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William Duke
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September 2, 2011

Dear Committee,

The dissertation of Gergely Harcos contains a number of important results about automorphic L-functions. More specifically it proves new sub-convexity results for various automorphic L-functions. These have applications to equi-distribution results of great arithmetic interest.

In Theorem 1.1 Harcos (with Blomer) proves a sub-convexity bound for twisted L-functions associated to $GL(2)$-automorphic forms that is uniform in both the s and conductor aspects. This bound is an analogue of Burgess' sub-convexity bound for Dirichlet L-functions in the conductor aspect. It is obtained by means of a serious improvement of a method of Bykovskii and represents the limit of the method. It implies an improved estimate for the Fourier coefficients of half-integral weight cusp forms, which has applications to distribution of integer points by ternary quadratic forms.

In Theorem 1.2 Harcos (with Blomer and Michel) provides a subconvexity bound for automorphic L-functions in level aspect when the nebentypus is not trivial. This work is a nice advance on a previous theorem of W. Duke, J. B. Friedlander and H. Iwaniec who obtained the subconvexity result in the case of primitive nebentypus. This refinement is used in the proof of a result by Einsiedler, Lindenstrauss, Michel and Venkatesh given in the Annals paper "The distribution of periodic torus orbits and Duke's theorem for cubic fields."

Perhaps the most important result is Theorem 1.3 (with Michel). This gives a general subconvexity bound for Rankin-Selberg L-function when one form is fixed and the other varies. The important aspect here is the level of the varying form. This result builds on previous results by various authors and gives a very useful general result. A striking application is to the equi-distribution of "small" sets of cm points with respect to the discriminant. This is a deep result and requires substantial new ideas beyond the proof of the result for all the cm points of a given discriminant.

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The candidate has an impressive set of results representing the state of the art in sub-convexity bounds. The results are correct as far as I can judge and are certainly original. While some of the techniques are developments of previous work by others, the candidate's contributions are substantial and new. The dissertation clearly describes the history of the problems and attributes the development of ideas very carefully. As a whole the work is without doubt important and represents a significant advance in our knowledge of L-functions and their arithmetic applications. I declare without hesitation my acceptance of the setting of the date for a public debate and of the dissertation's work.

Sincerely

W. D. Duke

William Duke
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