Report on the dissertation
1-motives and Albanese maps in arithmetic geometry
by Tamás Szamuely

In his dissertation Szamuely treats important topics in the theory of semi-abelian varieties and 1-motives. These objects play an essential role in arithmetic geometry and the results about them are of central importance. There are three main topics in the dissertation. These are the Albanese map on open subvarieties of smooth projective varieties, arithmetic duality theorems for 1-motives and rational points on principal homogeneous spaces of semi-abelian varieties. Although these topics are interrelated for the purpose of this report I will treat them separately.

The Albanese map is one of the most important constructions in algebraic geometry. It factors through the Chow group and a famous theorem of Roitman establishes that for smooth projective varieties the Albanese map is an isomorphism on torsion elements of order prime to the characteristic of the base field. Milne extended this to the p-part. In a joint paper with Spiess, Szamuely extended this theorem to open subvarieties using Serre's generalized Albanese map, a universal map to semi-abelian varieties. The formulation of the theorem involves the degree zero part of algebraic singular homology of Suslin. The proof is completely new, and is based on a conceptually very clean approach that is very satisfying. It relies on a beautiful commutative diagram that compares Suslin homology with etale cohomology as well as the Albanase variety of the variety in question. The commutativity is the hard technical part which requires the re-interpretation of the Albanese map in Voevodsky's derived category of motivic complexes. This approach by Szamuely and Spiess is very innovative and the result is of great importance.

The above result on the torsion points whose order prime to the characteristic is complemented by another theorem on the generalized Albanese map for open subvarieties of smooth projective varieties over the algebraic closure of a finite field, where the isomorphism is shown for the full torsion without restriction on the order. This result is more number theoretic in nature and relies on the class field theory of tame coverings of varieties over finite fields.

1-motives appeared already above and the next two topics are entirely about them. Firstly duality theorems for commutative groups schemes over local and global fields are among the most fundamental results in arithmetic. Tate had played a crucial role in developing this theory, and his is name is attached to many of the results including the Nakayama-Tate duality for algebraic tori, Tate's duality for Abelian varieties, Cassels-Tate duality for Tate-Shafarevich groups. It is perhaps worth to point out that for his work Tate was awarded the Abel prize in 2010.

Szamuely in joint work with Harari established common generalizations of the above mentioned results to the case of 1-motives. These are special 2-term complexes introduced by Deligne. Each 1-motive has a Cartier dual equipped with a canonical derived pairing. Szamuely and Harari establish a duality pairing on the Galois hypercohomology of these complexes over local and global fields. These include statement for a duality pairing over a local field into $\mathbf{Q}/\mathbf{Z}$ and Tate-Shafarevich groups of motives over number fields as well as more technical cohomological tools such as a twelve term exact sequence of hypercohomology groups.

Finally in another work with Harari Szamuely applied these duality statements to local-global principles for principal homogeneous spaces for semi-abelian varieties. The prototype of such a principal homogeneous space is Selmer's curve $3x^3 + 4y^3 + 5z^3 = 0$ which also have the property that it has points on every completion of the rationals but nevertheless has no rational point. Since Hasse showed that such examples do not exist in lower degrees this is usually expressed as the failure of the Hasse principle.

The failure of the Hasse principle is explained partly by Manin's obstruction. It is important to know when Manin's obstruction is the only one to the existence of a global point given the existence of local points. Szamuely and Harari proved a beautiful result in this respect. They showed that for semi-abelian varieties with finite Tate-Shafarevich group Manin's obstruction is the only obstruction to the Hasse principle. This gives a very satisfying complete picture for the triviality of principal homogeneous spaces for semi-abelian varietes.

The whole dissertation is filled with very important results, usually proven with great conceptual clarity and using the highest level and most modern tools of the theory. These results form the basis of further work of other scientists in the field. For these reasons I very strongly recommend the acceptance of the dissertation.

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