press, 1997.

[R46] G.W. Stewart: "On the early history of singular value decomposition", Technical Report TR–92–31, Institute for Advanced Computer Studies, University of Mariland, March 1992.

[R47] G.H. Golub and W. Kahan: "Calculating the singular values and pseudoinverse of a matrix", SIAM Journal on Numerical Analysis, 2:205–224, 1965.

[R48] Pál Rózsa: "Lineáris algebra" (Linear Algebra, in Hungarian), 2nd Edition, Műszaki Könyvkiadó, Budapest, 1976, p. 650.

[R49] P. Baranyi, L. Szeidl, P. Várlaki, and Y. Yam: "Definition of the HOSVD-based Canonical Form of Polytopic Dynamic Models", in the 3rd International Conference on Mechatronics (ICM 2006), Budapest, Hungary, July 3-5 2006, pp. 660-–665.

[R50] P. Baranyi, L. Szeidl, P. Várlaki, and Y. Yam: "Numerical reconstruction of the HOSVD based canonical form of polytopic dynamic models", in the Proc. of the 10th International Conference on Intelligent Engineering Systems, London, UK, June 26-28 2006, pp. 196-201.

[R51] E.F. Deprettere, editor. SVD and Signal Processing, volume Algorithms, Applications and Architectures. North-Holland, Amsterdam, 1988.

[R52] R. Vaccaro, editor. SVD and Signal Processing, volume II. Algorithms, Applications and Architectures. Elsevier, Amsterdam, 1991.

[R53] M. Moonen and B. D. Moor, editors. SVD and Signal Processing, volume III. Algorithms, Applications and Architectures. Elsevier, Amsterdam, 1995.

[R54] Z. Petres: Polytopic Decomposition of Linear Parameter-Varying Models by Tensor-Product Model Transformation. PhD Dissertation, Budapest University of Technology and Economics, 2007.

[R55] J. Madár, J. Abonyi, F. Szeifert: "Feedback Linearizing Control Using Hybrid Neural Networks Identified by Sensitivity Approach", Engineering Applications of Artificial Intelligence, 18:343-351.

[R56] Petros A. Ioannou and Jing Sun: "Robust Adaptive Control", Prentice Hall, Upper Slade River, NJ, 1996.

[R57] H.K. Khalil: "Nonlinear Systems", 2nd ed. Prentice Hall: Upper Saddle River; 1996.

[R58] B. Goodwine, & G. Stepan: "Controlling Unstable Rolling Phenomena", Journal of Vibration and Control 2000, 6:137-158.

[R59] D. Hilbert: "Mathematische Probleme", 2nd International Congress of Mathematicians, Paris, France, 1900.

[R60] V.I. Arnold: "On functions of Three Variables", Doklady Akademii Nauk USSR, 114:679-681, 1957.

[R61] D.A. Sprecher: "On the Structure of Continuous Functions of Several Variables", Trans. Amer. Math. Soc., 115:340–355, 1965.

[R62] G.G. Lorentz: "Approximation of Functions", Holt, Reinhard and Winston, 1966. New York.

[R63] E.K. Blum & L.K. Li: "Approximation Theory and Feedforward Networks", Neural Networks, vol. 4, no. 4, pp. 511-515, 1991.

[R64] V. Kůrková: "Kolmogorov's Theorem and Multilayer Neural Networks", Neural Networks, vol. 5, pp. 501-506, 1992.

[R65] B. Kosko: "Fuzzy Systems as Universal Approximators", in Proc. of the IEEE Int. Conf. on Fuzzy Systems, San Diego, 1992, pp. 1153-1162.

[R66] J.L. Castro: "Fuzzy Logic Controllers are Universal Approximators", IEEE Trans. on SMC, vol. 25, pp. 629-635, 1995.

[R67] B. Moser: "Sugeno Controllers with a Bounded Number of Rules are Nowhere Dense", Fuzzy Sets and Systems, vol. 104, no. 2, pp. 269-277, 1999.

[R68] D. Tikk: "On Nowhere Denseness of Certain Fuzzy Controllers Containing Prerestricted Number of Rules." Tatra Mountains Math. Publ., vol. 16, pp. 369-377, 1999.

[R69] E.P. Klement, L.T. Kóczy, and B. Moser: „Are fuzzy systems universal approximators?," Int. J. General Systems, vol. 28, no. 2-3, pp. 259-282, 1999.

[R70] D. Tikk, P. Baranyi and R. J. Patton: „Polytopic and TS model are nowhere dense in the approximation model space. In Proc. of IEEE Int. Conf. on Systems, Man and Cybernetics SMC 2002, pp. 150–153, Hammamet, Tunisia, October 6–9, 2002.

[R71] P. Gahinet, P. Apkarian, and M. Chilali: "Affine parameter-dependent Lyapunov functions for real parametric uncertainty", in Proceedings of Conference on Decision Control, pp. 2026-2031, 1994.

[R72] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan: “Linear Matrix Inequalities in Systems and Control Theory”, SIAM books, Philadelphia, 1994.
[R73] P. Gáspár and J. Bokor. “Progress in system and robot analysis and control design”, Springer, 1999.
[R74] D. Tikk, P. Baranyi, R.J. Patton, I. Rudas, J.K. Tar: “Design Methodology of Tensor Product Based Control Models via HOSVD LMIs”, Proc. of the IEEE Intl. Conf. on Industrial Technologies 2002, pp. 1290-1295.
[R75] R. Reed: “Pruning Algorithms - A Survey”, in: IEEE Transactions on Neural Networks, 4., pp. 740-747, 1993.
[R76] S. Fahlmann, C. Lebiere: “The Cascade-Correlation Learning Architecture”. in: Advances in Neural Information Processing Systems, 2, pp. 524-532, 1990.
[R77] T. Nabhan, A. Zomaya, A: “Toward Generating Neural Network Structures for Function Approximation”, in: Neural Networks, 7, pp. 89-9, 1994.
[R78] G. Magoulas, N. Vrahatis, G. Androulakis: “Effective Backpropagation Training with Variable Stepsize”, in: Neural Networks, 10, pp. 69-82, 1997.
[R79] C. Chen, W. Chang: “A Feedforward Neural Network with Function Shape Autotuning”, in Neural Networks, 9, pp. 627-641, 1996.
[R80] W. Kinnenbrock: “Accelerating the Standard Backpropagation Method Using a Genetic Approach”, in: Neurocomputing, 6, pp. 583-588, 1994.
[R81] A. Kanarachos, K. Geramanis: “Semi-Stochastic Complex Neural Networks”, in: IFAC-CAEA '98 Control Applications and Ergonomics in Agriculture, pp. 47-52, 1998.
[R82] K. Levenberg: “A Method for the Solution of Certain Problems in Least Squares”. Quart. Appl. Math., 2:164—168, 1944.
[R83] D. Marquardt: “An algorithm for least-squares estimation of nonlinear parameters”, SIAM J. Appl. Math., 11:431—441, 1963.
[R84] J. Kennedy, R. Eberhart: “Particle Swarm Optimization”, Proc. of IEEE Intl. Conf. on Neural Networks, Perth, pp. 1942-1948, 1995.
[R85] S. Kirkpatric, C. D. Gelatt, Jr., and M. P. Vecchi: “Optimization by simulated annealing”, Science, 220:671–680, 1983.
[R86] J.H. Holland: “Adaptation in Natural and Artificial Systems”, University of Michigan Press, Ann Arbor, 1975.
[R87] M. Vose: “Modeling simple genetic algorithms”, Proceedings of the Foundations of Genetic Algorithms Workshop, Vail, CO: Morgan Kaufmann, 63-74, 1992.
[R88] D. Whitley: “An executable model of a simple genetic algorithm”, Proceedings of the Foundations of Genetic Algorithms Workshop, Vail, CO: Morgan Kaufmann, 45-62, 1992.
[R89] F. Castillo, A. Kordon, G. Smits, B. Christenson and D. Dickerson: “Pareto Front Genetic Programming Parameter Selection Based on Design of Experiments and Industrial Data”, Proceedings of the GECCO’06 Conference, July 8–12, 2006, Seattle, Washington, USA, pp. 1613-1620.
[R90] J. Koza: “Genetic Programming: On the Programming of Computers by Means of Natural Selection”, MIT Press, Cambridge, MA, 1992.
[R91] W. Banzhaf, P. Nordin, R. Keller, and F. Francone: “Genetic Programming: An Introduction”, Morgan Kaufmann, San Francisco, CA, 1998.

[R92] R. Feldt and P. Nordin: “Using Factorial Experiments to Evaluate the Effects of Genetic Programming parameters”, Proceedings of EuroGP’2000 Conference, pp. 271-282, Edinburgh, UK, 2000.

[R93] V. Pareto: “Cour d’Economie Politique”, Librarie Droz, Geneve, (first edition in 1896), 1964.

[R94] K. Deb: “Multi-Objective Optimization Using Evolutionary Algorithms”, JohnWiley & Sons, 2001, 1 – 12.

[R95] C.A. Mattson, A. A. Mullur, and A. Messac: “Smart Pareto Filter: Obtaining a Minimal Representation of Multiobjective Design Space”, Engineering Optimization, Vol. 36, No. 6, 2004, pp. 721-740.

[R96] N. Srinivas and K. Deb: “Multiobjective Optimization Using Nondominated Sorting in Genetic Algorithms”, Journal of Evolutionary Computation 2(3) (1994) 221-248.

[R97] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan: “A fast and elitist multiobjective genetic algorithm: NSGA-II, IEEE Transactions on Evolutionary Computation 6(2) (2002) 182-197.

[R98] Yahya El Hini: “Comparison of the Application of the Symplectic and the Partially Stretched Orthogonal Transformations in a New Branch of Adaptive Control for Mechanical Devices”, Proc. of the 10th International Conference on Advanced Robotics – ICRA 2001", August 22-25, 2001, Budapest, Hungary, pp. 701-706, ISBN 963 7154 05 1.

[R99] Gy. Schuster: “Fuzzy Approach of Backward Identification of Quasi-linear and Quasi-time-invariant Systems”, Proc. of the 11th International Workshop on Robotics in Alpe-Adria-Danube Region (RAAD 2002), June 30-July 2 2002, Balatonfüred, Hungary, pp. 43-50

[R100] Gy. Schuster: “Adaptive Fuzzy Control of Thread Testing Furnace”, Proc. of the ICCC 2003 IEEE International Conference on Computational Cybernetics, August 29-31, Gold Coast, Lake Balaton, Siófok, Hungary, pp. 299-304

[R101] Gy. Schuster: “Improved Method of Adaptive Fuzzy Control of a Thread Testing Furnace”, Proc. of the 2006 IEEE International Conference on Computational Cybernetics (ICCC 2006), Tallinn, Estonia, August 20-22, 2006, pp. 125-129

[R102] Gy. Hermann: “Application of Neural Network Based Sensor Fusion in Drill Monitoring”, Proc. of Symposium on Applied Machine Intelligence (SAMI 2003), Herl'any, Slovakia, February 12-14, 2003, pp. 11-24.

[R103] M. Bucko, L. Madarász: “Use of Fuzzy Sets for Creation of Controlling Mechanism of Expert System”, *Bulletins for Applied Mathematics*, Balatonfüred, May 13-16, 1993.

[R104] I. Kováčová, L. Madarász, D. Kováč, J. Vojtko: “Neural Network Linearization of Pressure Force Sensor Transfer Characteristics”. Proc. of the 8th International Conference on Intelligent Engineering Systems 2004 (INES’04), September 19-21, 2004, Technical University of Cluj-Napoca Romania, pp. 79-82, ISBN 973-662-120-0 (2004).

[R105] J. Tick & J. Fodor: “Some Classes of Binary Operations in Approximate Reasoning”, Proc. of the 2005 IEEE International Conference on Intelligent Engineering Systems, (INES '05), Sept. 16-19, 2005, pp. 123-128.

[R106] J. Tick and J. Fodor: “Fuzzy Implications and Inference Processes”, Computing and Informatics 24(6):591-602.

[R107] I.J. Rudas, M.O. Kaynak: "Minimum and maximum fuzziness generalized operators", Fuzzy Sets and Systems, 98(1):83-94.
[R108] I.J. Rudas: "Evolutionary Operators: New Parametric Type Operator Families", Intl. Journal of Fuzzy Systems 2(4):236-243.
[R109] G.G. Hall: "Alkalmazott csoportelmélet" (Applied Group Theory, in Hungarian), Műszaki Könyvkiadó, Budapest, 1975. ISBN 10 0805 3.
[R110] J. Nossek: (University of Munich, Germany): An informal seminar held at the Computer- and Automation Institute of the Hungarian Academy of Sciences, Summer 1992.
[R111] J.K. Tar et al: "Unified Approach to Non-Linear Robot Control Based on RT Quasi-Diagonalization of Symmetric Matrices", in proc. of 24th Int. Symp. on Industrial Robots, 4-6 Nov., 1993, Tokyo, Japan, pp. 791-798.
[R112] I. Rudas: "Novel methods and principles for modeling and control of some non-linear systems" – Doctoral Thesis defended at HAS (MTA Doktori értekezés), 2003.
[R113] "The Mechatronics Handbook," Editor-in-Chief: H. R. Bishop, joint issue by ISA – The Instrumentation, Systems, and Automation Society and CRC Press, Boca Raton London New York, Washington D.C., ISBN: 0-8493-0066-5, 2002.
[R114] Ali Zilouchian & Mo Jamshidi (Eds.), Intelligent Control Systems Using Soft Computing Methodologies, CRC Press, Boca Raton London, New York, Washington D.C., USA, 2001.
[R115] W.H. Press, B.P. Flannery, S.A. Teukolsky, W.T. Vetterling: "Numerical Recipes", Cambridge Univ. Press, 1986.
[R116] B. Lantos: "New Principles and Methods in Control and Identification of Robots", *Doctoral Theses, Hungarian Academy of Sciences*, (1993), (in Hungarian).
[R117] Lajos Jánossy: "Relativitáselmélet a fizikai valóság alapján", (in Hungarian) Akadémia Kiadó, Budapest, 1973, p. 111. (The Lorenzt Transformation as Deformation, Chapter V)
[R118] J. Vaščák, L. Madarász: "Similarity Relations in Diagnosis Fuzzy Systems", Journal of Advanced Computational Intelligence, Vol. 4, Fuji Press, Japan, ISSN 1343-0130, pp. 246-250, (2000). "In the case of certain problem classes similarity relations can also be observed and utilized to simplify the design process."
[R119] Herbert B. Callen: "Thermodynamics and Introuction to Thermostatistics", 2nd Edition, John Wiley & Sons, New York, Chichester, Brisbane, Toronto, Singapore, 1985, ISBN 0-471-86256-8
[R120] Raffaello D'Andrea: "Control of Autonomous and Semi-Autonomous Sytems", in the Proc. of the 4th International Workshop on Robot Motion and Control, June 17-20 2004, Puszczykowo, Poland, pp. 11-15, ISBN: 83-7143-272-0.
[R121] J.M. Holland: "Basic Robotics Concepts", Howard W. Sams, Macmillan, Inc., Indianapolis, IN., 1983.
[R122] J. A. Tenreiro Machado: Fractional Calculus and Dynamical Systems, invited plenary lecture at the IEEE International Conference on Computational Cybernetics (ICCC 2006), Tallinn, Estonia, August 20-22, 2006.
[R123] S. Lacroix: Traité du calcul differentiel et du calcul intégral, Courciel, Paris, France, 1819.

[R124] J. Liouville, Mémoire sur le calcul des différentielles a indices quelconcues,' J. Ecole Polytechn., vol. 13, pp. 71-162, 1832.
[R125] A.K. Grünwald, Über 'begrenzte' Derivationen und deren Anwendung, Zeitshrift für angewandte Mathematik und Physik, vol. 12, pp. 41-480, 1867.
[R126] A. Gemant, Method of Analyzing Experimental Results Obtained from Elasto-Viscous Bodies, Physics, vol. 7, pp. 311-317, 1936.
[R127] A. Gemant, On Fractional Differentials, The Phylosophical Magzine, vol. 25, pp. 540-549, 1938.
[R128] K. B. Oldham and J. Spanier, The Fractional Calculus: Theory and Application of Differentiation and Integration to Arbitrary Order, Academic Press, 1974.
[R129] K. S. Miller and B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, John Wiley and Sons, 1993.
[R130] I. Podlubny, Fractional Differential Equations, Academic Press, San Diego, 1999.
[R131] P. J. Torvik and R. L. Bagley, On the Appearance of the Fractional Derivative in the Behaviour of Real Materials, ASME Journal of Applied Mechanics, vol. 51, pp. 294-298, 1984.
[R132] C. G. Koh and J. M. Kelly, Application of Fractional Derivatives to Seismic Analysis of Base-isolated Models, Earthquake Engineering and Structural Dynamics, vol. 19, pp. 229-241, 1990.
[R133] J. Machado, A. Azenha, Fractional-order hybrid control of robot manipulators, IEEE International Conference on Systems, Man and Cybernetics, pp. 788-793, 1998.
[R134] O. P. Agrawal, Solution for a Fractional Diffusion-wave Equation in a Bounded Domain, Nonlinear Dynamics, vol. 29, pp. 145-155, 2002.
[R135] R. Isermann, K. H. Lachmann, and D. Matko, Adaptive Control Systems, New York DC, Prentice-Hall, USA, 1992.
[R136] C. C. Nguyen, Sami S. Antrazi, Zhen-Lei Zhou, Charles E. Campbell Jr, “Adaptive control of a stewart platform-based manipulator”, Journal of Robotic Systems, volume 10, no. 5, pp. 657-687, 1993.
[R137] J. Somló, B. Lantos, P.T. Cát, Advanced robot control, Akadémiai Kiadó, Budapest, Hungary, 2002.
[R138] K. Hosseini-Suny, H. Momeni, and F. Janabi-Sharifi, “Model Reference Adaptive Control Design for a Teleoperation System with Output Prediction”, J Intell Robot Syst, DOI 10.1007/s10846-010-9400-4, pages 1--21, 2010.