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Heterogeneity in Stock Market Expectations and Portfolio Choice of American Households

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1 Introduction

1.1 Motivation

Decisions under uncertainty are shaped by the decision makers' preferences, the constraints they face, and their beliefs about the future. The conceptual separation of beliefs from preferences is perhaps one of the most important assumptions behind the economic theories of decision under uncertainty. The separation of beliefs from preferences is complete in expected utility theory with subjective probabilities. However, most nonexpected and non-Bayesian utility theories also separate the two, at least at a conceptual level (see, for example, Machina, 2002, for a review of nonexpected utility theory and Gilboa, 2010, for a critical review of decision theory).

Economic analyses typically focus on the effects of constraints on decisions. Constraints are considered more likely to be affected by policy decisions, while individual preferences are thought to be unaffected by policy decisions. Whether and how beliefs can be affected by policy decisions is typically not addressed in economics. An important exception is the role of inflation expectations in the Lucas critique of policies exploiting the Phillips curve (Lucas, 1976). One interpretation of the critique is that policies cannot fool people, and except for some descriptive research on inflation expectations and their relationship to central banks' credibility, most economists continued to assume away the impact of policies on beliefs. However, beliefs may be influenced by policy decisions, either directly or indirectly, more so than preferences. In any case, assumptions about the stability of, or effects of policies on, beliefs or preferences are empirical questions and are difficult to assess without adequate measurement.

However, measuring preferences and beliefs at the same time is difficult by observing actions only. Revealed preference theory (see, for example, Richter, 2008, for a review) is applicable to decisions under uncertainty only if one fully specifies the individual beliefs about the future. For example, in rational expectations equilibrium, people are assumed to make decisions based on beliefs about the probability distribution of future states of the world that will prove to be correct in the sense that people's subjective beliefs will in turn characterize the actual probability distribution of future states. Unfortunately, assumptions about people's beliefs are difficult to test by observing actions only, unless one is willing to assign fully specified preferences to the decision makers.

Therefore, it appears obvious that these questions prescribe and agenda for empirical research. We should learn about people's preferences and beliefs in many decision situations that are important for economic theory or policy.

There is a growing literature on uncovering preferences and on establishing relevant heterogeneity in these preferences Typically, this type of research examines people's decisions in experimental situations. The most important general preferences regarding decisions under uncertainty are related to risk and time. (Experiments in non-expected utility frameworks often aim at measuring other preferences such as ambiguity aversion or loss aversion.) In typical experiments that measure risk preferences, decision makers are presented with lotteries with fully specified probability distributions and are asked to choose from the available options. In effect, in risk preference experiments, the experimenter induces variation in the probability distribution while keeping preferences constant (because the probabilities vary for the same people). Assuming that people fully understand and internalize those probabilities, their choices can then be used to recover their risk preferences. A similar variation can be induced in terms of the time horizon of outcomes to measure time preferences.

Many such experiments were criticized for analyzing specific populations (often undergraduate students) and studying small samples. However, it is of course possible to conduct similar experiments in large representative samples, although such experiments are rather costly. Dohmen, Falk, Huffman and Sunde (2010) is an excellent example: they investigate correlations between intelligence ("IQ") on the on hand and risk preference and time preference the other hand in a sample that is representative of the adult population in Germany. This research finds that there is substantial heterogeneity in risk aversion, and that, on average, risk aversion is moderate.

Typical experiments involve outcomes with real money at stake. Therefore, these experiments are "incentivized," which means that people's decisions have consequences for them in terms of real money. For obvious reasons, however, the amount of money at stake in these experiments is usually small compared to the money at stake in the most important real-life decisions that concern economists.

An alternative method for eliciting preferences presents respondents with hypothetical gambles, again with fully specified probability distributions - and then asks respondents to make hypothetical decisions. The seminal paper measures risk preferences: Barsky, Juster, Kimball and Shapiro (1997) ask people to choose between hypothetical jobs that would result in different risks in terms of lifetime earnings and then recover the parameter of risk tolerance assuming constant relative risk aversion utility. Importantly, these types of hypothetical gamble questions allow for recovering "cardinal" (numerical) measures of risk preferences with some additional assumptions, while the more widely used simpler survey questions (how would you rate your risk tolerance?) allow for ordinal measures (ranking) only. A series of follow-up papers examined the stability of measured preferences, the effect of question wording and the role of measurement error (Sahm, 2007; Kimball, Sahm and Shapiro, 2008). They find substantial heterogeneity in risk aversion and that risk preferences are stable over time. As an important methodological contribution, they also find that there is substantial error in the survey measures, and they develop a method to address that error appropriately. They also find that most people are likely significantly more risk averse in these hypothetical situations than has been previously expected and observed in incentivized experiments, which is an important puzzle.

A widely aired criticism of the hypothetical gambles approach is that people may not make thoughtful decisions if they do not have the right incentives to do so. Consequently, the preference measures from this approach, and the heterogeneity therein, may have little to do with "real" preferences that are relevant in real-life situations. The evidence in this respect is mixed. On the more affirmative side, Dohmen, Falk, Huffman and Sunde (2010) find that an ordinal measure of risk preference from simple survey questions is strongly correlated with risk preference measures derived from decisions in an incentivized experiment. At the same time, Anderson and Mellor (2009) find that risk preference measures derived from hypothetical gambles of this type are often very weakly, if at all, correlated with risk preference measures derived from decisions in incentivized experiments. The authors also find that the situation described in hypothetical gambles matters with regard to the hypothetical decisions people make, and the strength of the correlation of the risk measures derived from those answers to the experimental measures vary with the described situation. Unfortunately, investigations of this type fall short of truly informative validations precisely because the large stakes involved in the most informative hypothetical gambles are impossible to implement in incentivized experiments.

Another concern with the hypothetical gamble approach is the cognitive difficulty of understanding hypothetical situations. It usually takes many complicated sentences to describe these situations, with many important details. Respondents are also asked to make quick decisions in situations that they most likely have never experienced. If the respondents were to experience such situations, their decisions would most likely take a lot of time, and they may consult other people, none of which is available in a survey situation. Note, however, that the cognitive difficulty also appears to be a problem in incentivized experiments: Dave, Eckel, Johnson and Rojas (2010) show that different wording can lead to different decisions among people with lower numerical skills, even in an incentivized experiment.

Therefore, it appears that the approach to measure preferences has made substantial progress but still has its problems.

A complementary approach aims at measuring beliefs. Here the experimental approach is not feasible: while it is possible, at least in principle, to place decision makers in situations with fully specified probabilities and then vary those probabilities to observe how decisions change, giving people preferences and varying those preferences is obviously impossible. This problem leaves researchers with one possibility if they want to measure beliefs: to ask decision makers directly about their beliefs.

A direct measurement of beliefs has potential problems that are similar to the hypothetical gambles approach to elicit preferences. In a typical survey situation, there is little time to answer the questions, and, beyond a spirit of cooperation, there are no incentives to get the answers right. Furthermore, there is an additional issue: asking about beliefs requires questions that people understand but that provide answers that are also useful in characterizing people's beliefs in a theoretically satisfactory way.

Manski (2004) argues that researchers should ask probabilities from decision makers if they are interested recovering decision makers' beliefs. If people have well-defined probabilities in their minds, asking for those probabilities is certainly the right approach. If people think about uncertainty in other ways, asking for probabilities may be more problematic. However, answers to probability questions may be informative even in that latter case.

We know little about how people actually think about uncertainty when they make economically relevant decisions. Furthermore, even if we knew more about this phenomenon, whether people can represent that uncertainty in probabilities when asked about it would be a different question. It is possible that, if necessary, not fully conscious accounts of uncertainty can be translated into probabilities even if decision makers do not make that transformation explicit in their thought processes when making decisions. In fact, the possibility of this transformation is the assumption of subjective probability theory. Of course, it is also possible that uncertainty is represented in ways that are impossible to translate into well-defined probabilities.

We know that people often make statements about uncertainty that do not conform with the laws of probability. Moreover, people often or make choices that do not satisfy the assumptions of subjective probability theory (see, for example, Ellsberg, 1961 or Tversky and Kahneman, 1974). This finding may imply that the expected utility theory and the subjective probability theory completely misrepresent the way people make decisions. Undoubtedly, failures in specific situations undermine the general nature of the theory. However, such failures do not necessarily undermine the theory's usefulness in certain situations: those theories may be sufficient descriptions of the way decisions are made in some situations but not all situations. For example, the fundamental thought processes may be significantly different from what expected utility theory would suggest in the sense that people do not calculate probabilities of future states of the world, attach utilities to each state, and/or multiply those probabilities with characteristics of states of the world and then add up those products. Instead, people may rely on heuristics and fast-and-frugal decision algorithms (see, for example, Gigerenzer, 2008). The expected utility theory with subjective probabilities may still be a good approximation to those decision rules in situations but not necessarily in all situations.

Whether probabilities are the way people actually think about uncertainty and the extent to which people's decision-making process can be appropriately approximated by expected utility theory, are very relevant questions that need further investigation. Unless the answer to those questions is very negative, asking probabilities from people about appears to be a sensible approach to making them characterize the uncertainty they face. There is a small but growing literature that makes use of people's answers to questions on probabilities of future events.

Research on expectations measured in probabilistic forms is made possible by the fact that some major surveys have begun to include questions on probabilities of future events. A pioneer in this approach is the Health and Retirement Study (HRS). The HRS is a large biannual panel household survey, representative of the American population 51 years of age or older and their households (see Juster and Suzman, 1995 and NIA, 2007, for reviews). The HRS has included several probability questions since its start in 1992. The inclusion of probability questions was initiated by the late Thomas Juster, the first principle investigator of the HRS - and a long-time advocate of eliciting beliefs by probability questions (see, for example, Juster, 1966).

The HRS includes questions on the probability of events such as living to certain age; working past a certain age; losing a job (if working) or finding a job (if unemployed); receiving an inheritance; and leaving an inheritance. Since 2002, the HRS has included one or more questions on the probability that the stock market would go up (or down) by some threshold values.

The example of the HRS itself led to similar surveys ("sister studies") around the world, including in Great Britain, Mexico and Japan. The Survey of Health, Ageing and Retirement in Europe (SHARE), a harmonized survey fielded in 19 European countries (and Israel), also closely follows the example of the HRS.[1] The success of the HRS and the spread of HRS-type surveys are in part due to the fact that population aging is one of the most important structural challenges of the developed world, and studying aging requires panel data with information in many domains. Another component of the HRS success is its organizational structure, which was closely followed by its sister studies. The HRS is governed by researchers as opposed to professional data collection agencies. Consequently, the content of the questionnaire is closely related to important research questions. Similarly to the HRS, its sister surveys also include questions on expectations, typically in the form of probability questions (although stock market expectations are typically not included in those other

[1] Hungary joined SHARE in 2011 in its 4^{th} wave.

surveys).

Hurd (2009) provides an overview of the first years of empirical research on expectations as measured by survey questions on probabilities. The conclusions of his overview are cautiously optimistic. It appears that there is substantial heterogeneity in people's expectations. It also appears that probabilistic measures of expectations fielded on surveys can capture a substantial part of that heterogeneity. Moreover, it appears that people's answers to these probability questions, on average, can be rationalized relatively easily in many important domains.[2]

However, these measures have their own problems. Quite naturally, the lack of incentives may be problematic because respondents may not put in the required effort and thus may not give well-grounded answers to these types of survey questions, a problem similar to questions on hypothetical gambles. Note that whether and how one could incentivize the elicitation of beliefs is a very difficult question that has not been addressed in the literature. People's ability to think in terms of probabilities may make these questions even more difficult to understand. One concern is the overall validity of expectation measures from such questions; another concern is the potential relationship between answer quality and respondents' cognitive capacity. This second concern is especially severe because an apparent correlation of expectation measures with real-life decisions may simply stem from, on the one hand, a correlation of those decisions with cognitive capacity, and on the other hand, the correlation of cognitive capacities with the quality of the probability answers, instead of from a genuine correlation between decisions and expectations.

Expectations, or beliefs about the future, are the subject of this dissertation.[3] I investigate ordinary individuals' expectations about returns attainable on the stock market. I focus on American Households because of data availability. The research I report on in this dissertation takes the measurement issues seriously. In fact, some of my research is in the forefront at addressing those concerns.

Stock market expectations are important for many reasons. These expectations should be relevant for the prices of stocks and other assets, the volume of transactions and other aggregate measures of asset markets. Differences between people regarding their stock market

[2] Important exceptions are massive overstatement of probabilities of rare negatve events such as natural disasters or terrorist attacks (Christelis and Georgarakos, 2010), or teen-agers almost absurd overestimation of the risks of major accidents or a premature death (Fischoff, 2008). Whether and how stock market expectations can be rationalized is a more complex question and a subject of this dissertation.

[3] I use "expectations" in a broad sense to denote beliefs about the future (as opposed to a more narrow use for "expected value"). Throughout the dissertation, I use the words "beliefs" and "expectations" interchangably.

expectations may be relevant for differences in portfolio choice behavior and, in turn, wealth accumulation. Furthermore, the existence of differences in stock market expectations is an important question in itself, from a theoretical perspective. Stock market expectations are expectations about market prices, with little room for private information. It is not obvious that individuals should exhibit substantial differences in stock market expectations; if they do exhibit substantial differences, we should understand the origins of such differences.

The primary focus of my dissertation is on the last question: why do people have different expectations about future stock market returns? I also address the important follow-up question: if people differ in their expectations, does that lead to differences in their portfolio decisions? The answers to these questions are also important for understanding asset prices and wealth differences.

Households' portfolio decisions determine the structure of assets that households chose to hold for their savings. In the United States, as in many other countries, fewer households hold stocks than standard theory would imply, at least if risk preferences are "sensible" (i.e., if the risk aversion is not extremely high) and beliefs are close to what historical evidence would suggest. This observation is the so-called "stockholding puzzle" (Mankiw and Zeldes, 1991; Haliassos and Bertaut, 1995; Campbell, 2006; Poterba, Rauh, Venti, and Wise, 2006). The stockholding puzzle is related to the equity premium puzzle, which states that returns on stocks observed in the past 100 years are difficult to reconcile with their historical risks (Mehra and Prescott, 1985; Kocherlakota, 1996; it appears that the recent financial crisis did not undo the equity premium puzzle, Damodaran, 2012).

There are three potential theoretical resolutions of the puzzle as well as an empirical resolution. The empirical resolution aims at showing that, for most people, stocks are significantly more risky than what the aggregate exercise by Mehra and Prescott suggests (the paper by Malloy, Moskowitz and Vissing-Jorgensen, 2008, is perhaps the most convincing of those attempts). The three theoretical directions are the following: many people may face very strong constraints that prevent them from investing in stock-market based assets; many people may be much more risk averse than what has formerly been judged as "sensible"; and many people may have beliefs about future stock returns that are characterized by substantially lower expected value and/or substantially higher perceived risks than what historical evidence would suggest.

There is an empirical literature on stock market participation that has focused on reduced-form effects of demography, education and wealth (Ameriks and Zeldes, 2000; Guiso, Haliassos, and Jappelli, 2002), cognitive capacity (Christelis, Jappelli and Padula, 2006), health (Rosen and Wu, 2003), or social interactions (Guiso, Sapienza and Zingales, 2004; Hong, Kubik and Stein, 2004). From a theoretical perspective, it is not always clear whether those factors should be understood as constraints or as determinants of preferences or beliefs. I think that it is obviously impossible to evaluate the three potential theoretical explanations without sensible measures for preferences and expectations. This dissertation reports on research that shows that a large part of the reduced-form relationships documented by the literature likely operates through expectations.

1.2 Structure of the dissertation

The dissertation consists of three chapters in addition to this introductory chapter and the chapter with concluding remarks. Chapter 2 characterizes the heterogeneity of stock market expectations among American households and aims at explaining that heterogeneity. The results of Chapter 2 suggest that an important part of the heterogeneity is the result of differences in learning histories, which are in turn caused by differences in returns to and costs of learning (as well as in initial conditions).[4] Chapter 3 provides additional empirical analysis to support the explanation put forward in Chapter 2, and it looks for other potential sources of heterogeneity in people's personality traits.[5] Chapter 4 examines the effect of the stock market crash of September 2008 on households' expectations.[6] The main text is kept relatively short to make it more accessible. Each chapter is complemented with its own appendix, with technical details and additional empirical results.

1.3 Methods

The three chapters ask different questions and use different samples of the Health and Retirement Study (HRS); however, the chapters are based on the same (or very similar) survey questions and use a common methodology. The measurement problem in each chapter is

[4] The title of Chapter 2 is "Heterogeneity in expectations about future stock returns, learning incentives and portfolio choice". It is an updated version of a previous paper coauthored with Robert J. Willis. The title of that previous paper was "Household Stock Market Beliefs and Learning," and the paper was published as NBER Working Paper 17614.http://www.nber.org/papers/w17614.pdf. The paper was featured in VOX, the policy and popular economic-research portal of the Centre for Economic Policy Research at http://www.voxeu.org/article/beliefs-and-stock-market.

[5] The title of Chapter 3 is "Financial knowledge, personality and expectations about future stock returns". It is new. Preliminary results from that chapter were presented at the "Formation and revision of subjective expectations" conference, held on November 8-9, 2012 in Québec city, Canada.

[6] The title of Chapter 4 is "Stock market crash and expectations of American households". It is an edited version of a paper coauthored with Péter Hudomiet and Robert J. Willis. The paper was published in The Journal of Applied Econometrics, 26: 393–415 (2011).http://onlinelibrary.wiley.com/doi/10.1002/jae.1226/abstract

to characterize expectations about stock market returns based on answers to probability questions.

All three chapters assume that people believe that yearly log returns are i.i.d. and normally distributed. The mean of log returns is denoted as μ and the standard deviation as σ. For example, $\mu = 0.1$ means that the mean return is approximately ten per cent. At a yearly frequency, the i.i.d. normal assumption for log returns is in line with historical data. In the period of 1945 to 2012, for example, yearly log nominal returns of the Dow Jones index were characterized by a mean of $\mu = 0.06$ and a standard deviation of $\sigma = 0.16$. Different windows can give lower and higher values of μ, and the value of σ is remarkably stable. Under the i.i.d. lognormality assumption, the beliefs of individual i about the stock market returns are fully characterized by her beliefs about the mean and the standard deviation, and we denote those subjective beliefs by $\tilde{\mu}_i$ and $\tilde{\sigma}_i$. (Index i refers to potential heterogeneity in the parameters, and the tilde refers to the subjective nature of the parameters.) We define $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ as the parameters that would characterize individual beliefs in investment situations. The goal in each paper is to characterize heterogeneity in $\tilde{\mu}_i$ and $\tilde{\sigma}_i$, understand the sources of that heterogeneity, and, in Chapter 2, establish its relationship to the heterogeneity in household portfolios.

$\tilde{\mu}_i$ and $\tilde{\sigma}_i$ are unobserved in the data. Instead, the HRS data includes answers to probability questions. In all three chapters we make use of the answers to two question. The first question is the same in each dataset: it asks what the respondent thinks is the probability that the market will go up. Answers to this question are denoted as p_0. In Chapter 2, the second question (p_{10}) asks about the probability that the market will go up by at least 10 percent. In Chapter 3, the second question (p_{20}) asks about the probability that the market will go up by at least 20 percent. In Chapter 4, the second question (p_c) asks about the probability that the market will go up by at least c per cent or go down by at least c percent ($c \in \{10, 20, 30, 40\}$) .

If answers to two probability questions are available, identifying the mean and standard deviation of log returns from the two probabilities is relatively straightforward under the normality assumption, by making use of the inverse normal c.d.f. Intuitively, higher $\tilde{\mu}_i$ corresponds to higher probabilities, while higher $\tilde{\sigma}_i$ pushes the argument of the c.d.f. toward zero thus pushing both probabilities towards 0.5.

To see the correspondence between the structural parameters ($\tilde{\mu}_i$ and $\tilde{\sigma}_i$) and the probabilities more intuitively, Figure 1.1 shows three probability distribution functions together with vertical lines at the cutoff points of 0 and 0.1 log returns that correspond to the p_0 and p_{10} questions. The continuous line shows a p.d.f. with historical moments between 1945 and 2002 ($\mu = 0.07$ and $\sigma = 0.15$) that is the relevant time period for the analysis in Chapter 2.

The dashed line corresponds to a mean-preserving spread (higher perceived risk), and the dotted line corresponds to a lower mean (more pessimistic beliefs).

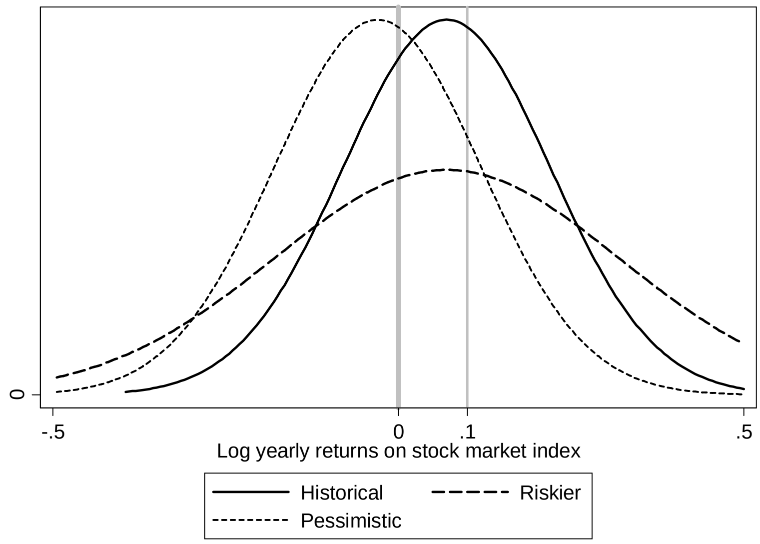

Figure 1.1. Examples for probability densities of normally distributed log returns, with the cutoff points for p_0 and p_{10}

If the probability answers were precise integrals of the relevant density, they would be equal to the area to the right of the corresponding bars at 0 and 0.1 log returns. We can denote those ideal probabilities with starts, such as p_0^*. The series of post-war returns up to 2002 (the year of the data used in Chapter 2) corresponds to $p_0^* = 0.68$, $p_{10}^* = 0.42$ and $p_0^* - p_{10}^* = 0.26$. Holding risk constant, more pessimistic beliefs result in smaller values of p_0^* and p_{10}^*. Therefore, we can think of the p^* variables as proxy variables for the perceived level of returns. A mean-preserving spread leads to a smaller area between the two vertical bars, which equals the difference $p_0^* - p_{10}^*$. The difference between the two answers may thus serve as a proxy for the inverse of the perceived risks. Analogous transformations could be used for other positive cutoff values instead of 10 per cent. These proxies are far from being perfect, though, partly because of the measurement issues (see below) and partly because, for example, a mean-preserving spread can also change p_0^* (see more details on that in Section 2.2).

The measurement problem means that, instead of the theoretical probabilities that we denoted by p^*, we are likely to observe something else in the data. There are strong theoretical reasons to believe that people's answers to the probability questions are not equal to the p^* transformations of these parameters. There is little time to answer the questions, and, beyond a spirit of cooperation, there are no incentives to get the answers right. It is therefore better to consider actual answers as "guesses" for what the p^* values may be, given recollections of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$.

The data shown in Chapters 2 and 3 (and the corresponding appendices) reveal answer patterns that strongly support this view. Almost all answers are rounded to the nearest 10, or to 25 or 75 percent. Focal values at 50 per cent account for an especially large part of all answers. Many respondents give the same answer to p_0 and p_c (for positive values of c), which, taken at face value, would imply infinitely large standard deviations of log returns. Rounding would allow for finite (but large) standard deviations to give that pattern. Some respondents give $p_0 < p_c$ (again, for positive values of c), which does not conform with the laws of probability. It may be that these respondents do not understand probabilities at all. It is also possible that these answers reflect inattention to one or both questions. The empirical evidence is in line with the latter interpretation. Fortunately, the data in the 2002 wave of the HRS allows for a direct assessment of survey noise because a small subset of the respondents answered the same probability questions a second time, in an experimental module. When these respondents were asked to answer the same probability questions a second time during the same interview approximately half an hour later, most gave different answers. Perhaps surprisingly, all three noise features (rounding, apparent violations of the laws of probability and test-retest noise) appear largely random. Some of these answer patterns make computing the p^* values impossible. All answer patterns indicate that actual answers are noisy transformations of relevant beliefs.

To address those problems, we developed a structural measurement model to estimate the heterogeneity in the relevant belief variables and to handle survey noise. The model relates the latent belief variables $(\tilde{\mu}_i, \tilde{\sigma}_i)$ to the observed answers to the probability questions (p_{0i}, p_{ci}) for any positive cutoff value c (negative cutoff values are straightforward to include in the analysis, and section C.2 in Appendix C. shows the details). The model accommodates all of the observed answer patterns and our intuition about how people answer difficult survey questions.

Our estimation strategy is structural in that it focuses on the theoretically relevant parameters and the relevant heterogeneity in those parameters (net survey noise). In particular, we estimate the moments of the distribution of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ in the population and in various subpopulations (analogously to Table 1 above), and we investigate the role of the heterogeneity of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ in the heterogeneity of stockholding. We model the differences between "theoretical" probabilities (the p^* variables above) and actual probability answers (the p variables) in two steps. The first step introduces survey noise, and the second step introduces rounding.

Noise is modeled as mean-zero additive components to the index $\tilde{\mu}/\tilde{\sigma}$ that enters the probabilities p_0 and p_c. The noise components, denoted by v_0 and v_c, are assumed to be jointly normal and potentially correlated. Let p_{0i}^{br} and p_{ci}^{br} denote hypothetical "before rounding" answers so that the observed answers p_{0i} and p_{ci} may be rounded versions of the former. Conditional on the draw of the noise variables, these hypothetical survey answers are then the following:

$$p_{0i}^{br} = \Phi\left(\frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{0i}\right) \tag{1}$$

$$p_{ci}^{br} = \Phi\left(\frac{\tilde{\mu}_i - c/100}{\tilde{\sigma}_i} + v_{ci}\right) \tag{2}$$

$$\begin{bmatrix} v_{0i} \\ v_{ci} \end{bmatrix} \sim N\left(0, \sigma_v^2 \begin{bmatrix} 1 & \rho_v \\ \rho_v & 1 \end{bmatrix}\right) \tag{3}$$

The noise components are assumed to be independent of any relevant heterogeneity, which is consistent with the randomness of the test-retest error and the near-randomness of the other noise features. The bivariate nature of the noise accommodates answers of $p_{0i} < p_{ci}$ if that phenomenon is due to inattention to the survey (which, as noted earlier, is supported by the near-randomness of its prevalence). The correlation coefficient between v_0 and v_c is related to average inattention. $\rho_v = 1$ would mean that all respondents answer questions p_0 and p_c with the same noise, which would not allow for answers such as $p_{0i} < p_{ci}$. At the other extreme, $\rho_v = 0$ would mean that all respondents forget their previous answers completely. The true value of ρ_v is likely to be in-between. Luckily we can use the test-retest evidence in HRS 2002 to us identify moments of the noise process (σ_v^2, ρ_v). We assume that the noise components in the core and module answers are independent, which is consistent with the evidence that we have.

Answers to the probability questions may differ from the hypothetical "before-rounding" probabilities p^{br} due to rounding. We accommodate rounding by an interval response model. An answer within a pre-specified interval can correspond to any probability p^{br} within that interval. Round numbers are in the middle of those intervals, which are defined in an exogenous fashion and are assumed to be the same for all respondents.

Formally, the vector of survey answers (p_{0i}, p_{ci}) is in the quadrant Q_{kl} if the vector of the hypothetical probabilities p_{ij}^{br} is in that quadrant:

$$\begin{pmatrix} p_{0i} \\ p_{ci} \end{pmatrix} \in \mathbf{Q}_{kl} \Leftrightarrow \begin{pmatrix} \Phi\left(\frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{0i}\right) \\ \Phi\left(\frac{\tilde{\mu}_i - c/100}{\tilde{\sigma}_i} + v_{ci}\right) \end{pmatrix} \in \mathbf{Q}_{kl} \tag{4}$$

$$\mathbf{Q}_{kl} = \begin{pmatrix} [q_k, q_{k+1}) \\ [q_l, q_{l+1}) \end{pmatrix} \tag{5}$$

In the implemented model, the intervals are defined, in percentage terms, as $[0,5)$, $[5,15)$, $[15,25)$,..., $[95,100]$. These intervals allow for rounding to the nearest ten. The interval response model is the simplest way of accommodating rounding that is compatible with the guesswork of calculating probabilities.

With additional assumptions on the cross-sectional distribution of in $\tilde{\mu}_i$ and $\tilde{\sigma}_i$, this model allows for estimating moments of the relevant heterogeneity in $\tilde{\mu}_i$ and $\tilde{\sigma}_i$. We assume that $\tilde{\mu}_i$ is normally distributed and $\tilde{\sigma}_i$ follows a two-point distribution. We estimate the conditional mean of the normal distribution, the conditional standard deviation of the normal distribution, and the probability of the low point conditional on the right-hand-side variables.

The expected value of $\tilde{\mu}_i$ across respondents is specified as a linear combination of right hand-side variables, with parameter vector β_μ. Unobserved heterogeneity in $\tilde{\mu}_i$ is assumed to follow a normal distribution with zero mean and standard deviation that is allowed to be related, in a linear fashion, to the right hand-side variables, with parameter vector β_u. This heteroskedasticity specification in $\tilde{\mu}_i$ allows for estimating differences in disagreement by groups defined by the observable characteristics. Heterogeneity in $\tilde{\sigma}_i$ is specified as a two-point distribution with the lower point fixed to the historical standard deviation, the upper point estimated as the same scalar for everyone, and the probability of the upper point specified as a probit model with parameter β_σ on the observable characteristics.

$$\tilde{\mu}_i = \alpha + \beta'_\mu x_i + u_i \tag{6}$$

$$u_{\mu i} \sim N\left(0, \exp\left(\beta'_u x_i\right)\right) \tag{7}$$

$$\tilde{\sigma}_i \in \left\{\tilde{\sigma}_{low}, \tilde{\sigma}_{high}\right\} \tag{8}$$

$$\Pr\left(\tilde{\sigma}_i = \tilde{\sigma}_{low}\right) = \Phi\left(\beta'_\sigma x_i\right) \tag{9}$$

The model can be estimated by Maximum Likelihood. The details of the likelihood function are provided in section A3 in Appendix A.[7]

[7] The specification of $\tilde{\sigma}_i$ is different between the three chapters, with a minor technical difference between Chapters 2 and 3 and a more substantive difference in Chapter 4. In The description above is correct for Chapter 2. In Chapter 3, heterogeneity in $\tilde{\sigma}_i$ is modeled as a two-point distribution, very similarly, only it is not the probability of the low point but the probability of the high point that is specified. Of course, that is a minor technical difference that affects the interpretation of the coefficients but not the identification or estimation. However, in Chapter 4, heterogeneity in $\tilde{\sigma}_i$ is specified as a log-linear function of right-hand-side variables, see equation (36). The parameters of that latter specification are more difficult to identify than parameters of the two-point specification. Consequently, the two-point distribution is the preferred specification. Despite its apparent restrictions, the two-point distribution is rather flexible (see, for example Heckman and Singer, 1984). Despite their differences, the two approaches yield qualitatively very similar estimates, as the parameters on the demographic right-hand-side variables demonstrate (compare,

With the exception of β_μ, the parameter estimates from the structural econometric model are not easy to interpret. At the same time, we can use the estimates to compute predicted values of $\hat{\mu}_i$ and $\hat{\sigma}_i$ for each respondent. The predictions use the estimates of the structural econometric model and the observable right hand-side variables as well as the observed probability answers. In formulae,

$$\hat{\mu}_i = \widehat{\mathbb{E}}\left[\tilde{\mu}_i | x_i, (p_{0i}, p_{ci}) \in \mathbf{Q}_{kl}\right] \quad (10)$$

$$\hat{\sigma}_i = \widehat{\mathbb{E}}\left[\tilde{\sigma}_i | x_i, (p_{0i}, p_{ci}) \in \mathbf{Q}_{kl}\right] \quad (11)$$

The conditional expectations are relatively straightforward to compute by Bayes' rule with the results of the structural model that specifies the full distributions for $\tilde{\mu}_i$ and $\tilde{\sigma}_i$. The predicted $\hat{\mu}_i$ and $\hat{\sigma}_i$ are then the sample analogues to those conditional expectations. The details of the derivation are provided in section A3. in Appendix A. This prediction method is analogous to the prediction of risk tolerance based on survey answers to hypothetical gambles by Kimball, Sahm and Shapiro (2008). The predicted values are different from the true values, creating measurement error in the variables. The measurement error is one of prediction error. The measurement error has zero mean and thus leads to an unbiased estimate of the population mean; however, the measurement error leads to an underestimation of the population standard deviation (because the predicted values are less dispersed than the true values). Using $\hat{\mu}_i$ and $\hat{\sigma}_i$ on the right-hand side of a regression leads to consistent estimates as long as all the covariates used in the predictions are also entered in the respective regression. The standard errors in this regression are inconsistent, though; thus, bootstrap standard errors are advised. If one uses the $\hat{\mu}_i$ and $\hat{\sigma}_i$ in regressions that have different covariates from the ones used in the prediction equations, OLS is inconsistent and a more sophisticated GMM procedure is appropriate (see Kimball, Sahm and Shapiro, 2008, for more details).

In all three chapters, the results from using the structural estimation method are qualitatively similar to, but stronger than, results using reduced-form linear regressions with the probability questions. The structural results with p_{20} as the second variable (in Chapter 3) are very similar to the results with p_{10} as the second variable (in Chapter 2) , except that the former show substantially larger variation in $\tilde{\sigma}_i$. Altogether, these results provide validity to the structural model. However, if the second probability variable is based on a negative threshold (as in the data description part of Chapter 4 and the robustness checks reported in Appendix C), the survey noise appears to have very different patterns (note that ρ_v, the noise

for example, the coefficient on the female variable in the equations for $\tilde{\sigma}$ in Tables 3.12, 4.4 and the female variables combined with marital status in Table A4.1).

correlation parameter, is estimated in such cases instead of using the calibrated value from HRS 2002). This fact highlights the importance of appropriate evidence regarding survey noise, which we do not have in relation to the negative-threshold stock market probability answers.

1.4 Preview of the results

The substantive aims of Chapter 2 are to characterize the heterogeneity of the stock market beliefs of American households, to understand the sources of that heterogeneity and to establish its relation to household portfolios. We hypothesize that heterogeneity is the result of differences in learning histories, which are in turn caused by differences in returns to and costs of learning (as well as in initial conditions). People learn about finance in general and the stochastic process of stock market returns in particular. The value of learning is proportional to savings, however, the costs are fixed. Consequently, people with higher earnings prospects should learn more than people with lower income prospects, especially if social security or defined benefit pension plans provide enough retirement income for the latter. Differences in the costs of learning and differences in general attitudes may also be heterogenous, creating additional heterogeneity in learning outcomes. Initial conditions matter as well. People with very low expectations will be less likely to learn and will see their beliefs unchanged. Ultimately, those who learn will revise their initial beliefs to be more precise, closer to what historical series would imply, and learning makes beliefs less heterogenous. This explanation is a human capital argument applied to financial knowledge (as in Delavande, Rohwedder and Willis, 2008). This explanation can be also viewed as an application of the information choice theory of Veldkamp (2011).

In line with the methods section above, beliefs are characterized by the subjective mean and subjective standard deviation of the one year ahead log return on the stock market index ($\tilde{\mu}_i$ and $\tilde{\sigma}_i$, respectively). The subjective mean and the subjective standard deviation are unobserved variables that we relate to the observed answers to two survey questions: one question about the probability that the stock market return would be positive (p_0) and the other question about the probability that the returns would be 10 per cent or more (p_{10}). The chapter describes the noise features in the probability answers in detail and argues for why the evidence is consistent with random survey noise.

We verify the implications of the learning theory by empirical evidence on stock market beliefs using a sample of 55 to 64 years old respondents of the Health and Retirement Study. Our sample consists of people who are at the peak of their asset accumulation process, and their beliefs and household portfolios are the result of their learning and investment history. We first show correlations and OLS regressions using observable answers to probability questions. Then we estimate the structural econometric model and estimate the theoretically interesting belief parameters conditional on the survey answers (analogously to the prediction of individual risk tolerance by Kimball, Sahm and Shapiro, 2009).

Our estimates show that respondents, on average, have low expectations and perceive risk to be high, however, there is substantial heterogeneity in expectations. Results from both the simple and the more structural analysis support the learning explanation. People who had stronger incentives to learn in the past indeed possess beliefs that are consistent with more learning. In particular, people with higher lifetime earnings, higher education, higher cognitive abilities, defined contribution as opposed to defined benefit pension plans, and those who are more optimistic and less uncertain about things in general have stock market beliefs that are less heterogeneous, somewhat less uncertain and considerably closer to levels that historical time series would imply. Our results also show that people who did not have strong incentives to learn in the past are very pessimistic about stock market returns.

Expectations are shown to be strongly related to the portfolio choice of American households. This result validates the survey's expectation measures as measures of "real" expectations as opposed to some artificial figures. This result also underlines the potential importance of expectations in resolving the "stockholding puzzle" and maybe also the related equity premium puzzle.[8]

Chapter 3 uses new data from the Health and Retirement Study to shed more light on heterogeneity in stock market expectations. In particular, this chapter considers two questions: (1) Are measures of financial knowledge, especially knowledge about past stock market returns, related to expectations about future stock market returns? (2) Are aspects of personality that are recognized to be important in psychology related to expectations about future stock market returns?

The first question is closely related to Chapter 2. The empirical analysis in that chapter shows that measures of incentives and personal characteristics are indeed related to expectations about future returns. However, a lack of explicit information on the knowledge of the

[8] In his review paper, Micheal Hurd (2009) cites a parapgraph from a previous version of Chapter 2 (Kezdi and Willis, 2008): "We estimated relevant heterogeneity [in stock market expectations] and related it to household investment behavior, with the help of a simple portfolio choice model. Our results confirm the validity of survey measures of expectations in predicting real behavior after measurement error is properly accounted for. A causal interpretation of the results suggest that heterogeneity in expectations leads to heterogeneity in stockholding, and low average expectations, high uncertainty, and large heterogeneity in expectations explain much of the stockholder puzzle." In order to make that statement more groudned, we need to estimate risk preferences together with expectations and analyze them jointly in portfolio choices.

history of stock market returns in the data used in that chapter prevents explicitly linking knowledge to future expectations. The results in this chapter fill that gap.

The results provide strong support to the argument that knowledge of the history of stock market returns is a major determinant of expectations about future stock returns. Those people who know that stocks have outperformed bonds and saving accounts in the past have beliefs about the distribution of future returns that are significantly closer to the characteristics of the historical return distribution. Their expectations are, on average, positive (whereas expectations of other people are, on average, negative). Their belief about risks are also closer to historical risks than the beliefs of other people, although their beliefs are also substantially higher than historical risks. The results on disagreement (heterogeneity in expectations) provide additional support to the learning argument. Expectations of those people who know that stocks have outperformed bonds and saving accounts in the past are less heterogeneous than the expectations of the rest of the sample.

Other aspects of financial knowledge are also shown to be strongly related to stock market expectations, even conditional on our measure of knowledge about the history of stock returns. It is possible that the measure of other financial knowledge is just another proxy for the knowledge of the history because the measure for the latter is very imperfect. If this assumption is true, the results provide no additional insight into the substantive question but instead it shows how imperfect the financial knowledge measures are. Another possibility is that this result shows that stock market expectations are influenced by other aspects of financial knowledge even conditional on knowledge about the history of stock returns. Thus, people with a perfect knowledge of the history of stock returns may form different expectations about future returns if they have different levels of financial knowledge. Without further evidence, it is impossible to separate the two explanations.

Question two asks what other factors may be behind the heterogeneity in expectations. This question focuses on standard aspects of personality. The psychology literature on personality aims at uncovering stable traits that affect thinking, feeling, and acting. The literature has come to a consensus that postulates that five major dimensions describe personality on a broad level: Agreeableness, Conscientiousness, Extraversion, Neuroticism, and Openness/Intellect. The collection of these dimensions is often called the "Big Five personality traits." According to this consensus view, an individual's scores in these 5 dimensions characterize stable patterns of thoughts and feelings, and those scores are widely used to predict individual behavior.

Another personality trait that is likely to be related to stock market expectations is optimism. General optimism, as a stable personality trait, is defined as "a generalized expectancy that good, as opposed to bad, outcomes will generally occur when confronted with problems across important life domains" (Scheier and Carver, 1985). The assumption that general optimism should be related to stock market expectations is motivated also by with earlier work of mine with Robert J. Willis on optimism (Kézdi and Willis, 2003). That analysis showed that a combined measure of optimism about various events is positively related to many positive life outcomes, even conditional on many other personal characteristics (including education or cognitive scores). A remarkable finding of that analysis is the association of sunshine optimism with many life outcomes. That measure was defined by comparing people's subjective probability assessment of the day following the interview being sunny to actual sunshine data for the day in question. Chapter 2 in this dissertation shows similar results: sunshine optimism is positively correlated with stock market expectations even conditional on many personal characteristics, including education, cognitive capacity and lifetime earnings.

The results with respect to personality are largely negative, except for optimism. Four of the Big Five personality traits (Agreeableness, Conscientiousness, Extroversion and Neuroticism) appear not to be related to stock market expectations. The fifth trait, Openness is associated with the level of expectations; however, that association becomes insignificant conditional on gender and education. In contrast, general optimism is significantly associated with the level of stock market expectations (but not perceived risk or disagreement). This last result is in line with intuition and our previous results on sunshine optimism. However, the relationship appears to be significantly stronger among people who do not own stock market-based assets. This finding suggests that the role of optimism is rather complex in shaping expectations and needs further investigation.

Altogether, the results of Chapter 3 imply that financial knowledge in general, and knowledge about the history of stock returns in particular, are an important determinants of expectations about future stock returns. There is substantial heterogeneity in expectations conditional on financial knowledge; however, understanding that variation proves to be difficult and requires further research.

Chapter 4 asks whether and how people's stock market expectations were affected by the stock market crash in September 2008 in the United States. The analysis uses data from the 2008 wave of the Health and Retirement Study to study the impact of the stock market crash on people's expectations. We estimate the effect of the crash on the population average of expected returns, the population average of the uncertainty about returns (subjective standard deviation), and the cross-sectional heterogeneity in expected returns (an indicator of disagreement). We show estimates from simple reduced-form regressions on probability answers as well as from a more structural model that focuses on the parameters of interest and separates survey noise from relevant heterogeneity. The measurement strategy makes use of the fact that the respondents of HRS 2008 answered the survey during 12 months from February 2008 to February 2009, a time period that includes the time of the stock market crash in early autumn. We show that the date of interview is largely independent of the respondents' past expectations about the stock market, so even if the date of interview is non-random it is unlikely to bias our results.

Our results imply a temporary increase in the population average of expectations right after the crash. At the same time, average uncertainty increased, perhaps as the result of increased stock market volatility. Our most robust finding is that cross-sectional heterogeneity in expected returns, an indicator of the amount of disagreement, increased substantially with the stock market crash. The effects are found to be largest among stockholders, those who follow the stock market, and those with higher than average cognitive capacity. The result on average expectations thus masks a wide distribution of effects of opposing signs. We also document the co-movement of stock market expectations with ex post returns, implied volatility and volume of trade.

Our finding suggests that there is heterogeneity in the cognitive processes (or mental models) people use to convert public news into personal probability beliefs, in accordance with some of the disagreement literature we mentioned above. The results on changes in heterogeneity complement recent empirical investigations that show substantial heterogeneity in stock market expectations of individual investors (Vissing-Jorgensen, 2003) as well as households (Calvet et al., 2007, 2009a,b; Dominitz and Manski, 2007; Kezdi and Willis, 2008; Hurd et al., 2011; Gouret and Hollard, 2011). The findings of this chapter add new results to this empirical literature by showing that the stock market crash and the financial crisis had significant effects on average expectations, average uncertainty, and, perhaps most importantly, the heterogeneity of expectations.

2 Heterogeneity in expectations about future stock returns, learning incentives and portfolio choice

Acknowledgement 1 *This section is an updated version of a previous paper coauthored with Robert J. Willis. The title of that previous paper was "Household Stock Market Beliefs and Learning" and it was published as NBER Working Paper 17614.*[9] *The paper was featured in VOX, the policy and popular-research economics portal of the Centre for Economic Policy Research*[10]*.*

[9] http://www.nber.org/papers/w17614.pdf

[10] http://www.voxeu.org/article/beliefs-and-stock-market

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2.1 Introduction

Beliefs about stock market returns are important determinants of households' investment behavior. Recent research has established the strong relationship between beliefs and stockholding, and it also documented substantial heterogeneity in those beliefs (Vissing-Jorgensen, 2004; Dominitz and Manski, 2005 and 2007; Amromin and Sharpe, 2006). This heterogeneity is puzzling since stock returns are publicly observable, and all of its history as well as many analyses are public information. Understanding the source of heterogeneity is important to understand heterogeneity in household finances, which is substantial (Campbell, 2006).

The goal of this paper is to characterize heterogeneity of the stock market beliefs of American households, understand the sources of that heterogeneity, and establish its relation to household portfolios. Our substantive contribution is to provide a more systematic account of the heterogeneity than the previous literature and relate it to a relatively simple explanation. Our methodological contribution is to estimate structural belief parameters from noisy survey answers to probability questions of the type advocated by Manski (2004).

We hypothesize that heterogeneity is the result of differences in learning histories, which are in turn caused by differences in returns to and costs of learning (as well as in initial conditions). People learn about finance in general and the stochastic process of stock market returns in particular. The value of learning is proportional to savings, but the costs are fixed. As a result, people with higher earnings prospects should learn more than people with lower income prospects, especially if social security provides enough retirement income for the latter. Differences in the costs of learning and differences in general attitudes may also be heterogenous, creating additional heterogeneity in learning outcomes. Initial conditions matter, too. Those with very low expectations will be less likely to learn and will see their beliefs unchanged. In the end, those who learn will revise their initial beliefs to be more precise, closer to what historical series would imply, and learning makes beliefs less heterogenous. This is a human capital explanation applied to financial knowledge (as in Delavande, Rohwedder and Willis, 2008). It is also an application of the information choice theory of Veldkamp (2011).

We characterize beliefs by the subjective mean and subjective standard deviation of the one year ahead log return on the stock market index. These are unobserved variables that we relate to observed answers to two survey questions: one about the probability that the stock market return would be positive and the other one about the probability that the returns would be 10 per cent or more. Our structural estimation model accounts for survey response error due to rounding, potential inattention, and the unwillingness or inability to make the necessary effort to give precise answers. A subset of the respondents in our sample answered the same pair of questions twice in the survey, about half an hour apart, which allows us to calibrate the moments of survey noise in a direct way.

We verify the implications of the learning theory by empirical evidence on stock market beliefs using a sample of 55 to 64 years old respondents of the Health and Retirement Study. Our sample consists of people who are at the peak of their asset accumulation process, and their beliefs and household portfolios are the result of their learning and investment history. We first show correlations and OLS regressions using observable answers to probability questions. Then we estimate the structural econometric model and estimate the theoretically interesting belief parameters conditional on the survey answers (analogously to the prediction of individual risk tolerance by Kimball, Sahm and Shapiro, 2008). Our structural model separates survey noise from relevant heterogeneity.

Our estimates show that respondents have low expectations and high perceived risk on average and substantial heterogeneity in expectations. Results from both the simple and the more structural analysis support the learning explanation. People who had stronger incentives to learn in the past indeed possess beliefs that are consistent with more learning. In particular, people with higher lifetime earnings, higher education, higher cognitive abilities, defined contribution as opposed to defined benefit pension plans, and those who are more optimistic and less uncertain about things in general have stock market beliefs that are less heterogeneous, somewhat less uncertain and considerably closer in levels to what historical time series would imply. Our results also show that the people who did not have incentives to learn are very pessimistic about stock market returns.

On top of the small literature on beliefs, many papers have looked at reduced-form associations of stock market participation with demography, education and wealth (Ameriks and Zeldes, 2004; Guiso, Haliassos, and Jappelli, 2002), cognitive capacity (Christelis, Jappelli and Padula, 2010), health (Rosen and Wu, 2003), or social interactions (Guiso, Sapienza and Zingales, 2004; Hong, Kubik and Stein, 2004). The results of our theoretical explanation and our empirical investigation are all in line with the results of that literature. They also suggest that part of those reduced form associations may operate through differential incentives for learning about attainable stock returns.

The rest of this chapter is structured the following way. Section 2.2 contains a brief characterization of stock market beliefs. Section 2.3 summarizes the setup and the most important implications of a simple theoretical model of household portfolio choices with learning. We then describe our data as briefly as possible in Section 2.4, and move on to descriptive evidence on the probability answers themselves in Section 2.5. Section 2.6 covers the estimation of the structural parameters of beliefs and their association with stockholding and the right hand-side variables. Section 2.7 concludes. Appendix A presents the details of our investigation, divided into four sections. Appendix section A.1 contains the formal structure of our theoretical model and its results. Appendix section A.2 shows more details of our data, descriptive statistics and results from linear regressions on observables. Appendix section A.3 contains the details of the structural estimation model, and Appendix section A.4 contains detailed estimation results and robustness checks.

2.2 Characterizing stock market beliefs

We assume that people believe that yearly log returns are i.i.d. and normally distributed. Throughout the paper we denote the mean of log returns as μ and the standard deviation as σ. For example, $\mu = 0.1$ means that the mean return is approximately ten per cent. At yearly frequency, the i.i.d. normal assumption for log returns lines up well with historical data available respondents to the 2002 wave of the survey we analyze. In the period of 1945 to 2002, yearly log nominal returns of the Dow Jones index were characterized by a mean of $\mu = 0.07$ and a standard deviation of $\sigma = 0.15$. Different windows can give lower and higher values of μ, and the value of σ is remarkably stable.

Under the i.i.d. lognormality assumption, the beliefs of individual i about the stock market returns are fully characterized by her beliefs about the mean and the standard deviation, and we denote those subjective beliefs by $\tilde{\mu}_i$ and $\tilde{\sigma}_i$. We define $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ as the parameters that would characterize individual beliefs in investment situations. The goal of this paper is to characterize heterogeneity in $\tilde{\mu}_i$ and $\tilde{\sigma}_i$, understand the sources of that heterogeneity, and establish its relationship to heterogeneity in household portfolios.

$\tilde{\mu}_i$ and $\tilde{\sigma}_i$ are unobserved in our data (the Health and Retirement Study). Instead, we observe answers to probability questions. In the larger part of the sample that we use to show descriptive statistics, one question was asked. This question (p_0) asked what the respondent thought the probability is that the market will go up. In the sample that we use for the structural analysis, we have answers to another probability question as well (p_{10}), about the probability that the market will go up by at least 10 per cent. The questions themselves were phrased the following way.

p_0 question: By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?

p_{10} question: By next year at this time, what is the chance they will have grown by 10 percent or more? [11]

When answers to both p_0 and p_{10} are available, identifying the mean and standard deviation of log returns from the two probabilities is relatively straightforward under the normality assumption. Let R denote one year ahead gross returns, which is a random variable with $\ln R \sim N(\mu, \sigma^2)$. In principle, one can relate these probabilities to the parameters of the lognormal distribution in a straightforward way. Let heterogeneity be denoted by an i index, the subjective nature of the probabilities by the tilde, and let stars denote theoretically correct probabilities derived from subjective beliefs; actual survey answers may be different, see later. Then,

$$p^*_{0i} = \tilde{P}_i[R \geq 1] = \tilde{P}_i[\ln R \geq 0] = \Phi\left(\frac{\tilde{\mu}_i}{\tilde{\sigma}_i}\right) \tag{12}$$

$$p^*_{10i} = \tilde{P}_i[R \geq 1.1] \approx \tilde{P}_i[\ln R \geq 0.1] = \Phi\left(\frac{\tilde{\mu}_i - 0.1}{\tilde{\sigma}_i}\right) \tag{13}$$

Observing p^*_{0i} and p^*_{10i} would allow for a simple computation of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ by making use of the inverse normal c.d.f. Higher $\tilde{\mu}_i$ corresponds to higher probabilities, while higher $\tilde{\sigma}_i$ pushes the argument of Φ toward zero and thus pushes both probabilities towards 0.5.

In order to see the correspondence between $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ on the one hand and p^*_{0i} and p^*_{10i} on the other hand in more intuitive ways, Figure 2.1 shows three probability distribution functions together with vertical lines at the cutoff points of 0 and 0.1 log returns that correspond to the p_0 and p_{10} questions. The continuous line shows a p.d.f. with historical moments between 1945 and 2002 ($\mu = 0.07$ and $\sigma = 0.15$). The dashed line corresponds to a mean-preserving spread (higher perceived risk), and the dotted line corresponds to a lower mean (more pessimistic beliefs).

[11] Note that the wording of the questions ("will be worth more") is somewhat vague. We interpret it as nominal returns without taking inflation, taxes or investment costs into consideration. If financially more sophisticated people have higher and more precise expectations, and, at the same time, they are more likely to think in real and/or after-tax terms, we shall underestimate heterogeneity in beliefs and its relation to variables that are related to financial sophistication.

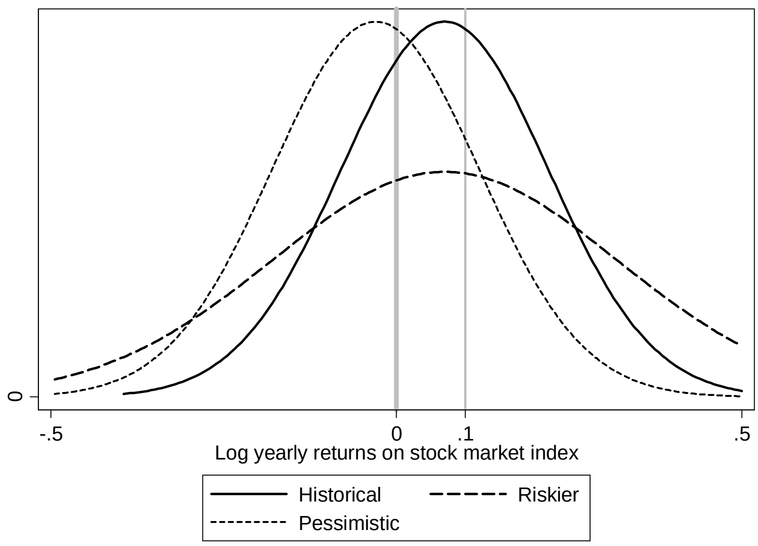

Figure 2.1. Examples for probability densities of normally distributed log returns, with the cutoff points for p_0 and p_{10}

p^*_{0i} and p^*_{10i} are equal to the area to the right of the corresponding bars at 0 and 0.1 log returns, respectively. The series of post-war returns to 2002 corresponds to $p^*_0 = 0.68$, $p^*_{10} = 0.42$ and $p^*_0 - p^*_{10} = 0.26$.

Holding risk constant, more pessimistic beliefs result in smaller values of p^*_{0i} and p^*_{10i}. We can therefore think of the answer to the p_0 (or the p_{10}) questions as proxy variables for the perceived level of returns. A mean-preserving spread leads to smaller area between the two vertical bars, which equals the difference $p^*_{0i} - p^*_{10i}$. The difference between the two answers may thus serve as a proxy for the inverse of perceived risks.

These proxies are not clean, though. The effect of risk on the probabilities can be ambiguous: higher risk corresponds to a smaller area to the right of a cutoff point if the mean is to the right (as for cutoff 0 when comparing the solid and the dashed curves), but it corresponds to a larger area if the mean is to the left (as for cutoff 0.1). Optimism/pessimism affects the difference between the probabilities, too, in ambiguous ways. For example, optimism decreases the difference if the mean is shifted outside the interval between the two bars from within the bars (as is the case for the dotted curve here), but the effect is the opposite if the mean is shifted towards to the middle of the interval. Simultaneous heterogeneity in the mean and the variance can lead to more complicated heterogeneity in the level and the difference of p^*_{0i} and p^*_{10i}.

Observing p^*_{0i} and p^*_{10i} would identify $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ at the individual level. Instead of p^*_{0i} and p^*_{10i}, however, we are likely to observe something else, as answers to the probability questions contain substantial noise with a complicated structure.

$\tilde{\mu}_i$ and $\tilde{\sigma}_i$ are the parameters that are relevant in investment situations. There are, however, strong theoretical reasons to believe that people's answers to the probability questions are not equal to the p^* transformations of these parameters. There is little time to answer the questions, and, beyond a spirit of cooperation, there are no incentives to get the answers right. It is therefore better to look at actual answers as "guesses" for what the p^* values may be, given recollections of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$.

The data reveals answer patterns that strongly support this view, and we shall document that later. Some of these answer patterns make computing the p^* values impossible. All of the answer patterns indicate that actual answers are noisy transformations of relevant beliefs. Our structural econometric model will address these problems.

2.3 Heterogeneity in beliefs and learning

In this section we briefly summarize the explanation we propose for the heterogeneity in stock market beliefs. It is in the spirit of the human capital literature and its application to financial knowledge (as in Delavande, Rohwedder and Willis, 2008), embedded in a standard life-cycle model with borrowing constraints. It can also be viewed as an application of the information choice theory of Veldkamp (2011). Section A.1 in Appendix A contains a formal model, its numerical solution and its most important comparative static results.

Heterogeneity in stock market beliefs is the result of differences in learning histories, which differences are in turn caused by differences in returns to and costs of learning (and, potentially, differences in initial conditions). Suppose that individuals live for three periods: young adulthood, mature adulthood and old age. Young people are endowed with some initial sets of beliefs about the mean and the standard deviation of log returns ($\tilde{\mu}_i$ and $\tilde{\sigma}_i$), and they share the belief that returns are i.i.d. lognormal.

In young adulthood, people work, earn wages, consume and save, but are subject to borrowing constraints. Mature adults look the same except that wages are considerably higher. In old age, people receive pension benefits and earn no wage. Importantly, pensions are from a defined-benefit system such as Social Security, and pension benefits are a concave function of lifetime earnings. As a result, people who earn below a certain threshold do not have an incentive to save for retirement, people who earn above that threshold do save, and their saving rate depends on their lifetime earnings. Savings can be held in risk-free bank accounts or invested in risky stocks. Borrowing and short sales are not allowed, making the share of stocks in per period savings between zero and one. The returns on stocks are governed by the i.i.d. lognormal process with historical moments, regardless of individuals' beliefs. But whether and how much an individual chooses to invest into stocks in any period depends on her beliefs about those moments at the beginning of the period.

The essence of the model is the possibility to learn about the parameters of the returns process. Learning is the result of a choice. If people choose to learn, they can observe a long historical series of log returns and update their subjective beliefs in a standard Bayesian fashion. The results are a posterior mean that is closer to the historical average and a smaller posterior variance. Learning is more general than updating beliefs: it is understanding the ways in which investment works. As a result, earning can increase the attainable expected return and/or reduce the risk of their portfolio on top of the effects of learning on beliefs about stock market returns. If they choose to learn, people have to pay a fixed cost. People may also learn in a more passive fashion: If they have high enough earnings in young age, they may invest those into stocks, and observing the returns will allow for updating beliefs.

The implications of this model are straightforward. The value of learning is proportional to intended savings, but the costs are fixed. Those who have higher lifetime earnings or lower learning costs will be more likely to learn. Similarly, if we allow for heterogeneity in initial belief endowments, higher initial subjective mean and lower initial variance will also lead to higher propensity to learn. Higher risk tolerance and more patience also lead to higher propensity to learn.

These results have important implications for the empirical analysis of heterogeneous stock market beliefs and household portfolios. Heterogeneity in lifetime earnings reflects heterogeneity in general human capital which, in turn, is the result of differences in the costs of and the benefits to investment based on probability beliefs about future returns. (Becker, 1964,1993; Willis, 1986; Card, 1999). If stock market beliefs are result of investment into a specific form of human capital, all personal characteristics that are related to lifetime earnings will also be related to beliefs as well, even conditional on lifetime earnings. This is another channel through which earnings and household finances are related, on top of the more traditional argument for the role of background risk (emphasized by, e.g., Viceira, 2001).

2.4 Data

In this section we give a brief overview of our sample and discuss the definition of the variables we use in the analysis. Section A.2 in Appendix A contains additional information.

We use data from the Health and Retirement Study (HRS), a large biannual panel household survey that follows older Americans (see NIA, 2007, for review). The HRS is representative of the American population 50 years of age or older, and their households. HRS has had a number of probability questions from 1992 on. It added questions on stock market beliefs in 2002. Besides subjective probabilities, HRS collects data on the amount and structure of savings, including tax-sheltered accounts such as 401(k), a rich set of demographic variables, and measures of cognitive functioning. In addition, retrospective earnings data from W2 tax forms are linked to a large subset of the HRS respondents for long time periods (the latter data are available in a secure data use setting). For the descriptive analysis in this paper, we use data from four waves of HRS, from 2002 through 2008[12]; for the structural analysis we use data from 2002 only.

We restricted the sample to people who were 55 to 64 years of age and whose spouse was also in that age range. The age restriction has both a theoretical and a practical reason. Households in this age group are around the end of the wealth accumulation phase of the life cycle but have not yet started decumulating their wealth. The cross-section of these households allows us to analyze heterogeneity in the results of learning and investment histories. The practical reason for the age restriction is the availability of retrospective earnings data from administrative sources, an important variable in the analysis. Sample sizes are in table A2.1 in Appendix A.

In 2002, the HRS asked the p_0 and the p_{10} questions, while in 2004 and 2006 only the p_0 questions. In 2008, the p_0 question was accompanied by a second question with eight randomized threshold values ranging from a decrease of 40 per cent or more to and increase of 40 per cent or more. In Chapter 4 of this dissertation, we use these probability variables from HRS 2008 to look at the effect of the crash of the stock market on households' beliefs. In this paper we use answers to the p_0 questions from all four survey waves and the p_{10} question from 2002.[13]

The 2002 wave of the HRS includes an "experimental module" with additional subjective probability questions about stock market returns. About five per cent of the respondents were randomly assigned to answer the questions in this module. Among others, the module included questions on p_0 and p_{10} once more. Typically, people answered the experimental module about 30 minutes and 60 questions after they answered the original p_0 and p_{10} questions. This small subsample allows for a direct analysis of measurement error in the probability answers, in the spirit of the test-retest reliability studies in the survey measurement literature.

Stockholding is measured at the level of households. In the HRS households are asked whether they had investments in stocks or mutual funds. If "yes," we call people in these households "stockholders outside retirement accounts". The survey asks about retirement accounts as well and the fraction of stocks in those (the latter in a simplified way until 2006).

[12] In Chapter 4 of this dissertation, we show that shortly after the fall of Lehman Brothers in September 2008 stock market beliefs of households changed substantially and in an unusual way. For this reason we decided to drop interviews that were made after September 2008 in this paper.

[13] The varying thresholds for the second probability question in HRS 2008 introduce econometric complications that we do not address in this paper.

Persons who lived in households in which someone had stocks or mutual fund investments in retirement accounts are labelled "stockholders in retirement accounts." The union of these two sets is labelled "stockholders."

The fraction of stockholders is 51 per cent in 2002. Conditional on stockholding, the share of stocks in portfolios held outside retirement accounts is 59 per cent, and it is 80 per cent on retirement accounts. Stockholding status declines between 2002 and 2008 and so does the fraction of stocks in the portfolio conditional on stockholding. The likelihood of being a stockholder increases in wealth (both total net wealth and financial wealth). Conditional on stockholding, the share of stocks in the portfolio seems unrelated to wealth. Tables A2.2 and A2.3 and Figures A2.1 through A2.4 in Appendix A show the details.

One of the most important variables is a proxy of lifetime earnings. The variable is defined as the cpi-adjusted mean earnings of households with individuals between age 40 and 55 based on the W-2 tax forms. The variable is from confidential data and is not available for part of the sample, which needed imputed values. Other right hand-side variables include standard demographics (age, gender, single or couple, years of education race and ethnicity) and wealth (measured in categories, separately for total net wealth and financial wealth).

Cognitive functioning is measured by the four short tests included in HRS (immediate word recall, delayed word recall, serial 7s (successively subtracting seven from one hundred) and dementia screening questions). We use the first factor of the four aggregate scores for each individual between 1992 and 2000. McArdle, Fisher and Kadlec (2007) argue that the first factor of these tests measures episodic memory.

We use three measures for general optimism/pessimism and one measure for general uncertainty as personal attitudes. Each of these measures is based on survey answers prior to the 2002 wave of the HRS. The first optimism variable is a dummy denoting positive errors in predicting sunny weather. HRS 1994 and 2000 included a "warm-up" question to the series of subjective probability questions about the probability that the day following the interview would be sunny. We obtained realized weather data for the day in question at the zip-code location of the interview, and we regressed the probability answer on sunny hours (their fraction to hours of daylight). The residual of this regression can be interpreted as a forecast error. The variable we use is a dummy indicating whether the respondent's average forecast error was positive on both of the two surveys. The use of the answers to the HRS sunshine question as a measure of optimism was first proposed by Basset and Lumsdaine (1999).

The second optimism variable is the individual's assessment of the likelihood that a major recession would occur the near future. The question was asked in HRS 1992, 1996 and 1998, and the measure we use is the average of those answers. This variable appears in the survey well before the stock market answers and is likely to reflect general pessimism about the economy. The third variable is a score created from the nine-item psychological depression tests administered to the respondents in all waves of the HRS between 1992 and 2000. This test lists symptoms of psychological depression, and we use the score as a measure of time-invariant general pessimism.

The measure for general uncertainty is the fraction of fifty per cent answers to all probability questions (except for the stock market questions) given by the individual in all of the surveys from year 1992 to 2002. The idea behind this measure is that a person's propensity to give 50-50 answers in many different domains indicates uncertainty in general. This variable is very similar to the one used in Hill, Perry and Willis (2005) and Sahm (2007).

The right hand-side variables include a proxy for risk tolerance for HRS respondents estimated by Kimball, Sahm and Shapiro (2008) from answers to hypothetical gambles over lifetime earnings in HRS 1992 to 2002. Using these measures, Sahm (2007) found a significant positive relationship between risk tolerance and stockholding in a larger sample of HRS respondents.

2.5 Descriptive analysis

Before turning to a more structural analysis, we show results from descriptive statistics and simple linear regressions using the answers to the probability questions. We first document survey noise in the probability answers; then we characterize observed heterogeneity in those answers; finally, we show that the probability answers predict stockholding in ways that are consistent with portfolio choice theory.

The answers to the stock market probability questions contain substantial noise. Tables A2.5 through A2.11 and Figure A2.5 in Appendix A show the detailed statistics.

95 per cent of the p_0 answers are rounded to ten or 25 or 75 per cent. Focal values at 50 per cent account for an especially large part of all answers. In the American context, the answer "fifty-fifty" to such a probability question may be interpreted as a synonym for "I don't know."[14] At the same time, 50 per cent is a frequent response to probability questions in Europe as well (Hurd, Rohwedder and Winter, 2005). The rounding in p_0 and p_{10} is typical for survey probability answers; see Manski (2004) for examples.

[14]Beginning in 2006, HRS has asked a follow-up question to respondents who answer the p_0 question with an answer of "50" to distinguish between those who believe that the stock market is equally likely to go up or down in the coming year from those who are "just unsure" about the probability. About two-thirds answer that they are unsure. See Bruine de Bruin and Carman (2011) for a more detailed analysis of the 50 per cent responses.

Many respondents give the same answer to p_0 and p_{10} that, taken at face value, would imply infinitely large standard deviations of log returns. Rounding would allow for finite (but large) standard deviations to give that pattern. Some respondents give $p_0 < p_{10}$, which does not conform the laws of probability. It may be that these respondents do not understand probabilities at all. It is also possible that these answers reflect inattention to one or both questions. Empirical evidence is in line with the latter interpretation.

The most direct evidence on survey noise comes from comparing answers to the p_0 and p_{10} questions in the core questionnaire and the experimental module. When the randomly selected small subset of the respondents were asked to answer the same probability questions once again during the same interview about half hour later, most gave different answers.

Perhaps surprisingly, all three noise features (rounding, apparent violations of the laws of probability, and test-retest noise) seem largely random (see tables A2.7 through A2.11 in Appendix A). The prevalence of these answer patterns are not related to stockholding or cognitive capacity. There are some weak associations between rounding and education, and the propensity to give the same answer to p_0 and p_{10} and education, lifetime earnings and wealth. Some demographic characteristics are also weakly predictive but no clear pattern emerges. The cross-sectional distribution of the probability answers in the experimental module is very similar to the cross-sectional distribution of the probability answers in the core questionnaire. The absolute difference between the core and module answers is unrelated to any observable variable.

Having established noise in the probability answers, we turn to relevant heterogeneity in them. The goal is to show variation in the probability answers across groups of respondents that, according to our argument, should have had different incentives for learning and thus should have different beliefs.

We focus on four statistics: the sample average of p_0 $(\bar{p}_0)$, the variance of p_0 in the sample $(V(p_{0i}))$, the average difference between p_0 and p_{10} $(\bar{p}_0 - \bar{p}_{10})$ and the fraction of missing p_0 answers. These statistics are computed using waves 2002 through 2008 of HRS, except for $(\bar{p}_0 - \bar{p}_{10})$, which is computed for 2002 only as p_{10} is not available in later years.

$\bar{p}_0$ can be thought of as a proxy for the mean level of stock market beliefs: higher values correspond to more optimistic beliefs, and the closer $\bar{p}_0$ is to 0.68 (or 0.61 for more recent years before 2002) the closer the level of beliefs is to what historical returns would imply. $V(p_{0i})$ is a measure of cross-sectional heterogeneity in expected stock returns, also called disagreement in the finance literature (Hong and Stein, 2007). $(\bar{p}_0 - \bar{p}_{10})$ is an inverse proxy for perceived risk: the larger the difference the lower risk is attributed to stock returns. The fraction of missing p_0 answers is a proxy for ignorance, which can be thought of as extreme uncertainty about stock returns.

Table 2.1 shows the descriptive statistics by lifetime earnings, father's occupation, education, cognitive capacity, risk tolerance and stockholding status. Those with higher lifetime earnings, education and cognitive capacity should have beliefs that reflect past learning because of stronger incentives to learn actively, both through its costs and benefits. Defined contribution (DC) pensions create higher incentives for learning than defined benefit (DB) pensions. Those with fathers who were managers or professionals grew up in families that were more likely to be exposed to stockholding or had higher levels of financial knowledge. Father's occupation is, of course, related to lifetime earnings as well, through intergenerational income links. Risk tolerance is also likely to be related to stock market beliefs both through passive learning (higher levels of risk tolerance lead to stockholding at least in case of favorable beliefs) and active learning (by increasing expected benefits). Finally, those who hold stocks towards the end of their active career have stock market beliefs that reflect past learning; either passive learning through earlier stockholding or active learning.

Learning should lead to beliefs that are characterized by levels closer to historical average, lower perceived risk, lower levels of ignorance. In addition, groups whose members learned more should be characterized by lower levels of disagreement. Translated to the proxy variables in Table 2.1, these would imply $\bar{p}_0$ closer to 0.68, lower $V\left(p_{0i}\right)$, higher $\bar{p}_0 - \bar{p}_{10}$ (and closer to 0.26) and lower fraction of missing p_0 answers.

Table 2.1
Descriptive statistics of the subjective probability answers to the stock market returns questions. HRS 2002 through 2008.

	$\bar{p}_0$	$V(p_{0i})$	$\bar{p}_0 - \bar{p}_{10}$	Fraction missing p_0
Top 25 per cent of lifetime earnings	0.56	0.067	0.113	0.03
Bottom 25 per cent of lifetime earnings	0.44	0.079	0.061	0.26
Education college or more	0.56	0.062	0.123	0.06
Education high school or less	0.45	0.074	0.062	0.23
Has DC pension (top 25% lifetime earnings)	0.60	0.059	0.148	0.02
Has DB pension (top 25% lifetime earnings)	0.55	0.064	0.137	0.03
Top 25 per cent of cognitive capacity	0.53	0.063	0.116	011
Bottom 25 per cent of cognitive capacity	0.42	0.082	0.053	0.31
Father was manager or professional	0.55	0.064	0.109	0.10
Father had other occupation	0.50	0.072	0.084	0.15
Top 25 per cent of risk tolerance	0.51	0.070	0.095	0.16
Bottom 25 per cent of risk tolerance	0.45	0.078	0.073	0.16
Stockholder	0.55	0.063	0.107	0.06
Not stockholder	0.45	0.074	0.063	0.24
Entire sample	0.50	0.071	0.086	0.16
Total number of observations	$11,259$	$11,259$	$3,532$	$13,408$

Sample: Health and Retirement Study, waves 2002, 4, 6 and 8 ($\bar{p}_0 - \bar{p}_{10}$ is from HRS 2002 only).
Respondents of age 55 through 64 with a spouse of the same age range (and singles)
p_0 is the answer to the probability of positive returns on stock markets by following year

The results are all consistent with the predictions of the learning model. Individuals with high lifetime earnings, DC pension plans, high levels of education, high cognitive capacity, high risk tolerance or who grew up in families that were exposed to stockholding (more likely if the father was manager or professional) have beliefs that reflect learning more than the beliefs of their complementary groups (non-stockholders, those with low lifetime earnings, DB pensions, low education, low cognitive capacity, low risk tolerance, non-managerial or non-professional father). Their beliefs are closer to historical probabilities, which also means more optimistic beliefs and lower perceived risk. There is less disagreement about stock returns in these groups, and there is less ignorance measured by the prevalence of missing answers.

The figures also imply that expectations are low, disagreement is substantial and perceived risks are high. Note however that the probability answers match historical frequencies well in combined groups for whom learning incentives should be the highest. College educated people with top 25 per cent lifetime earnings and DC pension plans have an average p_0 of 0.64, average $p_0 - p_{10}$ at 0.17, zero per cent missing answers and very little disagreement. Nevertheless, low expectations and high perceived risk among the general population, and among non-stockholders in particular, is remarkable.

Table A2.12 in Appendix A shows substantial variation of beliefs by the year of interview. Interestingly, average beliefs of stockholders exhibit remarkable stability over the years, and much of the cross-year variation is due to non-stockholders. The same is true for missing probability answers. Conversely, much of the cross-year variation in disagreement comes from stockholders.

OLS regressions reveal partial correlations that are very similar to the simple inter-group differences in Table 2.1 above (see table A2.13 in Appendix A). The belief-specific right hand-side variables predict stock market probabilities in expected ways, too. Sunshine optimism is positively related to the level of p_0 answers, while past beliefs about economic recession and depressive symptoms are negatively related. The propensity to have given fifty-fifty answers in the past is strongly negatively related to the difference between p_0 and p_{10}, indicating strong positive correlation with perceived stock market risk. There are strong differences among demographic groups as well, even after holding the other variables constant. Women, singles and African Americans give probability answers that indicate more pessimistic beliefs, higher perceived risks, and they and Hispanics are more likely to give missing answers.

The probability answers predict stockholding, as documented by OLS regressions in Table A2.14 in Appendix A. We estimated two separate linear regressions for stockholding, one for the probability of nonzero stock-market based assets in the household portfolio, $\Pr(s_i > 0)$ and one for the share of such assets if nonzero $E[s_i | s_i > 0]$. The two types of regressions allow for looking at the relation of beliefs and stockholding at the extensive margin and the intensive margin separately. This specification does not "handle" selection into stockholding but it is the simplest way to look at the two margins.[15] For each left hand-side variable, we estimated one regression on the entire sample (with the appropriate age restriction) that includes p_{0i} and the dummy for missing p_{0i}, and another one on the HRS 2002 sample that includes $p_{0i} - p_{10i}$.

The results are all consistent with the role of beliefs in portfolio choice. Stockholding

[15] Credible identification of a selection model would require exlcusion restrictions in the second regression, i.e. instruments that affect stockholding at the extensive margin but not the intensive margin. In principle, one would need variation in fixed costs that is exogenous to anything that affects investment choices that lead to variation in the fraction of stocks. We argue that fixed costs are mostly related to learning, the results of which naturally affect all investment choices. Valid instruments are thus hard to find in this case.

is strongly positively related to p_0 answers, negatively related to the propensity to give a missing p_0 answer, and it is positively related to $p_0 - p_{10}$, indicating a negative relation to perceived risk. Conditional on stockholding, the fraction of stock-market based assets in household portfolios are positively related to p_0 answers and $p_0 - p_{10}$, the latter again indicating a negative correlation with perceived risk. These results are strong because they are conditional on lifetime earnings, education, cognitive capacity and demographics. The results at the intensive margin are all the more remarkable because only beliefs and education have significant coefficients. Controlling for detailed measures of household wealth decreases the coefficients by half, but most remain significant. Wealth in this age group is endogenous as it is the result of savings and investment history, and thus these latter results are likely biased downward in magnitude.

The descriptive statistics and the linear regression results are consistent with the hypothesis that stock market beliefs are results of learning over the lifetime and predict stockholding. These results are robust in the sense that they are free from additional econometric assumptions. At the same time, they yield estimates that are hard to interpret, for two reasons. First, the probability answers and their simple transformations may be affected by heterogeneity both in the subjective mean ($\tilde{\mu}$) and the subjective standard deviation ($\tilde{\sigma}$). Second, measurement error is likely to distort the observed probability answers and thus the descriptive statistics derived from them. The next section presents a more structural measurement model that deals with these problems.

2.6 Structural analysis

We develop a structural measurement model to estimate heterogeneity in the relevant belief variables and handle survey noise. The model relates the latent belief variables $(\tilde{\mu}_i, \tilde{\sigma}_i)$ to the observed answers to the probability questions (p_{0i}, p_{10i}). It accommodates the observed noise features in the data and our intuition about the way people answer difficult survey questions.

Our estimation strategy is structural in the sense that it focuses on the theoretically relevant parameters and the relevant heterogeneity in those parameters (net of survey noise). In particular, we estimate the moments of the distribution of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ in the population and in various subpopulations (analogously to Table 2.1 above), and we investigate the role of heterogeneity of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ in heterogeneity of stockholding.

The assumption of i.i.d. normal log returns implies that individual beliefs are fully characterized by $\tilde{\mu}_i$ and $\tilde{\sigma}_i$. As we noted earlier, probability judgments elicited in a survey situation are likely to be very different from those that enter into a real life investment decision. The survey response to a probability question takes less than thirty seconds, on average, and there are practically no incentives to get the answers right.[16] We model the difference in two steps. The first introduces survey noise, and the second step introduces rounding.

Noise is modeled as mean-zero additive components to the index $\tilde{\mu}/\tilde{\sigma}$ that enters the probabilities p_0 and p_{10}. The noise components, denoted by v_0 and v_{10}, are assumed to be jointly normal and potentially correlated. Let p_{0i}^{br} and p_{10i}^{br} denote hypothetical "before rounding" answers so that the observed answers p_{0i} and p_{10i} may be rounded versions of the former. Conditional on a draw of the noise variables, these hypothetical survey answers are then the following:

$$p_{0i}^{br} = \Phi\left(\frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{0i}\right) \tag{14}$$

$$p_{10i}^{br} = \Phi\left(\frac{\tilde{\mu}_i - 0.1}{\tilde{\sigma}_i} + v_{10i}\right) \tag{15}$$

$$\begin{bmatrix} v_{0i} \\ v_{10i} \end{bmatrix} \sim N\left(0, \sigma_v^2 \begin{bmatrix} 1 & \rho_v \\ \rho_v & 1 \end{bmatrix}\right) \tag{16}$$

The noise components are assumed to be independent of any relevant heterogeneity, which is consistent with the randomness of the test-retest error and the near-randomness of the other noise features. The bivariate nature of the noise accommodates answers of $p_{0i} < p_{10i}$ if that phenomenon is due to inattention on the survey (which, as noted earlier, is supported by the near-randomness of its prevalence). The correlation coefficient between v_0 and v_{10} is related to average inattention. $\rho_v = 1$ would mean that all respondents answer questions p_0 and p_{10} with the same noise, which would not allow for answers like $p_{0i} < p_{10i}$. At the other extreme, $\rho_v = 0$ would mean that all respondents forget their previous answers completely. The true value of ρ_v is likely to be in-between.

We identify moments of the noise process (σ_v^2, ρ_v) by making use of answers from the experimental module. Recall that a small subset of the respondents answered the same probability questions once more, in an experimental module. We assume that noise components in the core and module answers are independent.[17] Comparing answers to the core

[16] At the same time, it is important to stress that more than a half century of survey research has shown that data from properly designed and executed sample surveys can be used to make valid inferences to population characteristics despite short response times and lack of incentive to tell the truth (or to lie).

[17] This is probably a lower bound to the noise variance, because any "noise" that would be specific to the entire survey situation but would not affect investment decisions (e.g. the experience of a bad day) would affect the "core" and "module" answers in similar ways and would not be measured by the test-retest difference.

and experimental module questions identifies the noise variance σ_v^2. Conditional on σ_v^2, joint moments of the p_{0i} and p_{10i} answers identify the correlation ρ_v. Section A.3 in Appendix A contains the details of identification and the calibration results. Our preferred estimates are $\sigma_v = 0.95$ and $\rho_v = 0.42$ or 0.61 (the latter depending on whether covariates are entered or not).

Answers to the probability questions may differ from the hypothetical "before-rounding" probabilities p^{br} because of rounding. We accommodate rounding by an interval response model. An answer within a pre-specified interval can correspond to any probability p^{br} within that interval. Round numbers are in the middle of those intervals, which are defined in an exogenous fashion and are assumed to be the same for all respondents.

Formally, the vector of survey answers (p_{0i}, p_{10i}) is in the quadrant Q_{kl} if the vector of hypothetical probabilities $\left(p_{0i}^{br}, p_{10i}^{br}\right)$ is in that quadrant:

$$\begin{pmatrix} p_{0i} \\ p_{10i} \end{pmatrix} \in \mathbf{Q}_{kl} \Leftrightarrow \begin{pmatrix} \Phi\left(\frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{0i}\right) \\ \Phi\left(\frac{\tilde{\mu}_i - 0.1}{\tilde{\sigma}_i} + v_{10i}\right) \end{pmatrix} \in \mathbf{Q}_{kl} \tag{17}$$

$$\mathbf{Q}_{kl} = \begin{pmatrix} [q_k, q_{k+1}) \\ [q_l, q_{l+1}) \end{pmatrix} \tag{18}$$

In the implemented model, the intervals are defined, in percentage terms, as $[0,5)$, $[5,15)$, $[15,25)$,..., $[95,100]$. These intervals allow for rounding to the nearest ten. The interval response model is the simplest way of accommodating rounding that is compatible with the guesswork of calculating probabilities.

With additional assumptions on their cross-sectional distribution, this model allows for estimating moments of the relevant heterogeneity in $\tilde{\mu}_i$ and $\tilde{\sigma}_i$. We assume that $\tilde{\mu}_i$ is normally distributed and $\tilde{\sigma}_i$ follows a two-point distribution. We estimate the conditional mean of the normal distribution and the probability of the low point conditional on right hand-side variables. In the benchmark model we estimate the variance of the normal distribution and the high value of σ unconditionally, and we set the low value of σ to the historical standard deviation of log returns. As a robustness check, we explore models in which the low value of σ is estimated as well.

$$\tilde{\mu}_i = \alpha + \beta_\mu' x_i + \gamma_\mu' z_{\mu i} + u_{\mu i}, \quad u_{\mu i} \sim N\left(0, \sigma_{u\mu}^2\right) \tag{19}$$

$$\tilde{\sigma}_i \in \left\{\tilde{\sigma}_{low}, \tilde{\sigma}_{high}\right\} \tag{20}$$

$$\Pr\left(\tilde{\sigma}_i = \tilde{\sigma}_{low}\right) = \Phi\left(\beta_\sigma' x_i + \gamma_\sigma' z_{\sigma i}\right) \tag{21}$$

In the equations, z_μ is the vector of optimism variables (positive error in forecasting sunshine, low expectations about the economy in the past, symptoms of clinical depression),

z_σ is the proxy variable of person-specific uncertainty (the fraction of fifty-fifty answers to all probability answers in the past), and x_i denotes the vector of all other right hand-side variables.[18] α stands for the constant in the equation for $\tilde{\mu}$. We estimated it as a vector, by allowing for a different constant for $\tilde{\sigma}_i = \tilde{\sigma}_{low}$ versus $\tilde{\sigma}_i = \tilde{\sigma}_{high}$, which allows for a correlation between $\tilde{\mu}_i$ and $\tilde{\sigma}_i$.

We estimated the model by Maximum Likelihood. The details of the likelihood function are in Section A.3 in Appendix A. We estimated the model with and without the right hand-side variables. The details of the parameter estimates are in table A4.1 of Appendix A.[19] Table 2.2 shows the estimates of the most important unconditional moments of the relevant heterogeneity in stock market beliefs.

Table 2.2.
Relevant heterogeneity in stock market beliefs. Estimates from the structural model

	Model without covariates		Model with covariates	
	Point estimate	SE*	Point estimate	SE*
Population average of $\tilde{\mu}$	−0.066	0.018	−0.050	0.015
Population standard deviation of $\tilde{\mu}$	0.197	0.019	0.218	0.027
Population average of $\tilde{\sigma}$	0.576	0.077	0.532	0.091

*Bootstrap standard errors

Sample: HRS 2002, 55 to 64 years old financial respondents (partner is also 55 to 64)

The results imply low expectations and high perceived risks on average and substantial heterogeneity in expectations. The population moment estimates are similar whether covariates are used or not used in the estimation, but the model with covariates indicates somewhat less pessimistic expectations and lower perceived risk. According to the estimates with covariates, the population average of $\tilde{\mu}_i$ is negative 5 per cent. The population standard deviation of $\tilde{\mu}$ is 22 per cent, indicating that over 40 per cent of the population has positive expectations, and almost 30 per cent have expectations at or above the historical average of

[18] Excluding z_σ from the equation of $\tilde{\mu}$ and excluding z_μ from the equation of $\tilde{\sigma}$ are motivated by the fact that they do not influence the belief proxies in the simple linear regressions (Appendix A, Table A2.13). These exclusions may in principle be important in identifying the association of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ on the one hand and stockholding on the other hand in the stockholding equations below. However, the inclusion of z_σ and z_μ in all equations (including the stockholding equations) does not change any of the results, see the additional estimates presented in Tables A4.9 through A4.11 in Appendix A.

[19] There we show additional results for the restricted sample of financial respondents and for a more flexible way of estimating heterogeneity in $\tilde{\sigma}_i$. Those results are qualitatively very similar to our benchmark esitmates, except that heterogeneity in $\tilde{\sigma}_i$ is less successfully pinned down in some cases.

0.07. The population average of the perceived standard deviation is over 50 per cent, to be compared to the historical standard deviation of 15 per cent.

Perhaps even more interesting are the moments conditional on the covariates. Instead of interpreting the estimated coefficients of the model, we show estimated moments of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ in various groups.

The moments within subgroups are based on predicted values of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$, which are conditional on observed survey answers p_{0i} and p_{10i} and the other covariates in the estimation model. The predicted subjective mean and subjective standard deviation are denoted by $\hat{\mu}_i$ and $\hat{\sigma}_i$, and they come from the following conditional expectations (where hats on the expectation operator mean estimates):

$$\hat{\mu}_i = \widehat{\mathbb{E}}\left[\tilde{\mu}_i | x_i, z_{\mu i}, (p_{0i}, p_{10i}) \in \mathbf{Q}_{kl}\right] \quad (22)$$

$$\hat{\sigma}_i = \widehat{\mathbb{E}}\left[\tilde{\sigma}_i | x_i, z_{\sigma i}, (p_{0i}, p_{10i}) \in \mathbf{Q}_{kl}\right] \quad (23)$$

The conditional expectations are relatively straightforward to compute by Bayes' Rule after the results of the structural model that specifies the full distributions for $\tilde{\mu}_i$ and $\tilde{\sigma}_i$. The predicted $\hat{\mu}_i$ and $\hat{\sigma}_i$ are then the sample analogues to those. The details of the derivation are in section A.4 in Appendix A.

This prediction method is analogous to the prediction of risk tolerance based on survey answers to hypothetical gambles by Kimball, Sahm and Shapiro (2008). The predicted values are different from the true values, creating measurement error in the variables. The measurement error is one of prediction error. It has zero mean and thus leads to an unbiased estimate of the population mean, but it leads to an underestimation of the population standard deviation (because the predicted values are less dispersed than the true values). Using $\hat{\mu}_i$ and $\hat{\sigma}_i$ on the right-hand side of a regression leads to consistent estimates as long as all the covariates that are used in the predictions are also entered in the regression in question. The standard errors in this regression are inconsistent, though, and thus bootstrap standard errors are advised. If one uses the $\hat{\mu}_i$ and $\hat{\sigma}_i$ in regressions that have different covariates from the ones used in the prediction equations, OLS is inconsistent and a more sophisticated GMM procedure is appropriate (see Kimball, Sahm and Shapiro, 2008, for more details).

Table 2.3 shows the average predicted belief variables in various groups in the sample. The table is analogous to Table 2.1 in the descriptive analysis, except that Table 2.3 does not have a measure of disagreement (because that is not estimated well by the dispersion of $\hat{\mu}_i$), and it does not repeat the fraction of missing answers.

Table 2.3.

Estimated mean of the structural parameters of stock market beliefs in various subpopulations. HRS 2002

	Average $\hat{\mu}_i$	Average $\hat{\sigma}_i$
Top 25 per cent of lifetime earnings	0.065	0.542
Bottom 25 per cent of lifetime earnings	−0.070	0.540
Education college or more	0.041	0.539
Education high school or less	−0.094	0.536
Has DC pension (top 25% lifetime earnings)	0.072	0.536
Has DB pension (top 25% lifetime earnings)	0.051	0.573
Top 25 per cent of cognitive capacity	0.023	0.551
Bottom 25 per cent of cognitive capacity	−0.138	0.517
Father was manager or professional	0.030	0.519
Father had other occupation	−0.049	0.550
Top 25 per cent of risk tolerance	0.008	0.515
Bottom 25 per cent of risk tolerance	−0.141	0.569
Financial respondent in couple	0.031	0.512
Non-financial respondent in couple	−0.051	0.558
Entire sample	−0.041	0.539
Total number of observations	3,314	3,314

Sample: HRS 2002. Respondents of age 55 through 64 (partner also 55-64)

$\hat{\mu}_i$ and $\hat{\sigma}_i$: predicted subjective mean and subjective standard deviation of the one-year ahead stock return

The inter-group differences in average expectations are large. Those with top 25 per cent lifetime earnings or top 25 per cent cognitive capacity believe that expected stock returns are 15 percentage points higher than those with bottom 25 per cent earnings or cognitive capacity. College educated respondents believe that expected returns are 13 percentage points higher than those with high school education or less. Average $\hat{\mu}$ is still below the historical average of 0.07 in these categories, but in combined categories it exceeds that (it is 0.11 for college educated respondents with top 25 per cent lifetime earnings and a DC pension plan). The differences by pension plan, the father's occupation, risk tolerance and financial respondent status are sometimes smaller but still substantial. These differences are all in line with the predictions of the learning model. On average, individuals who had higher incentives to learn believe that expected stock market returns are positive and closer to historical evidence than other individuals.

The estimated intergroup differences in perceived risk are smaller and do not always have the expected direction, because our model is less successful in capturing heterogeneity in $\tilde{\sigma}$.

Nevertheless, in most of the cases when the difference is substantial, the results are in line with the predictions of the learning model: those who had higher incentives to learn believe that risks are lower and closer to historical evidence.

Finally, we examine the association of predicted individual beliefs with stockholding. Stockholding is specified as a two-tier hurdle model. The extensive margin (whether a household holds any stock-market based assets at all) is a probit, and the intensive margin (how much it holds if it holds any) is a truncated regression. Subjective beliefs are entered in the right-hand side of these equations in the form of their predicted values $\hat{\mu}_i$ and $\hat{\sigma}_i$, in additive ways.

$$\Pr(s_i = 0) = \Phi(\alpha_1 \hat{\mu}_i + \beta_1 \hat{\sigma}_i + \delta_1' x_i) \quad (24)$$

$$s_i^* = \alpha_2 \hat{\mu}_i + \beta_2 \hat{\sigma}_i + \delta_2' x_i + u_{2i} \quad (25)$$

We estimated both models in two ways, first without the stock market belief estimates and second with them. Apart from the belief-specific variables, the same right hand-side variables are included in the structural model as in the stockholding equations. As a result the coefficients of the belief variables are consistently estimated (see our discussion above and also in Kimball, Sahm and Shapiro, 2008). Because of inconsistency of the analytical standard errors, we present bootstrap standard errors that are re-sampled at the level of households (in order to allow for within-household correlations that are obviously strong because of the common left hand-side variable).

Besides the coefficients of the belief variables, it is also interesting to see whether and to what extent coefficients of the other variables change with the inclusion of the belief variables. Table 2.4 shows the most important results from the stockholding equation (24). Table A4.2 in Appendix A contains all estimates.

Expected stock market returns ($\hat{\mu}_i$) are strongly predictive of the probability of stockholding and the share of stocks in household portfolios. Individuals who believe that stock market returns are higher by one percentage point live in households that are 0.7 percentage points more likely to own stocks, and if they own stocks, the share of stocks among their financial assets is 0.3 percentage points higher. The estimated correlation of perceived risk, $\hat{\sigma}$, is not significant in the equations.

Other right hand-side variables have strong associations with the probability of stockholding, and most are of the expected sign. They are, however, at most weakly predictive of the share of stocks in household portfolios, which makes the strong predictive power of expected returns even more remarkable.

Inclusion of the stock market beliefs decreases the association of stockholding and the other right hand-side variables. The association with education and cognitive capacity are cut by a third. Single men and women are significantly less likely to hold stocks than couples in the reduced form but not if we condition on their stock market beliefs. Females in couples are of course just as likely to hold stocks as the reference group of coupled men because stockholding is defined at the household level. Their beliefs are, however, a lot less optimistic, and that's why, conditional on their beliefs, they should be more likely to hold stocks according to the second model. African Americans are 23 percentage points less likely to hold stocks in the first model (conditional on all the other right hand-side variables), and the difference drops to 18 percentage points if beliefs are also controlled. The difference between Hispanics and non-Hispanics are 23 percentage points (again conditional on the other right hand-side variables), and is unaffected by the inclusion of beliefs.[20]

Overall, our findings suggest that those people who should learn about returns in the stock market do learn and, given their beliefs, those people who should invest do invest. It is important to emphasize that the estimated coefficients in Table 2.4 do not capture the causal effect of beliefs about stock returns on stock holding. Rather, as our theoretical model emphasizes, beliefs are the product of a process of learning (or failure to learn) that takes place over the life cycle with many feedback loops between observations of market returns, evolution of earnings and wealth and investments in learning.

[20] For robustness checks, we re-estimated all models with the financial respondents only, as well as with alternative specifications for the heterogeneity in $\tilde{\sigma}_i$. The results, shown in Tables A4.3 through A4.10 in Appendix A, are very similar to those presented above.

Table 2.4

Subjective stock market beliefs and stockholding at the extensive margin

	$\Pr(s_i > 0)$, partial effects		$E(s_i \mid s_i > 0)$	
	(1)	(2)	(3)	(4)
$\hat{\mu}_i$		0.734		0.302
		(0.088)**		(0.110)**
$\hat{\sigma}_i$		−0.050		0.144
		(0.108)		(0.131)
Log lifetime earnings	0.041	0.034	0.000	−0.003
	(0.010)**	(0.010)**	(0.012)	(0.012)
Education	0.033	0.023	0.011	0.008
	(0.004)**	(0.004)**	(0.05)**	(0.005)
Cognitive capacity	0.069	0.044	0.002	−0.009
	(0.012)**	(0.012)**	(0.015)	(0.015)
Log risk tolerance	0.023	−0.039	0.048	0.030
	(0.028)	(0.029)	(0.028)*	(0.029)
Single female	−0.110	0.021	−0.023	0.009
	(0.023)**	(0.032)	(0.029)	(0.037)
Single male	−0.094	−0.031	−0.022	−0.005
	(0.029)**	(0.031)	(0.037)	(0.038)
Female in couple	−0.003	0.087	0.010	0.029
	(0.016)	(0.024)**	(0.017)	(0.025)
African American	−0.233	−0.181	0.034	0.057
	(0.030)**	(0.030)**	(0.046)	(0.053)
Hispanic	−0.229	−0.225	−0.008	−0.004
	(0.044)**	(0.043)**	(0.066)	(0.064)
Other variables	YES	YES	YES	YES

Probit models (1) and (2); truncated regression models (3) and (4).

Clustered standard errors in parentheses; bootstrapped for models (2).and (4)

** significant at 1%; * significant at 5%

Sample: HRS 2002, 55 to 64 years old financial respondents (partner is also 55 to 64)

2.7 Conclusions

Using survey data on subjective probabilities and a rich set of personal characteristics, this paper estimates heterogeneity in stock market beliefs and proposes an explanation for the source of that heterogeneity. We show descriptive evidence and develop a structural measurement model to capture the theoretically important belief parameters and separate survey noise from relevant heterogeneity. We provide detailed evidence on survey noise and the measurement model accommodates all the noise features we document. The results are consistent with our proposed explanation for heterogeneity in stock market beliefs. They also reinforce previous results about the predictive power of beliefs on stockholding.

Our results establish the importance of belief heterogeneity in household finances. They show that survey answers to probability questions can be helpful in characterizing individual beliefs, but their analysis should recognize the importance of survey noise. Our econometric model is a simple but sensible attempt to deal with measurement error that may be a useful reference for further research in this direction.

Our structural estimation results on the subjective mean of stock returns are relatively strong, while our results on the subjective standard deviation are weaker. We explored different models with different assumptions about the form of heterogeneity in the subjective standard deviation, and the results were always qualitatively similar. It is possible that answers to more probability questions or probability questions that are defined for more distant horizons would result in stronger identification in the presence of substantial survey noise.

So, what can we learn from these results? First, the HRS survey data is consistent with a model in which beliefs about the stock market depend on financial knowledge and the acquisition of financial knowledge is costly. Although our results emphasize the importance of beliefs, on a cautionary note, they also suggest that the strong correlation between beliefs and stock market participation in the HRS and other surveys cannot be interpreted as a causal relationship. Second, our results in some ways support the recent emphasis on finding ways to improve financial literacy as potentially useful policy to help people prepare for retirement. It would be useful to know more than we do about the mechanisms by which people acquire financial knowledge. Our model suggests that feedback effects through learning by doing may have large cumulative effects in the long run. Thus, policies that encourage participation in stockholding at a small scale early in the life cycle may motivate people both to improve their knowledge of risks and returns and to increase their level of saving.

3 Financial knowledge, personality and expectations about future stock returns

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3.1 Introduction and theoretical motivation

Expectations about future returns attainable on the stock market vary considerably. In the previous chapter, Robert J. Willis and I argued that knowledge about the stock market in general, and historical stock market returns in particular, is an important determinant of stock market expectations. We contrasted various implications of that argument with empirical evidence using data from the Health and Retirement Study (HRS), but we did not have explicit measures of knowledge about the history of stock market returns or other elements of financial knowledge.

That work also left open the question of what else may affect expectations beyond financial knowledge. Two people with the same knowledge about the past and the same knowledge of the way the economy works may have very different expectations for the future. Those differences may be at least partly driven by differences in personality.

In this paper I use new data from the HRS to shed more light on heterogeneity in stock market expectations. In particular, I consider two questions:

1. Are measures of financial knowledge, especially knowledge about past stock market returns, related to expectations about future stock market returns?
2. Are aspects of personality that are recognized to be important in psychology related to expectations about future stock market returns?

The psychology literature on personality aims to uncover stable traits that affect thinking, feeling, and acting. The literature has come to a consensus that five major dimensions describe personality on a broad level: Agreeableness, Conscientiousness, Extraversion, Neuroticism, and Openness/Intellect. The combination of these dimensions is often called the "Big Five personality traits." According to this consensus view, an individual's score on these five dimensions characterize stable patterns of thoughts and feelings, and these scores are widely used to predict individual behavior.

The Big Five personality theory reduces individual differences in personality to the five categories and characterizes them accordingly. A more theoretical approach to the Big Five personality theory postulates that its five dimensions capture something fundamental about people's way of thinking and feeling. There are more complicated theoretical structures that include the Big Five personality traits to provide a better understanding and a better connection to neurobiological structures. For my analysis, these more complicated structures are not essential. However, it is important to remember that the Big Five theory is not simply and ad-hoc representation of observed personality but is a theory with a deeper structure. That structure is the subject of intensive research.

Agreeableness is associated with the degree of cooperation versus exploitation of others. It is therefore conceptually related to altruism as it is used in economics research. Conscientiousness is associated with the degree of control. It is also thought to be related to the ability to follow rules and to pursue goals with potentially delayed rewards. In a sense, it may be related to time preference, but that relationship is not obvious (see more later). Extraversion is associated with higher sensitivity to rewards. This can be understood at a broad level, including the anticipation of rewards and its effect on behavior as well as the enjoyment of actual rewards and its effect on behavior. Neuroticism, sometimes referred to by its reverse measure, Emotional Stability, is associated with an individual's reactivity to threats and punishments. Openness, sometimes called Intellect, is associated with higher willingness and ability to think in abstract ways and to explore and analyze information. This willingness may be closely related to one's capacity to think in abstract ways and to analyze information. Therefore, openness is closely related to cognitive capacities.

The measurement of the Big Five factors is typically based on answers to survey questions about self-assessed characteristics. These self-assessment measures have been validated by evidence provided by peers as well as relevant choices and behavior.

There is a growing body literature that attempts to connect the Big Five personality traits to the most important dimensions of individual preferences that are relevant for economic decision making. In a sense, that research is part of the broader literature that attempts to validate the Big Five personality structure. In another sense, that research aims to uncover the fundamental psychological bases of economic preferences.

Anderson, Burks, DeYoung and Rustichini (2011) find that Neuroticism is related to risk preferences conditional on cognitive skills. However, they find that other personality traits are not related to risk preferences or time preference. The lack of a relationship between Conscientiousness and time preference is surprising, but this negative result is common in the literature. However intuitive it is to relate Conscientiousness to time preference, little empirical evidence has linked the two.

In their survey, Becker, Deckers, Dohmen, Falk and Kosse (2012) find that Big Five personality measures are at most weakly related to time preference and risk preference. They, conclude that time preference is not significantly related to personality traits. Moreover, their conclusions on associations with risk preferences not very positive. They note that, although some studies find relationships between Neuroticism and risk preferences, similarly to Anderson, Burks, DeYoung and Rustichini (2011), others do not confirm that relationship. In contrast with time preference and risk preferences, social preferences (such as trust or reciprocity) are found to be strongly associated with Big Five personality traits. However, the role of social preferences is not straightforward in investment decisions.

The finding of a weak relationship between Big Five personality traits and economic preferences that are relevant for investment decisions is somewhat puzzling. Some of these personality traits are found to be related to individual behavior and outcomes that are likely affected by financial decisions. Using personality measures from earlier survey waves of the Health and Retirement Study (HRS), Duckworth and Weir (2010) found that Conscientiousness and Emotional Stability (the reverse of Neuroticism) are strongly related to lifetime earnings, conditional on cognitive skills and education. Even more importantly for household finance research, they found that Conscientiousness is related to retirement savings, conditional on cognitive skills, education and lifetime earnings.

It seems that personality, as measured by self-assessment survey questions, explains part of the variation in wealth conditional on lifetime earnings and part of the variation in lifetime earnings. However, personality seems to be, at most, weakly related to risk preferences and time preferences. These findings raise the question of what may be responsible for the association with wealth if not association with preferences. It is possible that personality differences are related to differences in expectation, which is part of the reason they explain variation in wealth.

Another personality trait that is likely to be related to stock market expectations is optimism. General optimism, as a stable personality trait, is defined as "a generalized expectancy that good, as opposed to bad, outcomes will generally occur when confronted with problems across important life domains" (Scheier and Carver, 1985). Optimism is also viewed as part of positive individual traits, along with hope and courage, for example (see, for example, the January 2000 issue of the American Psychologist and the introduction by Seligman and Csikszentmihalyi, 2000). Quite naturally, if general optimism exists, it is likely to emerge in expectations about specific positive events. Its relationship with stock market returns is more natural for stockholders, for whom higher stock returns are positive events, but not for other people.

Note that the relationship between general optimism and the Big Five personality traits is not straightforward and is the subject of ongoing research in psychology. Sharpe, Martin and Roth (2011) find that Agreeableness, Conscientiousness, Extroversion and Neuroticism are all related to general optimism, in rather complicated ways. Indeed, it is possible that general optimism captures aspects of personality that are conceptually different from the aspects that Big Five theory can capture. It is also possible that those aspects are at least as important for decision under uncertainty as the Big Five personality traits.

The association of general optimism with stock market expectations is motivated by earlier work of mine with Robert J. Willis on optimism (Kézdi and Willis, 2003), too. That analysis showed that a combined measure of optimism about various events is positively related to many positive life outcomes, conditional on many other personal characteristics (including education or cognitive scores).

A remarkable finding of that analysis was the association of sunshine optimism with many life outcomes. That measure was defined by comparing people's subjective probability assessments of the day after the interview being sunny to actual sunshine data for the day in question. The measure is a binary variable, indicating a positive residual when sunshine expectations were regressed on actual sunshine data (collected from weather station observations). The binary variable is therefore one for those provided a higher probability of a sunny weather for the day after the interview than for other respondents who faced the same actual sunshine the day after. In that analysis (Kézdi and Willis, 2003), we found that people with sunshine optimism had higher expectations for many events, including survival to old age and economic growth. The data we used for that analysis did not contain expectations about the stock market. However, we have shown that optimism predicts stockholding (as well as many other outcomes), which we have interpreted as indirect evidence for optimism affecting stock market expectations and, in turn, stockholding.

Indeed, these interpretations turned out to be correct when we analyzed stock market expectations directly in a later paper, included in the previous chapter of this dissertation. The results there show that sunshine optimism is positively correlated with stock market expectations, conditional on many personal characteristics, including education, cognitive capacity and lifetime earnings.

Taken together, the psychology literature and our earlier findings suggest that general optimism may be associated with expectations. Moreover, this association is likely to remain positive conditional on financial knowledge.

3.2 Data

I use two different subsamples of the Health and Retirement Study (HRS) for the analysis.[21] The initial sample is the 2010 wave of the HRS, which contains answers to the stock market expectations questions (see below for more detail). In each wave of the HRS, randomly selected subsamples are invited to answer a few more questions, collected in "experimental modules." In 2010, one experimental module contained questions on financial sophistication.[22] The first subsample of my analysis consisted of respondents in this experimental module.

The second subsample consisted of respondents to the "leave-behind" questionnaire of HRS 2010. This was a self-administered paper-and-pencil questionnaire that respondents were invited to complete and mail using pre-paid envelopes. In 2010, half of the sample received the "Participant Lifestyle Questionnaire" as a leave-behind questionnaire. This questionnaire contained measures of the standard Big Five personality dimensions as well as measures of optimism.

Altogether, 15,372 respondents participated in the 2010 wave of the HRS. Of these, 1545 answered the experimental module on financial literacy, and 8184 answered the personality measures in the leave-behind questionnaire.

I restrict the analysis to respondents 50 to 70 years of age. IN principle, the HRS is representative of the people 51 years old (the "age eligible") and their spouses. In practice, the sample is refreshed every six years to include new cohorts to represent the entire age distribution starting with age 51. In 2010, the age-eligibles were 56 years old or older. My sample of people 50 years or older therefore consisted of younger spouses (between age 50 and 55) and age-eligible respondents (between age 56 and 74).

The size of the 50- to 70-year-old sample was 685 for the experimental module on financial literacy and 3547 for the personality measures.

Throughout the analysis, I use control variables of gender, age, years of education, a measure for cognitive capacity (the standardized number series score) and wealth.

Cognitive capacity is measured by the number series score. The number series test was adapted from the Woodcock-Johnson (WJ-R) battery (McArdle, Fisher and Kadlec, 2007). The questions in this test present puzzles that must be solved by recognizing patterns, in numbers. The number series test is considered one of the most valid and reliable measures of fluid intelligence. The age-adjusted scores of pattern recognition tests, such as the number series test, are often referred to as "IQ."

[21] See the data section of the previous chapter for more details on the HRS.

[22] The module was designed by Annamaria Lusardi, Olivia Mitchell, Miles Kimball and Tyler Shumway.

Fluid intelligence is thought to represent reasoning and thinking in novel situations. Modern cognitive psychology distinguishes fluid intelligence (Gf) and crystallized intelligence (Gc) in classifying cognitive abilities. Crystallized intelligence is thought to represent acculturated knowledge, potentially as a result of individuals' investment in knowledge (Horn and McArdle, 2007). The number series measure is therefore conceptually different from knowledge, which is important to keep in mind throughout the analysis.[23]

The number series test was administered for the first time in the HRS in 2010. It was an adaptive test with up to six items. Adaptive tests adjust the difficulty of the question to the results of previous answers, thus increasing the power of the test in differentiating between people over a larger support of the underlying ability distribution. For this analysis, I use a standardized score from this test for the entire HRS sample. Approximately 15 per cent of the respondents had no valid number series score; I filled in sample means for these respondents for each analysis subsample separately. (Dropping observations with missing number series scores does not change any of the results).

Wealth is measured as the natural logarithm of total household wealth net of debts. Total household wealth includes financial wealth as well as housing and other non-financial wealth items own by members of the household, added up. About 10 per cent of the sample has non-positive wealth (most of them negative wealth, sometimes quite large negative wealth) and I use a binary variable in the analysis for people in such households.

The summary statistics of these control variables in the entire sample and the two subsamples of my analysis are shown in Table B.1 in Appendix B. The 50- to 70-year-old subsample has better cognitive abilities and higher education than the entire sample, due to the age restriction. The remaining differences between the entire HRS sample and the 50- to 70-year-old subsample are not very large. Further restrictions change the picture in different ways. The subsample with measures of financial knowledge is slightly more educated but has slightly lower abilities and lower wealth and includes more minorities than the entire age-restricted subsample, whereas the personality subsample is more educated and has higher average abilities and higher wealth and includes a lower fraction of minorities at the same time.

The analysis focuses on stock returns expectations. In 2010, three stock returns questions were asked:

[23] In ongoing work with Robert J. Willis, Susann Rohwedder and Péter Hudomiet ("Financial knowledge, fluid intelligence and investment decisions"), we use a more precise version of the number series score that has more items. That work uses other, substantially more detailed cognitive data. It shows that other potential measures of fluid intelligence, including, most importantly, Raven's matrices, are conceptually similar but less powerful measures of fluid intelligence in the analysis if financial decisions. Note that HRS does not have other measures of fluid intelligence. I describe our ongoing research in more detail in the next footnote.

p_0 question: By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?

p_{20} question: By next year at this time, what is the percent chance that mutual fund shares invested in blue-chip stocks like those in the Dow Jones Industrial Average will have gained in value by more than 20 percent compared to what they are worth today?

p_{ltn20} question: By next year at this time, what is the percent chance that mutual fund shares invested in blue-chip stocks like those in the Dow Jones Industrial Average will have fallen in value by more than 20 percent compared to what they are worth today?

Recall from the previous chapter that in 2002, HRS asked two questions, p_0 and p_{10} (the chance that stocks would gain in value by 10 per cent or more), and in 2004 and 2006 it asked p_0 only. In 2008, HRS asked two questions, p_0 and another question with varying threshold. (The 2008 data are analyzed in the study presented in the next chapter.) The 2010 stock market expectations questions differ from the 2002 questions in that they consist of three questions instead of two, and the second question is, p_{20} instead of p_{10}. Note, however, that p_{20} is similar to the original p_{10} question, only the threshold is higher. The third question, p_{ltn20}, is different in that it asks about a negative threshold and the probability that returns would be below that threshold.

These differences between the 2002 data and the 2010 data are important. First, the 2010 thresholds cover a larger part of the support of the returns distribution, 0, +20 and −20 per cent (compared to 0 and +10 per cent in 2002). Second, there are three points in the distribution compared to the two points in 2002. In principle, these differences can produce more precise estimates than the estimates that use p_0 and p_{10} only.

Recall from the previous chapter that answers to probability questions are likely to be plagued by missing observations and serious survey noise. The fraction of missing answers is substantially higher for the stock market probability questions than for other questions in the survey, including all other probability questions, too. Most of the valid answers are multiples of 10 per cent or 25 per cent, in the case of both stock market expectations and subjective probabilities of other events. Perhaps more importantly, some answer combinations for stock returns probabilities do not conform to the laws of probability. Finally, and perhaps most interestingly, comparing answers to the p_0 question asked twice in HRS 2002 (from a small subset of respondents who answered an experimental module) revealed a substantial test-retest difference.

The fact that more than one probability is elicited about stock returns allows for the identification of more features of the distribution, but it also poses additional problems. As discussed in the previous chapter, some combinations of answers violate the laws of probability, and other combinations imply zero probability mass between two points of the support. In the 2002 data, the laws of probability are violated if $p_0 < p_{10}$, and zero probability mass is implied by $p_0 = p_{10}$, apparently violating the normality assumption of log returns. The analogous violations in 2010 are $p_0 < p_{20}$ and $p_0 = p_{20}$, respectively. The third question asked the probability of returns falling below a threshold (of negative 20 per cent), which is a slightly different event. Here, the laws of probability are violated if $p_0 + p_{ltn20} > 1$, and zero probability mass is implied between 0 and negative 20 per cent if $p_0 + p_{ltn20} = 1$.

As we argued in the previous chapter, although those violations may indicate that many respondents simply cannot think in terms of probabilities, it is also possible that those apparent violations are the results of simple survey noise. Noise of this type may arise if respondents answer the two probability questions by making up answers without much thinking and/or without keeping in mind their answer to the previous question. In the previous chapter, we provided evidence that strongly support the survey noise argument. Moreover, we have also provided evidence that survey noise is largely random and independent of cognitive capacity or other relevant heterogeneity.[24]

Our first piece of evidence was that shows that test-retest noise seems completely independent of everything we measure about people, including their memory scores and numeracy measures. Among other things, this implies that anything that is due to survey noise should be unrelated to those characteristics. Second, we found that the propensity to give "funny" answer combinations to the stock return probability questions is very weakly related to observable characteristics, at least if we properly condition on the first probability answer. In this chapter, I present further evidence to support our claim that apparent violations of the laws of probability and zero mass events are largely due to random survey noise.

Table 3.1 summarizes the noise features in the 2010 data and compares them to the 2002 data (both samples refer to 50- to 70-year-old people at the time of the interview, which is different from the sample used in the previous chapter and table A2.5 of Appendix A). Table B.1 in the appendix to this chapter shows the corresponding regression results.

[24] The lack of association of survey noise with personal characteristics, especially cognitive capacity, may seem surprising. One would think that people with better memory or higher levels of fluid intelligence would be more able to remember their previous answers and, as a result, would be less susceptible to test-retest errors. This is not that obvious, however, if we recognize that answers to difficult survey questions of this sort are also a function of cognitive effort, which is a choice variable and is therefore endogeneous. It is possible that respondents with higher levels of ability put less effort into answering cognitively demanding questions than people with lower levels of ability, which may result in a lack of association between cognitive abilities and survey noise.

Table 3.1. Some patterns of survey answers in 2002 and 2010 in the HRS stock market probability questions. 50 to 70 years old respondents

Per cent of responses where	HRS 2002	HRS 2010
$p_0 = 0.5$	20.6	24.2
$p_0 = 0.0$ or $p0 = 1.0$	11.0	7.6
p_0 rounded other ten per cent	31.1	41.8
p_0 rounded 25% or 75%	7.3	7.6
p_0 not round number	4.8	5.5
p_0 missing	25.2	13.3
Total	100.0	100.0
$p_0 > p_{10}$	41.6	
$p_0 = p_{10}$	44.0	
$p_0 < p_{10}$	14.4	
Total	100.0	
$p_0 > p_{20}$		58.9
$p_0 = p_{20}$		30.2
$p_0 < p_{20}$		10.1
Total		100.0
$p_0 + p_{ltn20} < 1$		65.5
$p_0 + p_{ltn20} = 1$		14.6
$p_0 + p_{ltn20} > 1$		19.9
Total		100.0

We can draw three conclusions from the tables. First, rounding, apparent violations of the laws of probability, and zero probability mass answers show similar patterns in 2002 with the p_0 and p_{10} combination and in 2010 with the p_0 and p_{20} combination. Second, these patterns are consistent with the survey noise argument and that this noise is largely random. Third, the patterns with respect to p_{ltn20}, the probability that returns are less than the negative threshold, are quite different.

In detail, we can conclude that rounding in p_0 is pervasive both in 2002 and 2010. The fraction of 50, 0, and 100 per cent responses was 32 percent in both 2002 and 2010, with more 50's in 2010 and more 0's and 100's in 2002. The fraction of unrounded answers is similar in 2002 and 2010. The fraction of missing answers is approximately 25 per cent in 2002 but only 13 per cent in 2010. The decrease in missing answers may be due to familiarity with the survey question (the stock market questions were asked in 2002 for the first time) or decreased uncertainty (most missing answers reflect genuine ignorance, as we argued in the previous chapter).

The joint response patterns in p_0 and p_{10} in 2002 and in p_0 and p_{20} in 2010 are difficult to compare because of the different thresholds. The incidence of both zero mass answers and apparent violations is lower in 2010, which may be due to learning the survey questions or the fact that the thresholds are farther apart. The relationship of the propensity to provide these answer combinations to other personal characteristics is very similar in 2002 and 2010 (see Table B1.2)The negative probability question is different. Both the prevalence of the patterns of those answer combinations and their relationship to other personal characteristics is very different for the answer patterns of p_0 when combined with p_{ltn20}.

These results have important implications for our analysis. First, we can analyze the p_0 and p_{20} answers in 2010 in ways that are very similar to our previous analysis of the p_0 and p_{10} answer pair in 2002. On the contrary, incorporating the p_{ltn20} answer would be a mistake without a better understanding the answer patterns and the role of survey noise in those patterns. Unfortunately, a test-retest analysis is impossible to perform in 2010. As a result, I cannot say more about the noise patterns in p_{ltn20}.

As a result of these considerations, I do not use the p_{ltn20} variable in the analysis and concentrate on p_0 and p_{20} only.

The first substantive question of this paper is the relationship of stock return expectations to financial knowledge. In contrast with the 2002 data, in 2010, we have direct measures of financial knowledge. Financial knowledge is measured by using questions administered in Experimental Module 8, on financial literacy, in the 2010 wave of the HRS. The most important variable is knowledge of historical returns. The question did not ask for the respondents' estimate of past returns. Instead, it asked whether the respondent thought that stocks outperformed bonds and saving accounts. The exact wording of the question was as follows:

> Which asset do you think historically has paid the highest returns over a long time period, say 20 years or more – savings accounts, bonds, or stocks? (1. Saving accounts 2. Bonds 3. Stocks)

The distribution of the answers to this question is shown in Table 3.2. Fifty-four percent of the answers indicated that stocks to had the highest returns, 27 per cent indicated bonds, 11 per cent indicated savings accounts, and 8 per cent did not know. Not surprisingly, those who indicated stocks (the right answer) were more educated and had higher cognitive scores than the rest of the sample.

Table 3.2
Answers to which asset paid the highest returns over the past 20 years.
Distribution and some statistics ($n = 685$)

Which asset paid highest historical returns	Distribution (%)	Fraction female (%)	Avg. years of education	Avg. standardized number series score
Saving accounts	11	61	11.2	−0.45
Bonds	27	61	13.0	−0.04
Stocks	54	59	13.8	0.30
Does not know	8	77	11.7	−0.36
All	100	61	13.1	0.08

HRS 2010. Financial literacy experimental module, 50- to 70-year-old respondents.

I use four other questions from the financial literacy module that measure financial knowledge other than knowledge of past returns. There are more questions in the module but these four are the cleanest and least controversial measures. The questions are as follows

Whether it is a good idea to own stocks of foreign companies;
whether one should diversify individual stocks;
whether bond prices fall if the interest rate falls;
whether it is easy to pick stocks that will have better than average returns.

These questions are a subset of the financial knowledge module of the CogEcon survey that I used in a paper with Robert J. Willis, Susann Rohwedder and Péter Hudomiet.[25]

[25] "Financial knowledge, fluid intelligence and investment decisions." The first draft of that paper is being finalized now (January 2013) and will be available as a working paper in March 2013. The paper combines insights from cognitive psychology and economics to distinguish knowledge from fluid intelligence and examins their role in financial decisions. Using a battery of financial knowledge questions and intelligence tests administered in two surveys, CogEcon and another survey called the American Life Panel, we show that financial knowledge is strongly associated with stockholding and with better investment decisions both before and throughout the financial crisis. Fluid intelligence has predictive power, but the associations with financial knowledge remain strong when controlling for fluid intelligence, whereas the associations with fluid intelligence diminish substantially when financial knowledge is controlled. In the paper we argue that financial knowledge is likely to be the key factor that makes people with higher fluid intelligence make better financial decisions (see, for example, Agarwal, Driscoll, Gabaix and Laibson, 2009, and Grinblatt, Keloharju and Linnainmaa, 2011). Our results also imply that research on financial literacy (where literacy is usually meant to encompass knowledge, fluid intelligence and other potential inputs into financial decision making) could benefit from distinguishing fluid and crystallized elements both for better measurement and for more careful consideration of policies (see, for example, Lusardi and Mitchell, 2008; Guiso and Jappelli, 2009; Kimball and Shumway, 2010; van Rooij, Lusardi and Alessie, 2011).

CogEcon is a panel survey with observations from 2008, 2009 and 2011.[26] The first wave was administered by mail and internet to a national sample of 1,222 persons aged 51 and older and their spouses regardless of age. The 2008 CogEcon survey contained a 25-item battery of financial literacy questions. Survey participants were randomly assigned a true or a false version of each question. Of the 25 questions only 13 were adequate measures of financial knowledge (the remaining 12 either combine financial knowledge with cognitive skills or are worded in ambiguous ways). The four questions listed above are a subset of those 13.

Table 3.3 shows the fraction of correct answers to each of the four financial knowledge questions and summary statistics in the correct answer categories.

Table 3.3
The fraction of correct answers to each of the four financial knowledge questions and summary statistics in the correct answer categories ($n = 685$)

		Statistics in the correct answer categories		
	Fraction correct (%)	Fraction female (%)	Avg. years of education	Avg. standardized number series score
Foreign stocks	46	61	13.8	0.26
Diversify stocks	66	58	13.3	0.19
Bond prices	46	57	13.7	0.30
Picking stocks	70	59	13.5	0.20
Sample total	n.a.	61	13.1	0.08

HRS 2010. Financial literacy experimental module, 50- to 70-year-old-respondents.

In the spirit of the psychometric literature, and similar to our financial knowledge paper coauthored with Willis, Rowhedder and Hudomiet, I created a standardized score from the financial knowledge questions. Naturally, the measure can take on five values: the number of correct answers to the four questions. Table 3.4 shows the distribution of the score along with the descriptive statistics I have shown in the previous tables. The modal score is three correct answers out of four, and 60 per cent of respondents got two or three questions right. The financial knowledge score tends to be smaller among women, and it is positively related to education and the number series score.

[26] The CogEcon data are described in detail on the University of Michigan Cognitive Economics Project website, http://cogecon.isr.umich.edu/survey/index.html . The survey was designed by Daniel Benjamin, Andrew Caplin, Miles Kimball, Kathleen McGarry, Claudia Sahm, Matthew Shapiro, Tyler Shumway and Robert J. Willis.

Table 3.4
The financial knowledge score: number of correct answers to all questions and summary statistics in the score categories ($n = 685$)

Financial knowledge score: Number of correct answers	Distribution (%)	Statistics in the score categories: Fraction female (%)	Avg. years of education	Avg. standardized number series score
0	6	65	12.0	−0.44
1	19	68	12.1	−0.28
2	29	64	12.6	−0.08
3	31	56	13.6	0.30
4	15	54	14.8	0.61
Sample total	100	61	13.1	0.08

HRS 2010. Financial literacy experimental module, 50- to 70-year-old-respondents.

The second substantive question of my analysis is the relationship between stock return expectations and personality. The analysis focuses on the Big Five personality traits and optimism.

The Big Five personality traits are measured by a standard battery of self-assessment items. When answering these questions, respondents state the extent to which they think the listed adjectives characterize them. The adjectives used in the survey are the following (grouped by personality trait for this listing but not in the survey):

Agreeableness: helpful, friendly, warm, caring, softhearted, sympathetic
Conscientiousness: organized, responsible, hardworking, careless, thorough
Extraversion: outgoing, lively, active, talkative, adventurous
Neuroticism: moody, worrying, nervous, calm
Openness: creative, imaginative, intelligent, curious, broad-minded, sophisticated

The five items for each measure were converted into a score that was standardized for the entire sample. The resulting personality measures are, quite naturally, not independent of one another. The correlation is strong between four of the five measures, with Neuroticism as the only exception. It seems that the personality items are uncorrelated with the number series score, with the potential exception of Openness. Table 3.6 shows the pairwise correlations.

Table 3.5

Pairwise correlation of the Big Five personality measures and the number series score

	Agreeable-ness	Conscientious-ness	Extro-version	Neuro-ticism	Open-ness	Number Series
Agreeableness	1.00					
Conscientiousness	0.42	1.00				
Extroversion	0.62	0.25	1.00			
Neuroticism	−0.03	−0.14	−0.08	1.00		
Openness	0.57	0.33	0.65	−0.08	1.00	
Number series	0.01	0.05	0.09	−0.05	0.11	1.00

HRS 2010, "Participant Lifestyle Questionnaire’" 50- to 70-year-old respondents.

Table B1.3 in Appendix B shows the regression results with each of the personality measure on the left-hand-side and the other personality measure on the right-hand-side together with the number series score, education, demographic variables and wealth.

The regression results confirm the correlated nature of the personality items, although the strength of the correlation is reduced by half when the other right-hand-side variables are controlled. The number series score is, at most, weakly related to the personality measures, but education has significant explanatory power. Higher education is negatively associated with Agreeableness and Neuroticism and positively related to Extroversion and, especially, Openness. The correlation between number series and Openness becomes insignificant because education takes on a positive association. There are systematic gender differences in the personality measures: women have significantly higher scores for Agreeableness, Conscientiousness, Extroversion, and they have significantly lower score for Openness. Age effects are also significant for Extroversion (increasing), Neuroticism (decreasing) and Openness (decreasing, perhaps because of cognitive decline). Ethnic and racial differences are also significant, especially for Conscientiousness and Neuroticism.

Conscientiousness is positively related to wealth even after controlling for cognitive capacity and the usual right hand-side variables. This result is in line with the results of Duckworth and Weir (2010), who used earlier measures in the HRS and found that conscientiousness was a strong predictor of wealth, even after controlling for lifetime earnings. This result is an important motivation for my inquiries into whether personality measures in general, and conscientiousness in particular, can help to explain heterogeneity in stock returns expectations.

General optimism is measured using questions on the same "Participant Lifestyle Questionnaire" in the 2010 wave of the HRS that contains the Big Five personality measures. Optimism is measured as the score of answers to three questions, which asking respondents whether they

are optimistic about the future;
expect the best things in uncertain times;
expect more good things to happen.

These measures were developed and validated by Scheier, Carver and Bridges (1994). Each question is measured on a six-point scale. The measure of optimism used in this paper is the score from the answers to these three questions, standardized to the entire sample.

Table B1.4 in Appendix B shows the regression results to shed light on the (partial) correlations of the measure of optimism with various right-hand-side variables, which include the sunshine optimism measure we used in previous research.[27] That measure is derived from questions administered in 1994 and 2000. It is available for a subset of the respondents with measures of optimism from 2010 but not for the entire sample. Therefore, I show results from two regressions, one with the sunshine measure and another one without. The number of observations in the first regression is less than half of the number of observations in the second.

The most important conclusion from table B1.4. is that the measure of general optimism from HRS 2010 and the sunshine optimism measure from HRS 1994 and 2000 are significantly correlated, even conditional on the other right-hand-side variables. This is a remarkable result for two reasons. First, both measures are likely to be rather noisy; the general optimism measure is based on three questions, and the sunshine optimism measure is based on two weather forecast questions compared to actual weather data. Second, the two measures are separated from each other by ten years or more. The significant relationship between sunshine optimism and the 2010 measure of general optimism cross-validates each measure, giving them high credibility.

The remaining correlations are consistent across the two subsamples. Women are slightly more optimistic than men, more educated and wealthier people are significantly more optimistic, and so are minorities. The differences between racial and ethnic groups are, of course, conditional on education and wealth, which should be kept in mind when interpreting them.

[27] Recall that this measure was defined by comparing people's subjective probability assessment of the day after the interview being sunny to actual sunshine data for the day in question. The measure is a binary variable, indicating a positive residual when sunshine expectations were regressed on actual sunshine data. The binary variable is therefore one for those who erred on the positive side more than other people (i.e., those who predicted sunshine with a probability that was above the average probability of people who faced the same actual sunshine).

The result for non-positive wealth is a phenomenon that is quite general in the analyses of expectations. Those with non-positive wealth are more optimistic than those with low levels of positive wealth in many domains, and we see the same association here with respect to general optimism. This association is driven primarily by households that are leveraged, in that they have higher value of debt than assets. The remaining results are intuitive and are consistent with the psychology literature and our earlier results (Kézdi and Willis, 2003).

Table B1.5 in Appendix B shows the correlation between financial knowledge, Big Five personality items, and optimism. Financial knowledge is uncorrelated with Big Five personality traits but weakly positively correlated with optimism. This is important because the theoretically interesting associations between personality and expectations are conditional on financial knowledge, but the sample is very small for a joint analysis of personality and financial knowledge. When we look at associations of expectations with personality measures without conditioning them on financial knowledge, the results on the Big Five personality traits are likely to be very similar to conditioning them on financial knowledge.

3.3 Methods

The main analysis ignores the fact that HRS 2010 asked three probability questions on stock market returns and uses the answers to the first two questions, p_0 and p_{20}. The reason for this methodological decision is that we simply do not understand the survey noise in the third question, p_{ltn20}. Recall that p_{ltn20}, asked the question of whether the returns would be below a certain (negative) threshold, whereas the first two questions asked whether the returns would be above certain (nonnegative) thresholds. The analysis of the answer patterns, documented in the previous section, suggests that the properties of the survey noise that enter this third variable are different from the properties that enter the first two. Moreover, because we have no direct test-retest evidence for the third variable, we are unable to characterize the noise pattern in a credible way. On the contrary, our analysis of the first two questions (or a question-pair that is very similar) in chapter one made use of test-retest data and arrived at a credible characterization of the noise process.

The methods that use two probability answers are identical to the methods used in the previous chapter, with the exception that p_{20} is used in place of p_{10} in the structural analysis. For brevity, I do not elaborate on the method but simply summarize it.

The goal of the structural econometric model is to estimate heterogeneity in the subjective beliefs of the mean and standard deviation of log returns, $\tilde{\mu}_i$ and $\tilde{\sigma}_i$, respectively. Let R denote one-year-ahead gross returns, which is a random variable with $\ln R \sim N(\mu, \sigma^2)$. $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ denote the individual-specific beliefs about those parameters (the i index refers to potential heterogeneity, and the tilde refers to the subjective nature). These parameters characterize the subjective beliefs about future stock market returns in the situation of an investment decision.

In principle, one can relate the probability answers to the parameters of the lognormal distribution in a straightforward way. Unfortunately, observed answers to the probability questions are not suited for such a straightforward transformation. There are theoretical reasons to believe that people's answers to the probability questions are not equal to the probability transformations of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ if those parameters are interpreted as the subjective mean and standard deviation that are relevant in an investment decision. There is little time to answer the questions in a survey situation, and, beyond a spirit of cooperation, there are no incentives to get the answers right. It is therefore better to look at actual answers as "guesses" of what the probabilities may be, given recollections of the returns distribution, which would be characterized by $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ in an investment situation.

As documented in the previous section (as well as the previous chapter), answers to the probability questions reveal patterns that support this view. Some of these answer patterns make it impossible to compute the probabilities. All of the answer patterns indicate that actual answers are noisy transformations of relevant beliefs. The strongest evidence is the test-retest evidence documented in the previous chapter using data from HRS 2002.

The structural model addresses the two main problems, the transformation of the probability answers to the theoretically relevant belief parameters and survey noise, in a simultaneous fashion. It relates the latent variables $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ to observed responses in the probability answers and the right hand-side variables (gender, education,etc.). The procedure specifies the way heterogeneity in $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ is related to heterogeneity in the right hand-side variables, taking observed probability answers and survey noise into account.

The expected vale of $\tilde{\mu}_i$ across respondents is specified as a linear combination of right hand-side variables, with parameter vector β_μ. Unobserved heterogeneity in $\tilde{\mu}_i$ is assumed to follow a normal distribution with zero mean and standard deviation that can be related, in a linear fashion, to the right-hand-side variables, with parameter vector β_u. This specification for heteroskedasticity in $\tilde{\mu}_i$ allows for the estimation of differences in disagreement by groups defined by the observable characteristics. Heterogeneity in $\tilde{\sigma}_i$ is specified as a two-point distribution with the lower point fixed to the historical standard deviation (0.15), the upper point estimated as the same scalar for everyone, and the probability of the upper point specified as a probit model with parameter β_σ on the observable characteristics.

With the exception of β_μ, the parameter estimates from the structural econometric model are not easy to interpret. Therefore, as outlined in the previous chapter, I computed predicted values of $\hat{\mu}_i$ and $\hat{\sigma}_i$ for each individual. The predictions use the estimates of the structural econometric model and the observable right-hand-side variables as well as the observed probability answers.

Survey noise is taken into account in the following ways. Rounding is handled by looking at intervals of the probability answers instead of the answers themselves, where the intervals are defined as rounding neighborhoods around multiples of 10 per cent. For example, answers of 77 percent, 80 percent and 83 percent are treated in the same way; similarly, 0 percent, 2 percent and 4 percent are treated the same way.

Test-retest noise, or, theoretically, the notion that people are likely to respond to survey questions by fast-and-frugal guessing instead of thorough cognitive effort, is taken into account by allowing random noise to the theoretical parameters as they enter the probability transformation. The potential for inconsistent answers, or, theoretically, the possibility for inattention when answering multiple questions on the same underlying variable, is handled by allowing for this noise component to differ for the different probability answers. The noise components are allowed to be correlated: a correlation of 1 means that all respondents answer the multiple probability questions in a fully consistent way, and a correlation of 0 means that they answer these questions in fully independent ways.

The distribution of the noise component is an important assumption of the structural model, and the additive mean-zero i.i.d. normality assumption is maintained. The variance and the covariance of the noise components are calibrated using the test-retest evidence documented in the previous chapter. Importantly, that calibration is based on the test-retest p_0 and p_{10} variables, but I use the same numbers in this analysis with p_0 and p_{20}. However, note that the Maximum Likelihood method can identify the between-question correlation coefficient of the test-retest noise components (because the fraction of answers that do not conform to the laws of probability identify that parameter if everything else is specified). When allowing for such an identification, the estimated correlations are very similar to the calibrated correlations for p_0 versus p_{20}. When a similar exercise is performed with respect to p_0 and p_{ltn20}, the result is always a negative correlation coefficient, which is difficult to interpret. This highlights the need for a better understanding of the noise process with respect to p_{ltn20} before including it in the analysis. Nevertheless, a relatively straightforward generalization of the structural econometric model can incorporate the answer to the third probability question, by transforming the third answer into $p_{n20} = 100\% - p_{ltn20}$. For robustness checks, all models are re-estimated, with results provided in section B3 in Appendix B.

An alternative method that requires fewer assumptions considers heterogeneity in the probability answers directly. Holding risk constant, more optimistic beliefs result in higher values of the probabilities. We can therefore think of them as proxy variables for the perceived level of returns. A mean-preserving spread leads to smaller area of the density function between two points of the support, as shown in Figure 2.1 in the previous chapter. The difference between two probability answers may thus serve as an inverse proxy variable for the subjective belief about the standard deviation of suture stock returns, which is a measure of perceived risk.

As we argued in the previous chapter, these proxies are not clean. The effect of risk on the probabilities can be ambiguous; higher risk corresponds to a smaller area to the right of a cutoff point if the mean is to the right, but it corresponds to a larger area if the mean is to the left (as for significantly higher cutoffs). Optimism/pessimism affects the difference between the probabilities in ambiguous ways. Thus, although this more direct approach requires fewer assumptions, the results of such regressions are more difficult to interpret.

Nevertheless, I show results for the probabilities themselves in the spirit of the previous chapter. I look at categories of the main explanatory variable in question (financial knowledge, personality) and show the average p_0, the standard deviation of p_0, the average of the $p_0 - p_{20}$ difference and the fraction of missing p_0 answers.

3.4 Results on financial knowledge

I first examine the first question of the paper: whether and to what extent expectations differ for people who are characterized by different levels of financial knowledge. I consider two types of financial knowledge: first, the specific knowledge that, historically, stocks have outperformed bonds and savings accounts, and, second, more general financial knowledge.

The results with respect to the first dimension of financial knowledge can be thought of as testing the hypothesis in the previous chapter. The hypothesis in that paper states that knowledge about past returns is an important determinant of expectations of future returns. The experimental module in wave 2010 was the first time that HRS asked a question about historical stock returns. Unfortunately, the question was asked of a random 10 per cent of the entire sample, limiting the scope of the analysis because of sample size considerations. Moreover, the historical returns question did not ask about historical stock market returns per se but whether those returns were higher than returns on bonds or savings accounts. This limits variation in potential answers: those who think that stock returns were relatively low but still somewhat above bond returns should give the same answer as those who think stock returns were very high. At the same time, their expectations about future stock returns are likely to be very different.

Nevertheless, even this crude version of the historical returns question should be related to significant differences in stock market expectations if our hypothesis is correct. Table

3.6 shows the simplest evidence: the average of the p_0 answer (the probability of positive returns), its standard deviation (a measure of disagreement), the average of the difference between p_0 and p_{10} (an inverse measure of perceived risk), and the fraction of missing p_0 answers (an indicator of ignorance, extreme risk, or uncertainty that prevents respondents from quantifying their expectations).

Table 3.6

Statistics of the stock market expectation answers by respondents' answers to which asset paid the highest returns over past 20 years

Which asset paid highest historical returns	Average p_0 (in %)	Standard deviation of p_0 (%)	Difference $p_0 - p_{20}$ (%)	Fraction missing p_0 (%)
Saving accounts	37	28	4	13
Bonds	46	24	12	5
Stocks	51	25	19	4
Does not know	48	32	15	43
All	48	26	16	9
Observations	626	626	476	685

HRS 2010. Financial literacy experimental module, 50- to 70-year-old-respondents.

The figures in Table 3.6 provide strong support for our hypothesis of knowledge about past returns being an important determinant of expectations about future returns. In HRS 2010, respondents who thought that saving accounts provided the highest returns historically thought, on average, that the probability that stock prices would go up within a year of the interview was 37 per cent. Respondents who thought that bonds provided the highest returns historically, thought, on average, that the probability was 46 per cent. Respondents who did not know which asset class performed best historically thought, on average, that the probability was 48 per cent (equal to the overall average). However, those who answered, correctly, that stocks outperformed the other asset classes thought, on average, that the probability of positive returns would be 51 percent.

Knowledge about historical returns is also related to perceived risk and uncertainty. The crude measure of perceived risk shown in Table 3.6 is the difference between p_0 and p_{20}. The larger this difference is, the smaller the perceived risk (because a larger difference means more probability mass between the two points of the support and thus a steeper c.d.f. or a taller and thinner p.d.f.). Those who thought that stocks earned the highest return are, on average, characterized by lower perceived risk than other respondents. According to this measure, the highest perceived stock market risk is characteristic of the group that thought that bank saving accounts produced the highest returns. The fraction of missing answers can be thought of as a measure of extremely high perceived risk, and it may include uncertainty of the type that may make people unwilling to quantify expectations. The fraction of such people is smallest among those who thought that stocks produced the highest returns.

The cross-sectional standard deviation in the p_0 answers is also smallest among those who thought that stocks outperformed other assets historically. This fact can be interpreted as additional support for the learning hypothesis outlined in the previous chapter. Learning leads to lower levels of disagreement, which should appear in a lower cross-sectional standard deviation of the answers. Note that a lower standard deviation in his group is not the result of a mechanical relationship. In principle, expectations about stock returns may be as diverse among people who think that stock returns are higher than bond returns as among people who think that stock returns are lower than bond returns. The fact that those who think stock returns are higher are also more homogeneous in their expectations is evidence that this group has more knowledge supporting their beliefs.

The second question addresses other types of financial knowledge. Here, the theoretical predictions are less straightforward. First, the measure of other financial knowledge may be just another proxy variable for knowledge about historical returns. This is likely the case if we do not condition on our measure of historical returns analyzed above. However, it may be true even conditional on that measure because variation in this measure of financial knowledge may be related to variation in knowledge about past returns among those who gave the correct answer, that stocks had outperformed the other assets.

Second, expectations are likely to differ even among people who have the same knowledge about past returns. This additional variation may be completely random; it may be related to personality traits and general optimism, or may be related to other dimensions of financial (or other) knowledge.

Table 3.7 shows statistics for the probability answers analogously to the previous table, in categories of the financial knowledge score. Recall that the score is the number of correct answers to the four questions listed in the Data section above.

Table 3.7

Statistics of the stock market expectation answers by respondents' financial knowledge score

Financial knowledge score	Average p_0 (in %)	Standard deviation of p_0 (%)	Difference $p_0 - p_{20}$ (%)	Fraction missing p_0 (%)
0	50	30	13	40
1	44	27	12	9
2	45	26	12	9
3	51	25	20	4
4	52	24	18	3
All	48	26	16	9
Observations	626	626	476	685

HRS 2010. Financial literacy experimental module, 50- to 70-year-old-respondents.

The relationship between the mean p_0 and the score variable is almost monotonic, with the exception of zero correct answers being higher than the mean. The gradient is rather steep; those who gave one correct answer thought, on average, that stock prices would go up with a 44 percent chance, whereas those whose answers were all correct thought, on average, that stock prices would go up with a 52 percent chance. Perceived risk, as measured, in an inverse fashion, by the difference between p_0 and p_{20}, is nearly monotonically related to the financial knowledge score. The largest differences are again in terms of the fraction of missing answers. Although 40 percent of those with zero correct answers said "I don't know" to the stock market expectation question (p_0), this number was only 3 percent among those whose answers were all correct. Finally, and perhaps most interestingly, disagreement in expectations, as measured by the standard deviation of the p_0 answers, is inversely monotonically related to the financial knowledge score, providing additional support to the learning argument.

Of course, as discussed in detail in the previous section, answers to the probability questions are imperfect proxies of the parameters of expectations that are of interest. For that reason I turn to structural estimates of heterogeneity in the relevant belief parameters, $\tilde{\mu}_i$ (the belief about the mean returns) and $\tilde{\sigma}_i$ (the belief about the standard deviation of returns, a measure of their risk).

First, I show the parameter estimates of the structural model. β_μ denotes the association of the subjective mean of the returns with the right-hand-side variables. β_u denotes the association with unobserved heterogeneity in perceived mean returns, a measure of disagreement. β_σ is the association between right-hand-side variables and the probit index of high perceived risk. Although the magnitude of this last parameter is especially difficult to interpret, its sign indicates associations with perceived risk, and its statistical significance indicates the statistical significance of the association with perceived risks.

Table 3.8. shows the estimation results with the two measures of financial knowledge entered together. The first measure, "Historical returns," is a binary variables that is one if the respondent indicated that stocks outperformed bonds and saving accounts in recent history and zero otherwise. The second measure, "Other financial knowledge," is the standardized (mean zero, standard deviation one) score.

Table 3.8
Stock market expectations and financial knowledge. Parameter estimates of the structural econometric model ($n = 619$, log likelihood $= -2637.6$)

	Perceived mean (β_μ)	Log unobserved heterogeneity in (β_u)	Probit coefficient for high perceived risk (β_σ)
Historical returns	0.11	-0.83	-0.61
(S.E.)	(0.04)**	(0.30)**	(0.28)*
Other financial knowledge	0.05	-0.50	-0.17
(S.E.)	(0.02)**	(0.13)**	(0.13)
Constant	-0.15	-1.35	1.26
(S.E.)	(0.03)**	(0.20)**	(0.27)**

Historical returns: dummy for response stocks had higher returns than bonds and savings accounts.
Other financial knowledge: standardized score. See the methods section for details of the model.
Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.
HRS 2010. Financial literacy module, 50- to 70-year-old-respondents. No other RHS variables

The results are qualitatively similar to, but stronger than, the results of the direct analysis of the probability answers in Table 3.6. This finding lends validity to the structural model in general and its handling of survey noise in particular.

Those who thought that stocks outperformed other assets had, on average, 11 percentage points higher subjective expected value of future stock returns than those who did not think that stocks outperformed other assets. A one-standard-deviation higher score on the other financial knowledge test is associated with a 5 percentage points higher subjective expected value. Unobserved heterogeneity in beliefs about mean returns is significantly lower among those who thought stocks outperformed other assets, and other financial knowledge is significantly negatively associated with unobserved heterogeneity in expectations. Finally, knowing that stocks had higher historical returns than bonds and saving accounts is also significantly negatively associated with beliefs about the standard deviation of returns, our measure of perceived risk.[28]

Note that the model in Table 3.8 does not contain other right hand-side variables. When the number series score, education and other demographic variables are entered in the regression (table B2.1 in Appendix B), the coefficients on the historical returns indicator drop by almost half and lose their statistical significance, but the coefficients on the other financial knowledge score remain similar. The relatively small sample size ($n = 619$) may be largely responsible for the loss of significance. Also note that conditioning on variables such as education may bias the estimated associations with financial knowledge toward zero; two people with the same financial knowledge indicator but different levels of education may have different levels of true financial knowledge that are not captured by our indicator. For that reason I argue that the unconditional estimates, presented in Table 3.8, provide a better picture of the association between beliefs about stock market returns and financial knowledge. The more conservative estimates are very similar with respect to the other financial knowledge score but show an association with the historical returns indicator that is half as strong as the unconditional estimates.

To get a better sense of the magnitudes, Table 3.9 shows the statistics of the predicted $\hat{\mu}_i$ and $\hat{\sigma}_i$ variables. Recall that these are predicted values of the latent variables conditional on the right-hand-side variables (the two financial knowledge variables in this case) as well as the answers to the stock market probability questions (p_0 and p_{20} in this case). The table shows the statistics in four categories by people's assessments of historical stock returns (lower than other assets or higher than other assets) and other financial knowledge (lower than average or higher than average).

[28] Tables B2.6 and B2.7 in Appendix B show corresponding estimates from a structural model that uses the answers to all three probability questions. Those results are substantially weaker than the results reported above. However, as discussed in the data section above, using the answers to the third probability questions are problematic because the noise features regarding that survey answer are poorly understood.

Table 3.9

Stock market expectations and financial knowledge. Implied average of the subjective mean ($\hat{\mu}$), heterogeneity in the subjective mean ($\hat{\mu}$) and implied average of perceived risk ($\hat{\sigma}$) .

Beliefs about historical returns and other financial knowledge	Average of $\hat{\mu}$	Standard deviation of $\hat{\mu}$	Average of $\hat{\sigma}$
Low returns, low knowledge	−0.17	0.27	0.50
Low returns, high knowledge	−0.09	0.06	0.49
High returns, low knowledge	−0.07	0.09	0.46
High returns, high knowledge	0.02	0.03	0.41
All	−0.07	0.17	0.46

High/low returns: response of stocks higher/lower returns historically than bonds and savings accounts.

High/low other financial knowledge: standardized score above/below average.

Statistics of predicted values from the structural estimation model in Table 3.8.

HRS 2010. Financial literacy module, 50- to 70-year-old-respondents. No other RHS variables

The results show strong associations. They also show low expectations among people who thought that historically, stock returns had been lower than returns on bonds or saving accounts or who are characterized by lower financial knowledge than average. Indeed, only people with higher than average knowledge and who knew that stock returns were higher in the past had positive stock return expectations (28 percent of the entire sample). Perceived risk is also significantly negatively related to financial knowledge. Perceived risk is estimated to be high even among those with higher than average scores and those who know that stock returns have historically outperformed other assets; the 0.41 standard deviation of log returns is more than twice as large as the historical figure. Nevertheless, those in the lowest quarter in terms of financial knowledge are characterized, on average, by a significantly higher standard deviation of 0.50.

The standard deviation of $\hat{\mu}$ captures the magnitude of disagreement in its natural unit of measurement. It is perhaps the best measure of disagreement; it combines observed heterogeneity and unobserved heterogeneity (although in this particular case, with no other covariates, this feature is not important). Estimated disagreement is strongly associated with financial knowledge. In fact, estimated heterogeneity in the lowest financial knowledge group is extremely high, suggesting that people in that category may make wild guesses when they assess the prospects of the stock market.

3.5 Results on Big Five personality traits

After analyzing the associations with financial knowledge, I turn to the second question of the paper, whether heterogeneity in expectations is related to personality traits. In contrast to the role of financial knowledge, the theoretical predictions for the association between stock market expectations and personality traits are not straightforward. This is especially true if we consider personality conditional on financial knowledge. Consciousness and Openness may be related to expectations, but this may be because they are related to financial knowledge. We may find some relationship between the level of expectations or perceived risk because there is heterogeneity in expectations even conditional on financial knowledge. Note, however, that personality and disagreement are unlikely to be related. Disagreement is likely to be smaller among those with higher financial knowledge, but similar associations do not naturally arise with respect to personality.

Unfortunately, the small sample size does not allow for a joint analysis of personality and financial knowledge. Therefore, I look at associations with personality without conditioning on financial knowledge. In principle, therefore, these results should be interpreted as upward-biased estimates of associations between expectations and personality. These associations include the association between personality and financial knowledge. In practice, however, the bias should be negligible. As discussed earlier, the Big Five personality measures are uncorrelated with financial knowledge in the sample.

Table 3.10 shows the statistics for the observed probability answers by personality measures, with each personality measure transformed into a binary variable indicating a below average or above average score.

Table 3.10
Statistics of the stock market expectation answers by categories of the Big Five personality score.

Big Five personality trait	Average p_0 (in %)	Standard deviation of p_0 (%)	Difference $p_0 - p_{20}$ (%)	Fraction missing p_0 (%)
Agreeableness				
above average	48	26	15	9
below average	47	26	18	13
Conscientiousness				
above average	48	25	17	10
below average	47	26	16	12
Extroversion				
above average	48	26	17	9
below average	46	26	16	13
Neuroticism				
above average	47	26	15	12
below average	48	26	18	10
Openness				
above average	49	25	17	9
below average	45	26	15	14
Overall average	47	26	16	11
Observations	3169	3169	2390	3547

HRS 2010. "Participant Lifestyle Questionnaire", 50- to 70-year-old-respondents.

Agreeableness, Conscientiousness, Extroversion and Neuroticism do not seem to be related to stock market expectations. There are no differences in average p_0, suggesting that these personality traits, at least as measured in the HRS, are not associated with the level of expectations. Similarly, heterogeneity in p_0 is the same for low and high values of these measures.

The figures show some associations with respect to the average difference in the probability answers and the fraction of missing p_0 answers, but these associations are weak at best. More importantly, they do not show consistent patterns. The difference in the probability answers can be viewed as an inverse proxy for perceived risk. The fraction of missing answers can be thought of as an indicator of uncertainty (which can be interpreted in a Bayesian framework as extremely high perceived risk). Associations with perceived risk and uncertainty should be of similar directions; thus, associations should be of opposite signs with respect to the two measures. For example, as shown in Table 3.6, financial knowledge is positively related to the average $p_0 - p_{20}$ difference and negatively related to the fraction of missing p_0 answers. The same is not true for Agreeableness in Table 3.10, where the associations are in the same direction. There are no associations with Conscientiousness, and only missing answers are related, weakly, to Extroversion. Neuroticism shows consistent associations with the difference in the probabilities and missing answers, but the magnitude of the associations is small.

Only the Openness score shows a significant association with the level of expectations and some association with perceived risk and uncertainty. The Openness measure seems unrelated to disagreement.

The results on the probability answers suggest no relationship between stock market expectations and Big Five personality traits, with the potential exception of Openness. To confirm this conclusion, we must consider the structural estimates. Table 3.11 shows the parameter estimates from five models without other control variables. The full set of estimation results and alternative results with the right-hand-side variables are in Tables B2.1 through B2.5 in Appendix B. The personality measures were entered on the right-hand-side of the models one by one. In contrast with the models with financial knowledge measures, these models do not allow for unobserved heterogeneity to vary with the right-hand-side variable because, as we have seen in Table 3.10, the personality measures are not related to the dispersion of the probability answers.

Table 3.11

Stock market expectations and personality. Parameter estimates from five structural econometric models, estimated separately with each Big Five personality measure

Big Five personality trait	Perceived mean returns (β_μ)	Probit coefficient for high perceived risk (β_σ)
Agreeableness score above average	0.00	0.15
(S.E.)	(0.01)	(0.11)
Conscientiousness score above average	0.00	0.00
(S.E.)	(0.01)	(0.01)
Extroversion score above average	0.02	−0.03
(S.E.)	(0.02)	(0.11)
Neuroticism score above average	−0.01	0.14
(S.E.)	(0.01)	(0.11)
Openness score above average	0.05	0.02
(S.E.)	(0.01)**	(0.11)
Observations	3057	3057

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant Lifestyle Questionnaire" subsample, 50- to 70-year-old-respondents.

The models contain only the respective dummy variable on their right hand side.

The parameter estimates in Table 3.11 yield the same conclusions as the direct analysis of the probability answers. Except for Openness, the Big Five personality measures are not related to the level of stock market expectations, and none of them, including Openness, is related to perceived risk.[29]

Openness is conceptually related to education and knowledge. I have shown in Table B1.3 in Appendix B that it is significantly related to education in our sample. The observed associations with Openness may, therefore, reflect the effect of education. Table 3.12 shows results from the structural model with other right-hand-side variables entered together with the Openness measure. (The table also repeats the results without other right-hand-side variables.)

[29] Tables B2.8 through B2.12 in Appendix B show corresponding estimates from a structural model that uses the answers to all three probability questions. Those results are qualitatively similar to the results reported above.

Table 3.12
Stock market expectations and Openness. Parameter estimates of the structural econometric model.

	Perceived mean returns (β_μ)		Probit coefficient for high perceived risk (β_σ)	
Openness above average	0.05	0.02	−0.03	0.02
(S.E.)	$(0.01)^{**}$	(0.01)	(0.13)	(0.10)
Female		−0.08		0.58
(S.E.)		$(0.01)^{**}$		$(0.10)^{**}$
Age		−0.00		−0.01
(S.E.)		(0.01)		(0.01)
Education		0.02		−0.04
(S.E.)		$(0.00)^{**}$		(0.04)
Number series score		0.04		−0.33
(S.E.)		$(0.00)^{**}$		$(0.17)^{*}$
Constant	−0.10	−0.05	1.02	0.72
(S.E.)	$(0.01)^{**}$	$(0.02)^{**}$	$(0.11)^{**}$	$(0.27)^{**}$
Log likelihood	−13074	−12996		
Observations	3057	3057		

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant Lifestyle Questionnaire" subsample, 50- to 70-year-old-respondents.

The results indicate that the association between Openness and stock market expectations diminishes by more than half and loses its statistical significance if the other right hand-side variables are entered in the models. Comparing the results to the partial correlations in Table B1.3 in Appendix B, we can see that gender and education are primarily responsible for that drop. In other words, although people with a higher Openness score have higher stock market expectations on average, if we compare people of the same gender and the same level of education but different measures of Openness, there are no significant differences in stock market expectations.

It seems, therefore, that the Big Five measures are not significantly related to stock market expectations. Note that these negative results are rather strong; the sample size is large (five times the sample size for the financial knowledge estimates), the estimated coefficients are practically zero with relatively small standard errors, and the estimates are likely biased upwards in magnitude because the models do not condition on financial knowledge.[30]

[30] The significance is also biased upwards from a multiple testing point of view. Testing multiple hypotheses

3.6 Results on optimism

Finally, I analyze the relationship between the measure of general optimism and stock market expectations. The analysis is analogous to the previous ones. I first look at the statistics of the stock market probability answers in categories defined by the measure of optimism, and then I turn to the results of the structural econometric model. Table 3.13 shows the first set of results.

Table 3.13

Statistics of the stock market expectation answers by respondents' optimism score

Optimism score	Average p_0 (in %)	Standard deviation of p_0 (%)	Difference $p_0 - p_{20}$ (%)	Fraction missing p_0 (%)
Lower third	44	26	14	13
Middle third	49	25	18	10
Upper third	49	26	17	10
All	47	26	16	11
Observations	3159	3159	2526	3538

HRS 2010. "Participant lifestyle" questionnaire subsample, 50- to 70-year-old-respondents.

The general measure of optimism is related to the level of stock market expectations, especially at low levels. Respondents in the lower third of the optimism score gave, on average, responses that were percentage points lower to the p_0 question than the rest of the sample. There are no detectable differences between those who scored in the middle of the general optimism score and those who scored at the top.

There are no differences in terms of the cross-sectional standard deviation of the p_0 answers in the optimism score categories. Considering the difference between the two probability answers suggest that the may be a weak negative relationship between optimism and our measures of perceived risk. Similarly, considering the fraction of missing p_0 answers suggests that optimism maybe negatively related to extreme perceived risk or uncertainty.

As I argued in the introduction to this chapter, the relationship between general optimism and stock market expectations is more natural among stockholders than non-stockholders.

invokes corrections for the p-values that result in higher values (less likely rejections of the null hypothesis of no association). The significance test results shown above (in the form of stars to denote "significance") do not adjust for multiple testing. They are therefore overly ready to reject the null of no association. The negative results are strong because the tests fail to reject the null even without proper adjustments. (A suitable approach in exploratory analyses of this kind is the False Discovery Rate adjustment to the p-values, which can be approximated by the easy rule of thumb of multiplying simple p-values by two; see Anderson, 2008).

For stockholders, higher returns are a positive event, whereas for non-stockholders, higher returns are not necessarily a positive event. Table 3.14 shows the average of the p_0 answers by the three groups of the optimism score separately for people who live in households that own stock-market-based assets and people who live in households that do not own such assets. Note that the three optimism categories are defined for the whole sample and not by stockholding category. This does not matter because the stockholding rate is very similar in the different optimism categories.

Quite surprisingly, the relationship between general optimism and average p_0, the crude proxy variable for the level of expectations, is as strong among non-stockholders as stockholders. Stockholders, in line with our finding in the previous chapter, have substantially higher levels of expectations. However, more optimistic respondents have higher levels of expectations among non-stockholders as well as stockholders. A similar puzzle emerges if we relate average p_0 to the sunshine measure of optimism (not shown here): the relationship is similar among stockholders and non-stockholders.

Table 3.14.
Average p_0 answers by respondents' optimism score separately for stockholders and non-stockholders

Optimism score	Average p_0 (%) Non-stockholder household	Stockholder household
Lower third	41	53
Middle third	47	53
Upper third	47	55
All	45	54
Observations	2310	849

HRS 2010. "Participant lifestyle" questionnaire subsample, 50- to 70-year-old-respondents.

Taken at face value, these results for the entire sample are in line with the intuitive hypothesis of the association of general optimism with higher levels of stock market expectations. The fact that the relationship seems to exist among non-stockholders as well as stockholders is less intuitive. Similarly, the results with respect to perceived risk and uncertainty are less straightforward to interpret. The lack of association with respect to dispersion of beliefs suggests that, if there is any relationship between general optimism and stock market expectations, it is unlikely to be associated with learning. Note, however, that the associations with respect to perceived risk are not very strong, and the interpretation of all results may be problematic due to the complicated relationship between probabilities and the mean and standard deviation of stock returns.

The structural estimates help to provide more insight into the magnitudes in terms of relevant heterogeneity, and they can help determine whether the observed association with general optimism remains significant if we condition on other right-hand-side variables. Table 3.15 shows the parameter estimates of the structural model without and with other right-hand-side variables in the entire sample. Table 3.16 shows the parameter estimates for β_μ separately estimated for stockholders and non-stockholders. Similar to the Big Five personality analysis, the structural model here does not allow for heteroskedasticity in unobserved $\tilde{\mu}_i$.

Table 3.15
Stock market expectations and optimism. Parameter estimates of the structural econometric model

	Perceived mean returns (β_μ)		Probit coefficient for high perceived risk (β_σ)	
Optimism above average	0.05	0.04	−0.26	−0.14
(S.E.)	(0.01)**	(0.01)**	(0.14)	(0.11)
Female		−0.08		0.58
(S.E.)		(0.01)**		(0.14)**
Age		−0.00		−0.01
(S.E.)		(0.01)		(0.01)
Years of education		0.02		−0.05
(S.E.)		(0.00)**		(0.05)
Number series score		0.04		−0.34
(S.E.)		(0.00)**		(0.14)**
Constant	−0.10	−0.06	1.15	0.80
(S.E.)	(0.01)**	(0.02)**	(0.12)**	(0.21)**
Log likelihood	−13083	−13004		
Observations	3058	3058		

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.
HRS 2010. "Participant lifestyle" questionnaire subsample, 50- to 70-year-old-respondents.

Table 3.16

Stock market expectations and optimism. Parameter estimates of β_μ from the structural econometric model, separately for stockholders and non-stockholders

	Perceived mean returns (β_μ)			
	Not stockholder household		Stockholder household	
Optimism above average	0.08	0.08	0.01	0.01
(S.E.)	$(0.02)^{**}$	$(0.02)^{**}$	(0.01)	(0.01)
Female		-0.08		-0.05
(S.E.)		$(0.02)^{**}$		$(0.01)^{**}$
Age		-0.00		-0.01
(S.E.)		(0.02)		(0.01)
Years of education		0.01		0.01
(S.E.)		$(0.00)^{*}$		(0.01)
Number series score		0.02		0.01
(S.E.)		(0.01)		(0.01)
Constant	-0.20	-0.11	0.01	0.03
(S.E.)	$(0.01)^{**}$	$(0.03)^{**}$	(0.01)	(0.02)
Log likelihood	-9680	-9641	-3996	-3962
Observations	2315	2315	903	903

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant lifestyle" questionnaire subsample, 50- to 70-year-old-respondents.

The results are rather robust. They reinforce the relationship between the level of stock market expectations and the measure of general optimism. According to our point estimates, those who score above average on the optimism measure believe, on average, that the mean of future stock returns is 5 percentage points higher than those who score below the average. This association is true if we compare people with the same cognitive score, education and demographic characteristics. However, there are no significant associations with perceived risk.[31]

Surprisingly, according to our results in Table 3.16, the positive association between general optimism and the level of stock market expectations is a characteristic of people

[31] Table B2.13 in Appendix B shows corresponding estimates from a structural model that uses the answers to all three probability questions. Those results are substantially weaker than the results reported above. However, as discussed in the data section above, using the answers to the third probability questions are problematic because the noise features regarding that survey answer are poorly understood.

who live in households without stock market-based assets. General optimism is not (or, at most, weakly) related to the level of stock market expectations among people who live in households that own stock market-based assets This puzzle seems robust to the composition of stockholders versus non-stockholders in terms of education, cognitive capacity and demographic characteristics.

3.7 Conclusions

This chapter focused on two questions: whether expectations about future stock market returns are related to financial knowledge and whether they are related to personality traits.

The answer to the first question is positive. Stock market expectations are strongly related to financial knowledge. From a theoretical point of view, the most important aspect of financial knowledge is knowledge of historical stock returns distribution. The previous chapter presented the theoretical arguments for the role of incentives and some personal characteristics in the acquisition of knowledge about past returns of the stock market. It argued that knowledge about the distribution of past returns should be important in determining expectations about future returns. The empirical analysis in that chapter showed that measures of incentives and personal characteristics are related to expectations about future returns, as predicted by the theory. However, a lack of explicit information about knowledge of the history of stock market returns prevented us from explicitly linking knowledge to future expectations.

The results in this chapter filled that gap. They provided strong support for the argument that knowledge of the history of stock market returns is a major determinant of expectations about future stock returns. The results show that people who know that stocks have outperformed bonds and saving accounts in the past have beliefs about the distribution of future returns that are significantly closer to the characteristics of the historical return distribution. Their expectations are, on average, positive (whereas other people's expectations are, on average, negative). Their beliefs about risks are also closer to historical risks than are other people's beliefs, although they are substantially higher than historical risks. The results on disagreement (heterogeneity in expectations) provide additional support for the learning argument. The expectations of people who know that stocks have outperformed bonds and saving accounts in the past are less heterogeneous than the expectations of the rest of the sample.

Other aspects of financial knowledge are shown to be strongly related to stock market expectations, even conditional on our measure of knowledge about the history of stock returns. It is possible that the measure of other financial knowledge is another proxy for knowledge of history because the measure of the latter is very imperfect. If this is true, this result provides no additional insight into the substantive question, but it shows how imperfect the financial knowledge measures are. Another possibility is that this result shows that stock market expectations are influenced by other aspects of financial knowledge, even conditional on knowledge about the history of stock returns. In other words, people with perfect knowledge of the history of stock returns may form different expectations about future returns if they have different levels of financial knowledge. Without further evidence, it is impossible to separate the two explanations.

The answer to the second question on personality traits is largely negative. Four of the Big Five personality traits (Agreeableness, Conscientiousness, Extroversion and Neuroticism) do not seem to be related to stock market expectations. The fifth trait, Openness, is associated with the level of expectations, but that association becomes insignificant conditional on gender and education. In contrast, general optimism is significantly associated with the level of stock market expectations (but not perceived risk or disagreement). This last result is in line with intuition and our previous results on sunshine optimism. However, the relationship seems to be significantly stronger among people who do not own stock market-based assets.

Taken together, these results imply that financial knowledge in general, and knowledge about the history of stock returns in particular, are important determinants of expectations about future stock returns. There is substantial heterogeneity in expectations conditional on financial knowledge, but understanding that variation proves difficult. Standard measures of personality, at least as measured by the self-assessment survey questions, do not seem to be related to heterogeneity. General optimism does seem to be related, but there are some puzzling patterns in this relationship. The origins and consequences of the relationship between general optimism and stock market expectations are difficult to assess without further research.

A methodological conclusion of this analysis is that the structural model developed in the previous chapter adequately captures the most important aspects of systematic variation in stock market expectations, at least with respect to $\tilde{\mu}_i$, the belief about the mean of future returns. Variation in perceived risk ($\tilde{\sigma}_i$; the belief about the standard deviation of future returns) is better captured using two points of the support if those points are farther from each other (returns above zero and returns above 20 per cent as opposed to returns above zero and returns above 10 per cent in the previous chapter). Although more points of the support, especially in the negative domain, should further help to identify perceived risk, the response patterns suggest that the particular question on the HRS in the negative domain (returns below negative 20 percent) is characterized by survey noise that is qualitatively different from the noise we characterized in the previous chapter. Further evidence is needed on the nature of survey noise in the negative domain before such variables can be incorporated into the analysis.

4 Stock market crash and expectations of American households

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4.1 Introduction

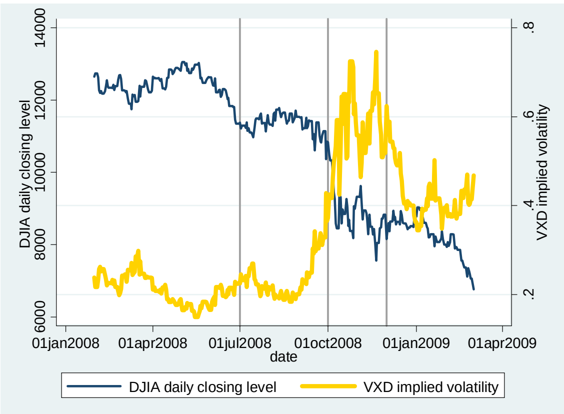
The stock market crash of 2008 and the financial crisis constitute a rare episode whose scope and implications fall outside the life experience of most American households. Whether and how those events affect people's expectations is an important question. To the extent that expectations guide investment behavior, substantial changes in expectations due to the financial crash can lead to substantial changes in investment. Besides average beliefs of 'the representative household', the crisis may have an impact on heterogeneity of such beliefs.

This study uses data from the 2008 wave of the Health and Retirement Study (HRS) to study the impact of the crisis on people's expectations. We estimate the effect of the crash on the population average of expected returns, the population average of the uncertainty about returns (subjective standard deviation), and the cross-sectional heterogeneity in expected returns (an indicator of disagreement). We show estimates from simple reduced-form regressions on probability answers as well as from a more structural model that focuses on the parameters of interest and separates survey noise from relevant heterogeneity. The measurement strategy makes use of the fact that the respondents of HRS 2008 answered the survey during 12 months from February 2008 to February 2009, a time period that includes the time of the stock market crash in early autumn. We show that the date of interview is largely independent of the respondents' past expectations about the stock market, so even if the date of interview is non-random it is unlikely to bias our results. Our analysis looks at changes in expectations during the HRS sampling period of February 2008 to February 2009. It may be useful to recall some of the important events during this period. The subprime mortgage crisis began well before 2008, but the Dow Jones peaked in October 2007 above

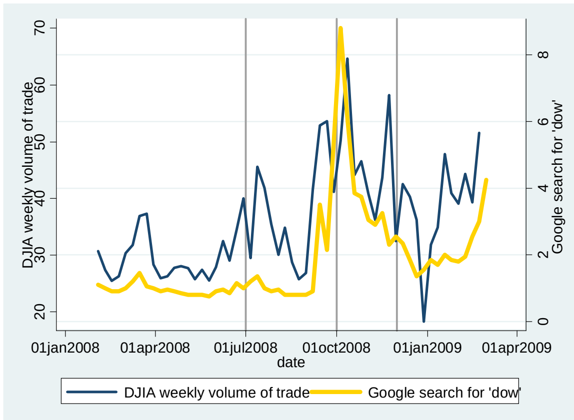
[32] http://onlinelibrary.wiley.com/doi/10.1002/jae.1226/abstract

14,000. By early 2008, though, the Dow was down to 12,000, and the rest of the year was characterized by a general decline until the crash of October. March 2008 saw the failed bailout of Bear Sterns and its subsequent sale to JP Morgan, but the rest of the Spring and the Summer went relatively quietly. On 15 September Lehman Brother filed for bankruptcy. The financial system was thought to be in severe danger, and it took a few weeks of uncertainty and heated debates before the US Congress passed the TARP bill on 3 October. The fall of 2008 also witnessed the run-up to the Presidential election on 4 November, which focused many people's attention towards economic issues, but it also led to a natural uncertainty about future economic policy.

Figure 4.1 shows time series of four stock market variables over the course of the HRS sampling period. We divided the sampling period into four sub-periods on the figure: February to June, July to September, October to November, and December to February 2009. We shall use these sub-periods throughout our analysis; their definition was based on the stock market time series we discuss below.



Level and volatility



Volume of trade and Google search for 'Dow'

Figure 4.1. Level of the Dow Jones Industrial Average (daily closing), the VXD annualized volatility index, weekly volume of trade in billions of dollars and Google search for "Dow' from the US in the sampling period of HRS 2008 (February 2008 through February 2009)

The left panel of Figure 4.1 shows the level of the Dow Jones Industrial Average and the VXD annualized volatility index.[33] After initial ups and downs, the level of the index started a substantial but gradual decline in June that stopped in August. The stock market crash hit in early October with a 3000-point drop in the Dow. The stock market experienced large swings in October and November, and the Dow reached a 6-year low of 7500 in late November. After some recovery and a brief period of stability, the Dow experienced another period of

[33] The VXD index is derived from prices of options on the DJIA, and it measures the future (30-day) expected volatility of investors. The details about this index can be found at http://www.cboe.com/micro/vxd/

steady decline in the first months of 2009. During the entire period, volatility showed the mirror image of the time series in levels, except that its increase started in September, and it reached its maximum in October and November. The right panel of Figure 4.1 shows the weekly volume of trade of the shares of the Dow Jones Industrial Average (DJIA) together with the trend of searches for the term 'Dow' on Google.[34] The latter variable is an indicator for the attention people give to news about the stock market. The figure shows a strong co-movement of the two time series: increased attention to stock market news coincided with increased volumes in March and July of 2008, February of 2009, and, especially, October of 2008. The Google index is normalized so that its 5-year average is one. The maximal 8.8 value in the first week of October means that almost nine times as many searches were made from the USA for the Dow Jones Industrial Average than in normal times. Looking at the two panels together, we can see that the volume of trade was the highest at times when the stock market index was decreasing, when uncertainty was increasing and when people paid a lot of attention to news about the market.

The main question of this paper is whether and how expectations changed during the stock market crash in early October 2008 and the following months. We compare post-crash expectations to those earlier in 2008. It is important to keep in mind that the baseline period was characterized by early signs of the crisis and a depressed stock market. Nevertheless, the comparison can shed light on the effect of a large and perhaps qualitatively different event compared to the more 'normal' declining market.

The crash may affect the population average of expected returns for various reasons. If people are unsure about the parameters of the returns process, they may use recent realizations to update their beliefs. In such a case, the crash would have a negative effect on everyone's expectations. If, on the other hand, people believe in mean reversion in stock market prices, the effect may be of the opposite sign. Of course, people may not want to update their beliefs if they don't learn from the returns. Besides stock prices, the political and policy news may have also affected people's expectations about the future of the economy and the financial sector in general, and the stock market in particular.

Empirical papers about stock market expectations usually find that average expectations track recent changes in the level of the stock market. When the stock market is increasing, average beliefs become more optimistic and conversely. See, for example, Kézdi and Willis (2008) about American households and Hurd et al. (2011) about Dutch households. According to Kézdi and Willis (2008), it took a 500-point gain in the Dow Jones to generate a one percentage point gain in expected yearly returns in 2002. With such a relationship, expected returns of respondents in November 2008 should be more than five percentage points lower

[34] http://www.google.com/trends

than expected returns of respondents 2 or 3 months earlier. On the other hand, the financial crisis of 2008 may have affected people's expectations in qualitatively different ways from the more gradual changes witnessed in 2002, especially if people had different views about the condition of the economy in 2002 and in 2008. People may expect asset prices to change in different ways after large sudden changes than gradual trends. This is the conclusion of Calvet et al. (2009b) who, using Swedish data, found that people tend to invest in well-performing mutual funds but also tend to dispose of winning individual stocks at the same time.

The effect of the crisis on average uncertainty is more predictably positive. Stock market risk increased dramatically, as indicated by the trend in volatility on Figure 4.1. Even those who do not follow the stock market could become more uncertain about the future of the economy in general and the stock market, in particular, as general uncertainty has been 'in the air' throughout the crisis.

The crisis may also affect the cross-sectional heterogeneity in households' beliefs. Heterogeneity and potential subjectivity of people's beliefs about future stock market returns has been the focus of recent developments in finance theory (see Hong and Stein, 2007, for an overview of disagreement models in finance). Harris and Raviv (1993) and Kandel and Pearson (1995) show that public announcements can increase disagreement about the fundamental value of assets if people interpret the news in different ways (see also Kondor, 2012). As Hong and Stein (2007) observe, this pattern is precisely the opposite of what one would expect based on a simple rational expectations model with heterogeneous priors, where public information should have the effect of reducing disagreement, rather than increasing it. Similar mechanisms may increase disagreement after the stock market crash as well. Dominitz and Manski (2011), for example, assume that the population is a mix of people who believe in the random walk hypothesis, who believe in the mean reversion of stock-prices, and who believe in the persistence of trends on the financial markets. When the crash hit the economy and stock prices fell sharply, people holding these various views should have interpreted its implications in different ways, and consequently the disagreement among them should have increased. Indeed, a potential explanation of the trading pattern shown in Figure 1 is that the increase of disagreement created space for trade as more optimistic traders wanted to buy and more pessimistic traders wanted to sell. Note that potential heterogeneity in the effect of the crash implies that the average effect could go either way.

Our results imply a temporary increase in the population average of expectations right after the crash. At the same time, average uncertainty increased, perhaps as the result of increased stock market volatility. Our most robust finding is that cross-sectional heterogeneity in expected returns, an indicator of the amount of disagreement, increased substantially with the stock market crash. The effects are found to be largest among stockholders, those who follow the stock market, and those with higher than average cognitive capacity. The result on average expectations thus masks a wide distribution of effects of opposing signs. We also document the co-movement of stock market expectations with ex post returns, implied volatility and volume of trade.

Our finding suggests that there is heterogeneity in the cognitive processes (or mental models) people use to convert public news into personal probability beliefs, in accordance with some of the disagreement literature we mentioned above. The results on changes in heterogeneity complement recent empirical investigations that show substantial heterogeneity in stock market expectations of individual investors (Vissing-Jorgensen, 2004) as well as households (Calvet et al., 2007, 2009a,b; Dominitz and Manski, 2007; Kezdi and Willis, 2008; Hurd et al., 2011; Gouret and Hollard, 2011). This paper adds new results to this empirical literature by showing that the stock market crash and the financial crisis had significant effects on average expectations, average uncertainty, and, perhaps most importantly, the heterogeneity of expectations.

4.2 Data

We use stock market expectations data from HRS-2008. Before turning to our analysis, it is helpful to provide some background on the evolution of the HRS stock market expectation questions.

In 2002, HRS introduced probabilistic expectations questions about returns in the stock market to the battery of subjective expectation questions that have been asked in HRS since it began in 1992. One motivation for adding these questions is that expectations about stock returns are a key component in determining retirement saving and portfolio choice. In addition, stock market expectations are of methodological interest because the history of stock returns and their daily realizations are public information, enabling researchers to investigate how news affects the updating of beliefs without the need to adjust for differences in private information.

Like other HRS probability questions, stock market expectations are asked as a percent chance based on a '0' to '100' scale where the respondent is told that:

> '0' means that you think there is absolutely no chance, and '100' means that you think the event is absolutely sure to happen.

The instruction goes on to say:

> For example, no one can ever be sure about tomorrow's weather, but if you think that rain is very unlikely tomorrow, you might say that there is a 10 percent chance of rain. If you think there is a very good chance that it will rain tomorrow, you might say that there is an 80 percent chance of rain.

Beginning in 2002, the HRS introduced a question about stock market expectations that has been asked in every wave of HRS since 2002. We call this the p_0 question. It reads:

> We are interested in how well you think the economy will do in the next year. By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?

Like other HRS subjective probability questions, many answers to the HRS stock market questions are heaped on '50' (Hurd and McGarry, 1995) and, unlike most other probability questions, a substantial number of people fail to answer the stock expectation questions at all. A number of researchers have suggested that '50' is an indicator of 'epistemic uncertainty' or imprecise probability beliefs (Fischoff and Bruine de Bruin, 1999; Lillard and Willis, 2001). Of course, it is also possible that some people who answer '50' mean that the event in question has a 50% chance of occurring or that they think that the probability falls within some range such as 40 – 60% and give '50' as a rounded approximation (Manski and Molinari, 2009).

Beginning in 2006 the HRS added an 'epistemic' follow-up question to several probability questions, including the p_0 question, to help understand the meaning of '50' answers:

> Do you think that it is about equally likely that these mutual fund shares will increase in worth as it is that they will decrease in worth by this time next year, or are you just unsure about the chances?

We now turn to a discussion of the 2008 data that we use in this paper. In 2008, HRS continued to ask the 'epistemic' follow-up to persons who answered '50 'to p_0 . For those who did not respond '50 'to p_0 or, if they answered '50', indicated that the shares were equally likely to increase or decrease in value, HRS added a follow-up question:

> By next year at this time, what is the chance they will have grown by x percent or more? (For negative values of x: By next year at this time, what is the chance they will have declined by -x percent or more?)

where the probability of a gain of $x\%$ from the set of $\{10, 20, 30, 40\}$ or a loss of $-x\%$ from the set of $\{-10, -20, -30, -40\}$ is randomly assigned.[35] We denote the answer to this question as p_{x+} if the random value of x is positive and as p_{x-} if it is negative. Note that p_{x+} denotes the probability that returns would be greater than x, while p_{x-} denotes the probability that they would be less than $-x$.

The full sample consists of 17,217 individuals from 11,897 households. We restricted the sample to those 14,735 persons who participated in the last three waves of HRS (2004, 6, and 8). In 2004, the sample was refreshed by a new, younger cohort. Out of the 14,735 people, 2,850 (19%) did not answer the p_0 question (the majority answered 'I do not know'), leaving us a sample of 11,885 people. As we indicated earlier, HRS did not ask the p_{x+} or p_{x-} questions from those who stated that they were unsure in response to the 'epistemic' follow-up question (2005 individuals).

Answer to the p_{x+} or the p_{x-} question is missing for another 486 individuals, and education was missing for an additional 45 individuals. Putting all these restrictions together, we ended up with a sample of 9348 individuals. The average age is 68 years, and 90% of the sample is 55 – 89 years old. We divide the sample into four subsamples based on the date of the interview (see Figure 4.1). These subsamples are very unbalanced in terms of the number of observations. 6285 respondents gave interview between February and June 2008, 2286 between July and September 2008, 556 in October and November 2008, and 211 between December 2008 and February 2009.

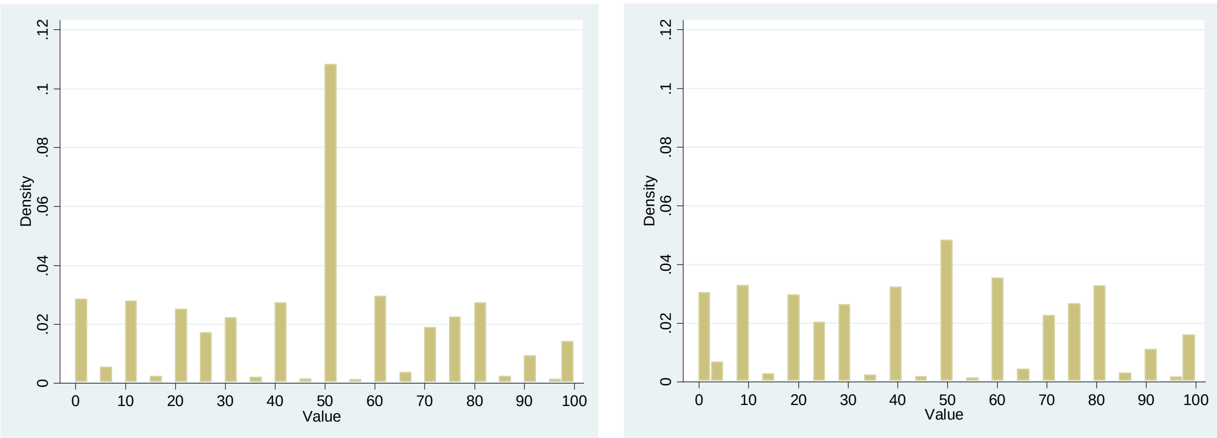
As we see, there are many missing values in the HRS stock market expectation data. Of the 14,735 people asked, only 9348 (63%) gave adequate answers to both questions. The two main sources of missing values are the 'I do not know' answers to any of the questions and being 'unsure' after giving a 50% answer to p_0 . In the analysis we shall ignore the missing values. We think that their omission does not invalidate our results for two reasons. First, people who 'do not know' or are 'unsure' might not have meaningful expectations about the stock market and thus they are not part of the population we would like to represent. These questions are not easy to answer, and if someone has no stocks and is sure that she will never have to deal with financial assets, she does not have to form expectations about the 1-year-ahead returns asked in the survey. Second, our goal is to analyze the changes in expectations after the crash. As long as the crash itself did not result in an increase or decrease of missing answers, the sample selection problem does not influence our main results. Analysis of the time series of missing answers reveals that the stock market crash did

[35] Randomization of x was not complete in the survey: those who gave 0 percent for the p_0 question were assigned to get a random x with x<0 but not x>0, while those who answered p_0=100 percent were assigned to get a random x with x>0 but not x<0.

not bring about more 'don't know' or 'unsure' answers. There is a small temporary decrease in the fraction of 'don't know' answers in October, but the decrease is both quantitatively small and statistically insignificant.

The distribution of p_0 answers is shown in the histograms of Figure 4.2. In part (a) we see the above-mentioned heaping at 50. Part (b) shows that the heaping disappears if we only leave in those 50 respondents who think that shares were equally likely to increase or decrease in value (rather than being unsure). Note that HRS did not ask the follow-up p_x questions from the 'unsure' people, so later in the analysis we will only use people from the panel (b).

Figure 4.2 also highlights another interesting issue: excessive rounding. Table 4.1 shows that almost 99% of the answers are multiples of 5, and more than 80% are multiples of 10. The fact that people give approximate answers to these probability questions is not surprising, since it is very hard to compute these numbers more precisely. A careful analysis should therefore incorporate this feature of the data.



All P_0 answers, N = 11885 P_0 answers in our final sample, N = 9348

Figure 4.2. Histogram of the p_0 answers in the total sample (on the left) and in the final sample (on the right)

Table 4.1, however, highlights an even more important problem: that of inconsistent answer pairs to the probability questions. Strongly inconsistent answers are those that contradict the laws of probability: $p_0 < p_{x+}$ or $p_0 + p_{x-} > 100$. Zero mass answers are the ones that imply zero probability of returns between the asked probabilities: $p_0 = p_{x+}$ and $p_0 + p_{x-} = 100$. Nearly 17% of the answers are strongly inconsistent, and more than 21% imply zero mass. On top of these problems, Kezdi and Willis (2008) document that many HRS respondents do not give the same answer to the same probability question (say, p_0) when it is asked twice within the survey 20 minutes apart. Analyzing stock market expectations in another dataset, Gouret and Hollard (2011) show that few people give answers that imply the same expectations if they are asked in two slightly different ways within the same survey. Perhaps surprisingly, both Kézdi and Willis (2008) and Gouret and Hollard (2011) find no relationship between personal characteristics and the propensity to give problematic answers, with the potential exception of income and expectations themselves.

We argue that such answers are due primarily to question-specific survey noise due to inattention. Survey responses are the results of individual behavior under circumstances that differ from circumstances when making an actual investment decision. Answers are given in a matter of seconds and there are practically no incentives to get the answers right. Therefore, we would be wrong to assume that the survey answers are equivalent to the probabilities that represent people's subjective return distribution which forms the basis for their investment decisions. In Section 4.4 we propose a method to separate survey noise from relevant heterogeneity in expectations.

Table 4.1

Fraction of rounded and inconsistent probability answers. HRS 2008.

		fraction
Rounding	p_0 is a multiple of 10	0.806
	p_0 is 5, 25, 75, or 95	0.140
	p_0 is a multiple of 5	0.986
	Strongly inconsistent answers*	0.169
	Zero mass answers**	0.215
N		$9,438$

*: $p_0 < p_{x+}$ or $p_0 + p_{x-} > 100\%$

**: $p_0 = p_{x+}$ or $p_0 + p_{x-} = 100\%$

4.3 Descriptive analysis

In this section we analyze the answers to the probability questions in a direct way. This should be viewed as preliminary descriptive analysis that cannot estimate the magnitude of the effect of the stock market crash, for two reasons: first, the probabilities themselves are not the objects of interest; second, survey noise can lead to biased estimates (especially on the heterogeneity of beliefs). At the same time, the descriptive analysis is free of additional assumptions that we need to make in order to recover more meaningful statistics.

Before the descriptive analysis, it is instructive to discuss how probabilities p_0 and p_x are related to the parameters of interest. Standard portfolio choice models include first and second moments of the (perceived) distribution of future returns as opposed to the probabilities themselves. With the help of additional distributional assumptions, answers to two probability questions can help identify the subjective mean and variance of the returns. Recall that the object of interest is the distribution of the 1-year-ahead returns of the stock market as viewed by the respondent. If we assume that people believe that the distribution of percentage returns is normal, two points in the subjective distribution identify the entire distribution and thus both the mean and the variance. Figure 4.3 shows a normal cumulative distribution function (c.d.f.) that is identified by the two points. The figure depicts the case where the mean of returns is 0.07 and standard deviation is 0.15 — numbers close to the post-war moments of nominal yearly returns on the Dow Jones (ending with year 2007). The probability of positive returns is around 68% (1 − 0.32), while the probability of returns of at least 20% (0.2) is around 20% (1 − 0.8). A respondent with the postwar – pre-2008 distribution in mind would answer p_0 to be 68% and p_{20} to be 20%.

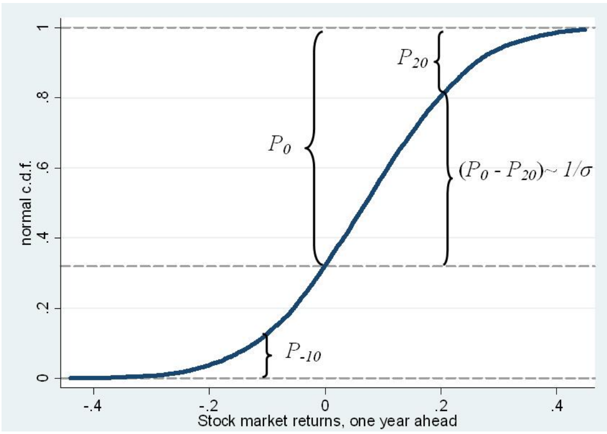


Figure 4.3. Standard normal c.d.f. (μ=0.07, σ=0.15), with p_0 and p_{20} shown.

Using answers to the two probability questions, one can in principle derive the mean (μ) and the standard deviation (σ) of the beliefs of individual i. Intuitively, the mean is identified from the level of the answers, while the standard deviation is identified from the distance between the two answers (larger distance means smaller variance). Formally, we can take the inverse of the appropriate probabilities:

$$p_{0i} = \Phi\left(\frac{\mu_i}{\sigma_i}\right) \tag{26}$$

$$p_{x+i} = \Phi\left(\frac{\mu_i - x/100}{\sigma_i}\right) \tag{27}$$

$$p_{x-i} = \Phi\left(\frac{x/100 - \mu_i}{\sigma_i}\right) \tag{28}$$

where, p_0 is the answer to the probability of positive returns, p_{x+} is the answer to the probability of returns at least x percent, and p_{x-} is the answer to the probability of losses of at least x percent. Note that a mean-preserving spread in uncertainty (σ_i) pushes the probabilities towards 0.5, because an increase in σ moves the index towards zero. This is very much in line with the casual interpretation of a "fifty-fifty" answer as reflecting ignorance. Using the example of positive x returns, inverting the probabilities would give this simple nonlinear but exactly identified system of two equations and two unknowns (μ_i and σ_i):

$$\Phi^{-1}(p_0) = \frac{\mu_i}{\sigma_i} \tag{29}$$

$$\Phi^{-1}(p_{x+i}) = \frac{\mu_i - x/100}{\sigma_i} \tag{30}$$

$$\Phi^{-1}(p_{x-i}) = \frac{x/100 - \mu_i}{\sigma_i} \tag{31}$$

Unfortunately, survey answers to the probability questions are not suited for such a direct transformation at the individual level. The excessive rounding and the relatively high fraction of inconsistent probability answers discussed in the previous section would invalidate such an analysis. In the next section, we propose a method for modeling both rounding and survey noise within a structural model. Before that, we present some basic descriptive results in this section.

In order to see if the stock market crash brought about changes in expectations about stock market returns, we estimate simple OLS regressions with crude proxies for the subjective mean (μ_i) the subjective standard deviation (σ_i) and the heterogeneity of expectations. In each regression, the right-hand side variables include three dummies for the four periods we focus on: February through June 2008 is the reference category, the first dummy is for July through September 2008, the second dummy is for October through November 2008, and the third dummy is for December 2008 through February 2009.

We estimate regressions with the probability answers themselves on the left-hand side in order to assess the effects on the population average of the level of the return distribution. If people become more pessimistic on average, we expect their answers to both the p_0 and the p_{x+} question to drop on average. If the second probability question has a negative threshold, their answer p_{x-} would go up on average. We therefore run two regressions, one with p_0 on the left-hand-side, and one with p_{x+} or $1 - p_{x-}$ on the left-hand-side. In order to partial out any threshold-specific factors that may bias answers to the second question, the second regression includes dummies for the different thresholds. The reference category is $x = +10$.

In order to see the effect of the crash on the cross-sectional heterogeneity of expectations (which we call disagreement), we look at regressions in which the left-hand side variables are the absolute values of the residuals from the previous regressions. If disagreement increases, the residuals from the previous regression would become more dispersed, and their absolute value would therefore go up.

The effect of the crash on the population average of subjective uncertainty is approximated by a regression with the difference in the two probability answers on the left-hand side. Recall from Figure 2 that the difference between p_0 and p_{x+} is inversely related to the standard deviation of the subjective distribution. Another way to see the connection is in terms of the p.d.f.: a larger difference would imply a larger probability mass concentrated on the support between p_0 and p_{x+}, which implies a less dispersed distribution. If the threshold of the second probability question is negative, the probability mass between p_0 and p_{x-} is given by $1 - (p_0 + p_{x-})$. In order for an increase in uncertainty to show up with a positive sign in the regressions, we used the negative of the differences for left-hand-side variables: $p_{x+} - p_0$ for positive thresholds and $[(p_0 + p_{x-}) - 1]$ for negative thresholds.

Before we turn to the results of the regressions, we address the question of whether the date of the interview is exogenous to prior stock market expectations. This is our most important identifying assumption in analyzing the effect of the stock market crash. The interview date was not randomly assigned. The HRS released the names of all sample households to its national field staff of interviewers at the beginning of the field period in February, 2008. Interviews were then completed in a sequence determined by each interviewer in consultation with regional field supervisors over the entire field period which ended in February, 2009. Sample members who are hardest to locate, most difficult to schedule and most reluctant to be interviewed tend to receive interviews relatively late in the field period. Ultimately, over 90 percent of eligible sample members were interviewed.

In the 2004 and 2006 waves of the survey, HRS collected data on p_0 from respondents in our sample (but there were no second probability questions asked on stock market expectations). Using these variables we can look at whether the date at which people were interviewed in 2008 is related to their answers to the p_0 questions in previous interviews. We estimated four regressions with stock market expectation variables from 2004 and 2006 on the left hand side and interview date in 2008 on the right hand side. The first two regressions have p_0 on their left hand side, while the third and fourth regressions have the residuals from those regressions (in each pair one is for 2004 and the other is for 2006). According to the discussion above, these regressions estimate the "effect" of interview date in 2008 on the average level of expectations prior to 2008 and heterogeneity of those expectations prior to 2008, respectively. The results from these "placebo" regressions are shown in Table 4.2. The only significant correlation with interview date in HRS 2008 and previous expectations is in column [2]: those who answered HRS 2008 between October and November gave slightly higher p_0 answers in 2004 on average. At the same time, no such relationship was found in more recent 2006. Overall, the results suggest that the date of the interview in HRS 2008 was largely exogenous to stock market expectations prior to 2008, a result that is especially robust in terms of disagreement.

Table 4.2
Placebo regression results: OLS estimates with proxies for the level (columns 1-2) and heterogeneity (columns 3-4) of expectations in 2004 and 2006 as dependent variables, the time of the interview in HRS 2008 as right-hand side variables.

Dependent variable	p_0 in 2006	p_0 in 2004	$\|u_{p_0}\|$ in 2006	$\|u_{p_0}\|$ in 2004
	(1)	(2)	(3)	(3)
Constant	49.38	50.76	20.41	20.61
	$(0.36)^{**}$	$(0.35)^{**}$	$(0.23)^{**}$	$(0.22)^{**}$
Interview between	0.73	1.31	−0.71	−0.09
July 2008 and Sep 2008	(0.70)	(0.68)	(0.81)	(0.43)
Interview between	0.79	2.49	−0.71	0.20
Oct 2008 and Nov 2008	(1.28)	$(1.25)^{*}$	(0.91)	(0.78)
Interview between	3.90	−1.82	1.39	1.24
Dec 2008 and Feb 2009	(1.99)	(2.00)	(1.26)	(1.26)
N	7941	8444	7941	8444

Robust standard errors in parentheses. * significant at 5%; ** significant at 1%

We can now turn to the effects of the interview date in 2008 on expectations in 2008. Table 4.3 shows the results. The dependent variables in columns (1) and (2) are the probability answers, our proxies for the population average of the level of the expectations. The results from the two regressions are very similar. The summer of 2008 brought no changes, and the average level was similar to the reference period in December 2008 through February 2009 as well. However, October and November 2008 saw a significant, if temporary, increase in the average level of expectations

Table 4.3

OLS regressions with proxies for the level (columns 1-2), heterogeneity (columns 3-4) and uncertainty (column 5) of expectations. HRS 2008.

Dependent variable	p_0	P_x	$\lvert u_{p0}\rvert$	$\lvert u_{Px}\rvert$	$P_x - p_0$
	(1)	(2)	(3)	(4)	(5)
Constant	45.63	64.81	24.49	21.43	-24.53
	$(0.36)^{**}$	$(0.79)^{**}$	$(0.18)^{**}$	$(0.43)^{**}$	$(0.97)^{**}$
Interview between	-0.14	0.57	-0.39	0.17	1.70
July 2008 and Sep 2008	(0.70)	(0.63)	(0.36)	(0.34)	$(0.77)^{*}$
Interview between	4.92	3.65	1.71	1.76	4.34
Oct 2008 and Nov 2008	$(1.25)^{**}$	$(1.213)^{*}$	$(0.64)^{**}$	$(0.62)^{**}$	$(1.39)^{**}$
Interview between	0.04	1.06	2.56	2.33	-0.18
Dec 2008 and Feb 2009	(2.00)	(1.80)	$(1.02)^{*}$	$(0.98)^{*}$	(2.21)
N	9348	9348	9348	9348	9348

Robust standard errors in parentheses. * significant at 5%; ** significant at 1%

Notes. "p_x" is defined as p_{x+} for positive thresholds and (1-p_{x-}) for negative thresholds

"p_x- p_0" is defined as (p_{x+}-p_0) for positive thresholds and (p_0+p_{x-}-1) for negative thresholds.

Columns (3) and (4) report the results on the absolute value of the residual from the previous regressions, which are our proxies for disagreement. The estimates imply that disagreement stayed constant before October 2008, but it increased significantly after the crash. Contrary to the average level of expectations, the increase in disagreement lasted to the end of the sampling period. Column (5) shows the estimates on the difference between the two probability questions, which proxy the effects on the population average of uncertainty. Uncertainty seems to have increased already during the summer, and the crash brought about a substantially larger increase. Similarly to the average level of expectations, though, average uncertainty returned to its baseline level in the last period.

The results from these regressions suggest that on average, people became more optimistic but also more uncertain after the crash, but those increases were temporary. Cross-sectional heterogeneity in expectations also increased after the crash, and that remained high a few months later as well. Unfortunately, as we highlighted earlier, these results are not suited for drawing quantitative conclusions for two reasons: they use crude proxies for the left hand side variables of true interest, and they do not incorporate the complex survey response problems shown in the previous section.

4.4 Structural estimation

In the previous section we derived the relation between the probability answers and the first two central moments of the subjective return distribution under the assumption of normally distributed returns. Because of rounding and response error, as discussed earlier, these relations cannot be mechanically applied to the data. We incorporate rounding and survey noise in our model in two steps.

Assume that when making an investment decision, individual i thinks of one-year ahead returns as R_i^* with mean μ_i and standard deviation σ_i. Throughout the analysis, we assume that R_i^* is normally distributed. (Results are robust to alternative distributional assumptions of Student-t and log-normal as presented in section C.2 of Appendix C.) The survey answers of individual i are, however, based on a noisy version of R_i^* that we denote as R_{ji} (where j denotes the question so that $j = 0$, $x+$ or $x-$). The noise is assumed to be additive: the mean of R_{ji} is $\mu_i + V_{ji}$, where v_{ji} is a mean-zero noise variable specific to question and individual. The idea behind this assumption is that in a survey situation individuals have little time and no incentives to retrieve their subjective distribution of stock market returns. As a result, the subjective distribution they have in mind when answering the questions is likely to be different from the subjective distribution they would consider in an investment situation. We allow the noise terms to be different for the two probability questions (p_0 and p_x) but correlated across questions: $Corr\left(v_{0i}, v_{xi}\right) = \rho_x$. The estimation model will allow for estimating both the variance of the survey noise and the correlation. When estimating the noise variance, we assume that it is proportional to subjective uncertainty σ_i. The intuition behind this assumption is those who have more diffuse expectations are likely to have a harder time retrieving those expectations. A consequence of this assumption is that $Var\left[R_i^*\right]/Var\left[R_{ji}\right]$ is constant. That is, this assumption ensures that the signal-to-noise ratio is constant in terms of perceived stock market returns.[36]

A second feature of our model is that we consider interval responses instead of the reported probabilities themselves. If the reported probability (p_{ji}) is in a pre-specified interval or 'bin' $[b_1, b_2]$ then the "true" probability (including the noise component v_{ji}) is assumed to be in the same bin but not necessarily the reported probability itself. Because a large fraction of the answers are multiples of 10 (see section 4.2), we have defined 10 percentage point wide bins: [0, 5); [5, 15); ... [95, 100). One consequence of this assumption is that a round answer can represent any expectation that would lead to probabilities around the particular round

[36]Note that this assumption is the same as the one we made in the previous chapters, but the notation is different. In the previous chapters, we had $\left(\frac{\mu_i}{\sigma_i} + v_{ij}\right)$, whereas here we have $\left(\frac{\mu_i + v_{ji}}{\sigma_i}\right)$. Using the first notation we assumed that $Var\left[v_{ij}\right]$ is constant. Using the second notation, here we assume that $Var\left[v_{ij}\right]/\sigma_i$ is constant. The two are equivalent.

number.

The two assumptions are combined to

$$
\begin{aligned}
p_0 &\in [b_1, b_2) \Leftrightarrow b_1 \leq \Phi\left(\frac{\mu_i + v_{0i}}{\sigma_i}\right) < b_2, && (32)\\
p_{x+i} &\in [b_1, b_2) \Leftrightarrow b_1 \leq \Phi\left(\frac{\mu_i + v_{x+i} - x/100}{\sigma_i}\right) < b_2, && (33)\\
p_{x-i} &\in [b_1, b_2) \Leftrightarrow b_1 \leq \Phi\left(\frac{x/100 - \mu_i - v_{x+i}}{\sigma_i}\right) < b_2 && (34)
\end{aligned}
$$

where, as before, p_0 is the probability of positive returns; p_{x+} is the probability of returns of at least x percent; and p_{x-} is the probability of losses of at least x percent.

Using interval responses is quite common in the literature dealing with subjective probabilities, but the explicit modeling of survey noise and the maximum likelihood approach is not. For example, Manski and Molinari (2009) argue that, because of rounding, the parameters of interest are only partially identified, and they propose an alternative estimator based on the theory of partial identification and set estimation. Their conservative strategy resulted in very wide estimated parameter sets, especially on the HRS data, probably because of excessive rounding. To avoid this problem we have chosen instead to specify the model fully with distributional assumptions on all the unobserved random variables (see later).

We specify heterogeneity in the subjective mean and variance of returns by equations in two latent left-hand-side variables μ_i, σ_i, of the form

$$
\begin{aligned}
\mu_i &= \alpha'_\mu w_i + \beta'_\mu x_i + \gamma'_\mu z_{\mu i} + u_i && (35)\\
\ln(\sigma_i) &= \alpha'_\sigma w_i + \beta'_\sigma x_i + \gamma'_\sigma z_{\sigma i} && (36)
\end{aligned}
$$

In the equations, w is the vector of date of interview dummies; x is the vector of covariates such as race, gender, age, education, and cognitive capacity; the z vectors are equation-specific variables. We say more about them later when we discuss identification.

An important issue addressed in this paper is the possibility of increased cross-sectional heterogeneity in expectations, which may be labeled as disagreement. In order to capture disagreement, we let unobserved heterogeneity in μ vary with the date of the interview. Variance in u (unobserved heterogeneity in μ) measures the heterogeneity of expected returns among individuals who share the same x and z_μ variables. Formally, we let the standard deviation of u be related to the date of interview dummies (w) and the other covariates (x):

$$\ln(Std(u_i)) = \alpha'_u w_i + \beta'_\sigma x_i \tag{37}$$

The last equation is for the standard deviation of the noise, v, which is assumed to be proportional to σ_i:

$$Std\left(v_{ji}\right) = \lambda\sigma_i \tag{38}$$

Equations (35) and (36) describe the parameters of interest as effects on (or correlations with) the expected value of latent variables (μ_i, σ_i), and equation (37) captures the effects on (or correlations with) the standard deviation of the latent variable μ_i. These latent variables are mapped to the probability answers as specified by the interval response model in equations (32) to (34), which include additive question-specific noise components (v_0 and v_x), as well. The model is completed by distributional assumptions on unobservables u and v. We assume that u, v_0 and v_x are jointly normally distributed and that unobserved heterogeneity, u, is independent of survey noise. However, we allow for v_0 and v_x to be correlated, and we estimate their correlation. One can argue that the correlation can be different for positive versus negative thresholds in the second question, and thus we estimate two correlation coefficients, one for v_0 and v_{x+} and one for v_0 and v_{x-}.

With these elements the model is complete and can be estimated using Maximum Likelihood. Before we turn to the results, it is worthwhile to spend some time on identification issues. For simplicity, assume for a moment that there are no covariates on the right hand side of (35)-(37). In this unconditional model we would have six parameters to estimate: μ, σ, $Std(u)$, λ, $Corr\left(v_0, v_{x+}\right)$ and $Corr\left(v_0, v_{x-}\right)$. In order to estimate them we need at least six moments. Interesting moments are $E\left[p_{ji}\right]$, $V\left[pji\right]$, $E\left[p_{0i} - p_{xi}\right]$, $V\left[p_{0i} - p_{xi}\right]$ and the fraction of inconsistent answers. Intuitively $E\left[p_{ji}\right]$ and $E\left[p_{0i} - p_{xi}\right]$ help identify $E\left[\mu_i\right]$ and $E\left[\sigma_i\right]$, while $V\left[p_{ji}\right]$, $V\left[p_{0i} - p_{xi}\right]$ and the fraction of inconsistent answers help identify $Std(u)$, λ and the correlations.

The estimation models include covariates and some exclusion restrictions as well. We use two instruments for μ (z_μ in equation 35) and one for σ (z_σ in equation 36). The first instrument for μ is the average probability that respondents assigned to the possibility of an economic recession in the near future in the previous two waves, 2004 and 2006, of the survey. The second instrument is an average score on nine questions about depressive symptoms of the interviewees in 2004 and 2006, such as feeling lonely or feeling sad, etc. again from the previous two waves of the survey. The instrument for σ is the fraction of 50 probability answers in 2004 and 6. The idea behind using this variable is that people who are generally uncertain tend to give a lot of 50-50 answers to probability questions.[37]

[37] This approach was first suggested by Lillard and Willis (2001) and has also been used by Sahm (2007) and Pounder (2007).

4.5 Results of the structural model

The main question addressed by our analysis is how structural parameters of stock market expectations changed through the sample period. Using the model outlined in the previous section, we estimate changes in the population average of μ_i (the subjective expected value of returns), the population average of σ_i (the subjective standard deviation of returns), and the population standard deviation of u_i (unexplained heterogeneity in the subjective expected value). We capture the change of expectations in time by dummy variables for the four periods: February to June 2008 (the reference period, characterized by relatively high level of stock market indices and low volatility); July to September 2008 (gradual decline, relatively low but increasing volatility); October to November 2008 (the aftermath of the stock market crash and subsequently low levels and high volatility); and December 2008 to February 2009 (low levels with some further decline, and lower volatility). In order to help interpret the coefficients, all right-hand side variables except the interview date dummies are normalized to have zero mean. As a consequence, the regression constant shows the expected value of the left-hand side variable in the reference period (February through June 2008) for an average respondent in the sample. Note that the mean of the right-hand-side variables in the reference period is very close to the overall sample mean. As a result, the regression constant is very close to the actual average response in the reference period. The results are shown in Table 4.4.

The estimates are in line with the reduced-form OLS results of Table 4.3. Average optimism about stock market returns increased temporarily in October-November: on average, people seemed to expect a recovery during this period. By December, the average expectations returned to where they were prior to the crash. Average uncertainty about stock market returns increased by 11 percent during the summer, and it increased again in October-November, by almost an additional 20 percent. However, average uncertainty seemed to return to its initial level afterwards. Unobserved cross-sectional heterogeneity in expectations increased by 13 percent in late summer as well, and it increased substantially in the fall. By October and November, the cross-sectional standard deviation was more than 50 percent larger than it was at the beginning of the year. Heterogeneity decreased somewhat after December, but it remained larger than before the crash.

Table 4.4

Date of interview in 2008 and average subjective expected value of yearly stock returns (μ), average subjective standard deviation $(\log(\sigma))$ and unobserved cross-sectional heterogeneity in expectations $(\log Std(u))$. Results from structural regressions. HRS 2008.

	μ	log(s)	log(Std(u))
Constant	-0.088	-0.606	-1.23
	(0.006)**	(0.023)**	(0.079)**
July 08 to September 08	0.001	0.113	0.131
	(0.010)	(0.037)**	(0.047)**
October 08 to November 08	0.062	0.292	0.569
	(0.025)*	(0.088)**	(0.099)**
December 08 to February 09	-0.028	0.019	0.38
	(0.033)	(0.119)	(0.135)**
Female	-0.062	0.235	0.146
	(0.009)**	(0.034)**	(0.044)**
Single	0.004	0.04	0.121
	(0.010)	(0.039)	(0.048)*
Black	-0.017	0.589	0.56
	(0.025)	(0.093)**	(0.096)**
Hispanic	0.002	0.387	0.332
	(0.027)	(0.107)**	(0.114)**
Age	-0.002	-0.005	-0.004
	(0.000)**	(0.002)**	(0.002)
Years of education	0.002	-0.034	-0.031
	(0.002)	(0.007)**	(0.009)**
Above average cognition	0.031	-0.101	-0.198
	(0.010)**	(0.037)**	(0.048)**
Follow the stock market	0.049	-0.129	-0.073
	(0.010)**	(0.038)**	(0.047)
Stockholder	0.072	-0.058	-0.18
	(0.010)**	(0.038)	(0.050)**
P(economic recession)	-0.003		
2004-2006 average	(0.000)**		
Depressive symptoms	-0.017		
2004-2006 average	(0.005)**		
Ratio of fifty-fifty answers		1.512	
2004-2006 average		(0.191)**	
Log-likelihood	-42277		
N	9348		
Mean(μ)	-0.085		
Mean(s)	0.616		
Mean(Std(u)	0.343		
$s^2/(s^2+V(v))$	0.645		
Rho(v0,vx-)	-0.491		
Rho(v0,vx+)	0.252		

Robust standard errors in parentheses. * significant at 5%; ** significant at 1%
Reference categories: Interview date March to June, male, non-Black and non-Hispanic, married

The coefficients on the other right-hand side variables indicate that women are significantly more pessimistic and more uncertain about stock returns; minorities are substantially more uncertain; older people are less optimistic and less uncertain; more educated people are less uncertain; and smarter people are more optimistic and less uncertain. Stockholders and those who follow the stock market are significantly more optimistic, and the latter are also less uncertain. The results imply that heterogeneity in beliefs is also different in different groups; those that are characterized by higher uncertainty on average tend to be more diverse in their beliefs. The sign on the variables that serve for exclusion restrictions are intuitive: those who were more pessimistic about the economy in the past have lower expectations on average, and the same is true for those with more depressive symptoms. The fraction of fifty-fifty probability answers in the past is a strong predictor of uncertainty about stock market returns.

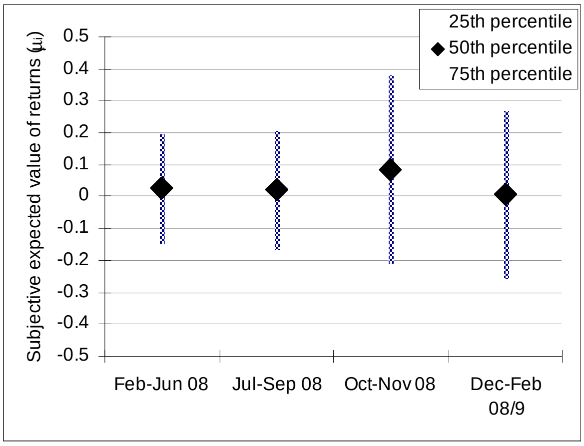
The last three lines of Table 4.4 contain estimates for the technical parameters. The ratio of true uncertainty to total variance that includes uncertainty as well as survey noise $(\sigma^2/(\sigma^2+V[v]))$ is constant by assumption and is estimated to be0.645. This implies that the noise variance is almost as large as true uncertainty. The noise terms are allowed to be correlated across questions. The correlation is positive when both probability questions ask about returns higher than a particular threshold value. It is negative when the second question is about the probability of returns smaller than the predefined values.

At first sight, it is surprising that the population average of expected returns is negative during the baseline period. Note, however, that male stockholders who follow the stock market and have above average cognitive capacity expect substantial positive returns on average; their average μ of 0.06 is close to the pre-2007 historical mean of 0.07.

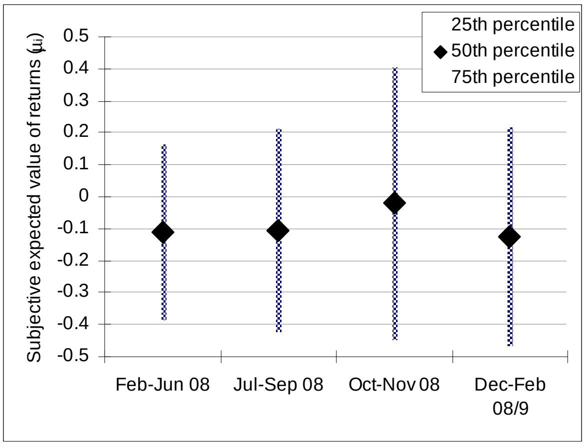
The coefficients on the interview date dummies in Table 4.4 show overall changes in the average level, average uncertainty, and heterogeneity of expectations. It is interesting to see whether those changes were different in different groups. In order to examine such possibilities, we estimated the model with full interaction using dummy variables that split the sample into two parts. The first model with interactions distinguishes between stockholders and non-stockholders. The second model looks at those who follow the stock market versus those who do not. The third model looks at those whose cognitive capacity is above the average versus those below average. The coefficient estimates of the three models are in Appendix C.1. The main results are summarized below with the help of three figures.

We first look at stockholders versus non-stockholders. Stockholders include all those who owned stocks directly, through mutual funds or in tax-sheltered accounts such as 401(k) accounts. Since asset holdings are defined at the household level, members of the same household were assigned the same stockholder status. From an asset pricing point of view, the effect of the crash on stockholders is more interesting than the effect on other households. Note that stockholding may be endogenous to the financial crisis. Therefore, we used the pre-crash stock-holding status from the 2006 wave of HRS. Figure 4 shows the results from the interaction model. The figure shows the 25th, 50th and 75th percentile of the population distribution of subjective expected returns (μ_i) for the four sampling periods. The distributions are recovered from the empirical distribution of the covariates and the normal assumption for the unobservables.

The results of Figure 4.4 indicate that stockholders have substantially higher and less uncertain expectations, consistently with standard portfolio choice models. Note that the differences are not captured in full by the interaction of stockholding status with interview date dummies presented in Table C1.1 in Appendix C, as the two groups differ in terms of the covariates as well (e.g. demographics and education). The median of the expected return distribution among stockholders is positive throughout the sample period, while the median of the non-holder distribution is negative. Changes in average μ (and average σ, see appendix) are similar in the two groups, heterogeneity among stockholders reacted to the stock market crash more in relative terms. In October and November 2008, the estimated inter-quartile range in expected returns rose from about 35 percentage points to almost 60 percentage points among stockholders and from around 60 percentage points to slightly more than 80 percentage points among non-stockholders.



Stockholders



Non-stockholders

Figure 4.4. Cross-sectional distribution of expected returns among stockholders and non-stockholders. Quartiles of the distribution by the date of the interview, estimated from the structural model with interactions (detailed results in Table C1.1 in Appendix C)

Next we look at the results for the better informed versus less informed individuals. HRS

2008 asked how closely the respondent follows the stock market. 8.5 percent answered "very closely", and another 36 percent answered "somewhat closely." The rest answered "not at all" or did not know or refused to answer. We merged the "very closely" and "somewhat closely" categories and called the subsample "informed respondents." The rest we call "uninformed respondents." Being informed and stockholding are of course correlated, but the correlation is far from being perfect. 70 percent of stockholders claim to follow the stock market (and 30 percent do not), while 30 percent of non-holders claim to follow the stock market (and 70 percent do not). Note that, similarly to stockholding status, whether one follows the stock market is potentially endogenous to the stock market crash. Unfortunately, only HRS 2004 contains the information for our sample, and it is missing there for quite a few individuals. We decided to use the 2008 measures for the analysis despite its potential endogeneity. The results are very similar if one uses the 2004 measures instead.

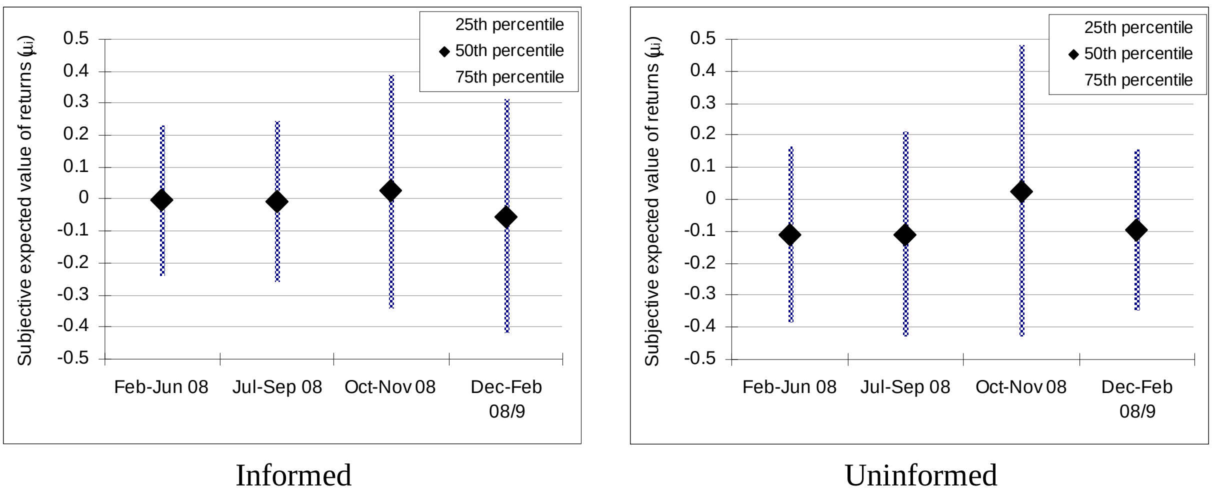
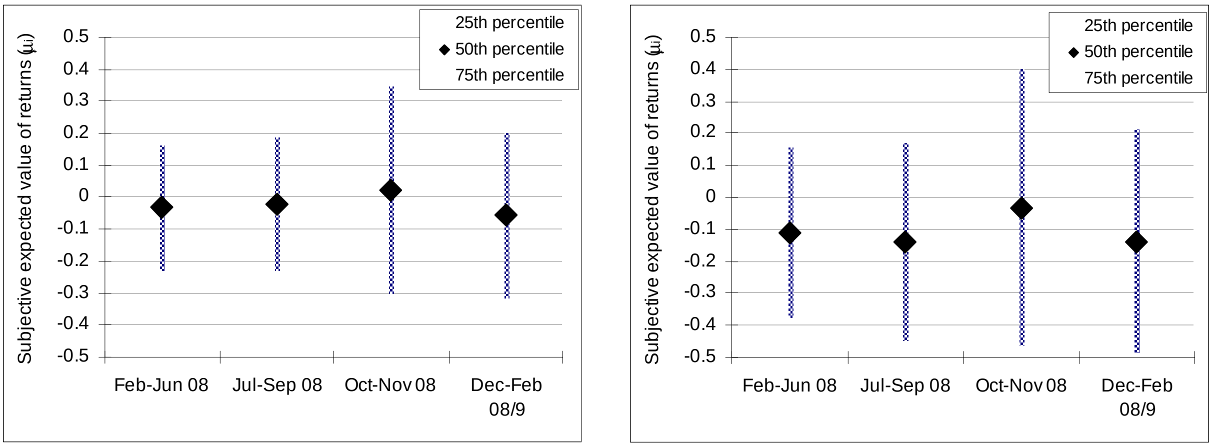


Figure 4.5. Cross-sectional distribution of expected returns among those who follow the stock market (informed respondents) and those who do not (uninformed respondents). Quartiles of the distribution by the date of the interview, estimated from the structural model with interactions (detailed results in Table C1.2 in Appendix C)

Figure 4.5 shows the results on the quartiles of expected returns in a way similar to the previous figure. The results are similar to the stockholder versus non-stockholder comparison, with some qualifications. The two groups differ less in terms of the initial level and heterogeneity of expectations than stockholders versus non-stockholders. The increase in the median of the distribution in October and November is more pronounced among the uninformed respondents, while the increase in the inter-quartile range is only marginally larger among the informed people. These results show that actual stockholding status is more strongly related to the effect of the stock market crash on expectations than whether one follows the stock market.

The third comparison is between those with above average cognitive capacity versus those with below average cognitive capacity. Cognitive capacity is measured by the principal component of various measures from HRS 2008. The measures include categories of self-rated memory, the score on immediate and delayed word recall and serial subtraction of seven from one hundred, and answers to three computing exercises, one of which is about compound interest rate. Cognition is correlated with whether one follows the stock market, but the correlation is not extremely strong (66 percent of informed respondents are above average in terms of cognitive scores, compared to 40 percent of uninformed respondents). Figure 4.6 shows the results again in terms of the estimated quartiles of subjective expected returns. In terms of the median, the patterns are more similar to what we found for informed versus uninformed people, while the patterns in terms of the inter-quartile range are closer to the patterns by stockholding.



Above average cognitive capacity   Below average cognitive capacity

Figure 4.6. Cross-sectional distribution of expected returns among people with above average cognitive capacity and those with below the average. Quartiles of the distribution by the date of the interview, estimated from the structural model with interactions (detailed results in Table C1.3 in Appendix C)

The results of the estimates suggest that the effect on the stock market crash on expectations was different in different groups of the population. The most pronounced differences are found between stockholders and non-stockholders. Differences between informed and uninformed people or higher cognition versus lower cognition people are similar but weaker.

They all suggest, however, that disagreement increased more among those who initially agreed more.

The final question we investigate in this paper is whether the changes brought by the stock market crash are close to what one would predict by changes in different stock market indices. We seek to answer two questions. The first question is whether the patterns of stock market expectations found above are related to the evolution of the stock market. The second question is whether the link between the stock market indices and expectations broke after the crash.

We have created three indicators, all based on the Dow Jones index. The first is the monthly log-return, defined as the log of the average DJIA index from the five days before the interview minus the same lagged by one month. The second indicator is the average of the VXD annualized volatility measure from the five days before the interview. The third measure is the log of the average daily volume of trade of shares in the DJIA index, again from the five days before the interview. These indicators are defined from the same data as the series on Figure 1, but their exact definition is somewhat different. The first indicator enters the equation of expected returns (μ), the VDX indicator enters the equation of uncertainty (σ), and the trading volume indicator enters the equation of disagreement ($Std(u)$). The rationale for the last inclusion is that high trading periods might be the ones when traders disagree about the fundamental price of assets and thus volume patterns might be able to predict disagreement.

Table 4.5 contains the estimates from two different specifications. Specification [1] is identical to the specification of Table 4.4 above, except that the stock market indicators are entered instead of the date of interview dummies. Specification [2] differs from specification [1] by allowing for an interaction of the stock market indicators with a dummy variable that is one if the interview date is after the crash (October 2008 through February 2009) and zero otherwise. If the relationship between the stock market indicators and expectations are the same before and after the crash, their coefficients should be stable across the specifications, and all the interaction terms should be zero.

Table 4.5
The effects of recent returns and volatility of the stock market index and the daily volume of trade of the shares of the DJIA, before and after the crash. HRS 2008.

	Specification (1)			Specification (2)		
	μ	$\ln\sigma$	$\ln St\,(u)$	μ	$\ln\sigma$	$\ln St\,(u)$
Constant	−0.09	−0.57	−8.96	−0.08	−0.79	−8.45
	(0.01)**	(0.04)**	(2.21)**	(0.01)**	(0.09)**	(2.23)**
Monthly log returns	0.05			0.33		
	(0.08)			(0.10)**		
VDX volatility index		0.01			0.98	
(avg. prev. 5 days)		(0.15)			(0.41)*	
Log volume of trade			0.35			0.33
(avg. prev. 5 days)			(0.10)**			(0.10)**
Post-crash dummy				−0.01	0.21	19.29
(Oct 08 to Feb 09)				(0.03)	(0.25)	(6.10)**
Post-crash interacted				−0.72		
with log returns				(0.26)**		
Post-crash interacted					−0.62	
with volatility					(0.60)	
Post-crash interacted						−0.84
with log volume						(0.27)**
Other covariates	YES	YES	YES	YES	YES	YES
Instruments	YES	YES		YES	YES	
Log likelihood	−42301.2					
N	9347					

Robust standard errors in parentheses. * significant at 5%; ** significant at 1%

The results for μ and σ are rather clear. They indicate that their relation to the stock market indicator (monthly returns and volatility, respectively) changed dramatically after the crash. Coefficients in specification [2] suggest that before the crash, an increase in the DJIA of 1 percent was followed by the population average of μ higher by 0.3 percentage points (i.e. 0.003). This magnitude is broadly in line with previous findings by, for example, Kézdi and Willis (2008). The post-crash relationship is just the opposite; there, the coefficient implies that the same one percent increase would be followed by a drop of 0.4 percentage points (=0.335-0.721). This negative relationship is most likely identified from the fact that within a one month window from the crash, the monthly log returns indicator was large and negative, while average expectations were higher than before. Specification [1] shows no relationship between returns and average expectations, because it shows the mixed results of a positive relationship between changes in the DJIA and the average level of expectations in the pre-crash period and the temporary increase in expectations after the crash. The population average of uncertainty shows a similar pattern; it tracks the volatility index before the crash in specification [2] closely, but its increase after the crash is smaller than what the large increase of the VDX would have implied. Again, specification [1] mixes the two and produces an insignificant estimate.

The results in Table 5 are less clear on the association between disagreement and the volume of trade. Our interpretation of the results is that it was largely similar before and after the stock market crash. An increase in the volume by one percent was associated with a subsequent increase in unobserved heterogeneity by somewhat over 0.3 percent before the crash in both specifications. The post-crash coefficients suggest a reverse association but also a huge increase in the intercept. Taken literally, they would imply that disagreement increased by astronomical magnitudes right after the crash, and from there it tracked trading volume with a negative coefficient. Recall though that trading volume jumped substantially right after the crash (see Figure 1 above), and it decreased considerably and steadily for most of the following time in our sampling frame. At the same time, disagreement stayed substantially higher after the crash than before, and it may have even increased in the immediate aftermath of the crash when volume dropped the most (although we would not have enough power to detect that). We argue that the post-crash association between volume and disagreement was dominated by the large increase in both right after the crash (a strong positive connection), and subsequent movements are of second order importance.

All the results presented in this section are based on the assumption of normally distributed subjective yearly returns. We checked the sensitivity of our results to this functional form assumption by considering two alternatives, the Student-t distribution with various degrees of freedom and the shifted log-normal distribution. The Student-t has fatter tails than the normal. It is motivated by the model of Weitzman (2007) who showed that, if agents have imperfect knowledge about the "true parameters" governing the stochastic process of stock market returns, and the parameters are evolving over time, then the posterior distribution of subjective returns can be Student-t. The shifted log-normal form is motivated by finance theory. While the log-normal is practically identical to the normal for small values of the return, it is quite different for larger values. All of our important results are robust to these alternative assumptions. The detailed results are available in section C.2 in Appendix C.

4.6 Conclusions

Using survey data on households' subjective probability beliefs about the one-year-ahead return on the Dow Jones stock market index, we estimated the effect of the stock market crash on the population average of expected returns, the population average of the uncertainty about returns (subjective standard deviation) and heterogeneity in expected returns. We presented estimates both from reduced-form OLS regressions and a structural model that can estimate relevant heterogeneity in subjective expectations and incorporates survey noise at the same time.

We used data from the Health and Retirement Study that was fielded in February 2008 through February 2009. We identified the effect of the crash from the date of the interview, which we showed to be exogenous to previous stock market expectations. The estimated effects are qualitatively similar in the reduced form regressions and from the structural model, and they are robust to the functional form assumption for the distribution of stock market returns. The results show a temporary increase in the population average of expectations and uncertainty right after the crash. The effect on cross-sectional heterogeneity is more significant and longer lasting, which implies substantial long-term increase in disagreement. Stockholders were found to have more positive, less uncertain and less heterogeneous expectations than non-stockholders, but the stock market crash led to a larger increase in disagreement among them than among non-stockholders. We found similar but smaller differences between those who follow the stock market and those who don't, as well as between those whose cognitive capacity is above the average and those whose cognition is below the average.

The large positive effect of the crash on disagreement suggests that there is heterogeneity in the cognitive processes (or mental models) people use to convert public news into personal probability beliefs, in line with the models of Harris and Raviv (1993) and Kandel and Pearson (1995). The differential effects on stockholders versus non-stockholders, and similar differences between informed and less informed or by cognitive capacity, may be due to the fact that those different groups receive different signals or process the signals in very different ways. These results provide empirical evidence for future research on heterogeneous beliefs in finance theory.

Another natural question for further research is whether the changes in expectations we document lead to changes in asset allocation. Data from HRS 2009 and 2010 will allow for a thorough analysis of the effect of the crisis on the reallocation of household portfolios and the role of expectations.

5 Concluding remarks

The three chapters of this dissertation examined ordinary people's expectations of returns attainable on the stock market. All chapters used the same methodology to uncover relevant aspects of people's expectations from their answers to survey questions. The questions were fielded in different years of the Health and Retirement Study (HRS), a survey representative of the 51-year-old and older American population. The focus of this dissertation was on the sources and consequences of heterogeneity of expectations. Each chapter examined a different aspect that heterogeneity.

Chapters 2 and 3 provided detailed evidence on survey noise, and the measurement model accommodates all the noise features we document. The results are consistent with our proposed explanation for heterogeneity in stock market beliefs: financial knowledge in general, and knowledge about the history of stock returns in particular, are important determinants of expectations about future stock returns. They also reinforce previous results about the predictive power of beliefs on stockholding. The results of Chapter 3 suggest that there is substantial heterogeneity in stock market expectations conditional on financial knowledge. Thus, two people with the same financial knowledge can have different expectations about future stock market returns. However, that extra heterogeneity in expectations is difficult to relate to standard psychological measures of personality. The only exception is general optimism, which appears to be significantly related to expectations of stock market returns, but the origins and consequences of the relationship between general optimism and stock market expectations are difficult to assess without further research.

In line with findings in the previous literature, the results of Chapter 4 suggest that macro events affect people's stock market expectations. Our results highlight the differential effect of the same macro event on different people's expectations. The large positive effect of the stock market crash on disagreement, documented in Chapter 4, suggests that there is heterogeneity in the cognitive processes (or mental models) people use to convert public news into personal probability beliefs. These results provide empirical evidence for future research on heterogeneous beliefs in finance theory.

The results in Chapter 2 establish the importance of belief heterogeneity in household finances. They show that survey answers to probability questions can be helpful in characterizing individual beliefs, but their analysis should recognize the importance of survey noise. Although our results emphasize the importance of beliefs, on a cautionary note, they also suggest that the strong correlation between beliefs and stock market participation in the HRS and other surveys cannot be interpreted as a causal relationship.

A causal interpretation of the results would suggest that heterogeneity in expectations leads to heterogeneity in stockholding, and low average expectations, high uncertainty, and large heterogeneity in expectations explain much of the stockholder puzzle. In principle, the identification of causal effects would require exogenous variation in expectations. However, as we emphasized in Chapter 2, the lifelong learning aspects of financial knowledge, together with the role of cognitive capacities and the complex incentives for savings, makes it difficult to find appropriate exogenous variation. Nevertheless, I believe that the results discussed in this dissertation, combined with better measurement of preferences and constraints, can shed more light on their respective role in households' investment decisions in general, and the stockholding puzzle in particular. That sets an agenda for future empirical research.

Appendices

A Appendix to Chapter 2

A.1 Details of the theoretical model

In this Appendix we present and solve a simple three-period life-cycle model of consumption and saving with risky assets, heterogeneous beliefs about the parameters of the distribution of returns, and potential learning about those parameters. The model is built on the "small scale" model of Haliassos and Michaels (2002), and we add to it elements that are connected to ideas in the human capital literature (e.g., Becker, 1964), its application to the acquisition of financial knowledge (Delavande, Rohwedder and Willis, 2008), and the theory of information choice (Veldkamp, 2011).

Consider individual i who lives for three periods. Period 1 contains her young active years (e.g., age 20 through 40), period 2 her active years in mature age (e.g., age 40 through 60), and in period 3 she is retired. In periods $t = 1, 2$ she receives labor income Y_{it}. In period 3 she receives pension benefits that are a function of her previous earnings $Y_{i3} = \pi(Y_{i1}, Y_{i2})$. We abstract away from taxes and non-labor income other than pensions. Importantly, pensions are from a defined-benefits-type system such as Social Security, and pension benefits are a concave function of lifetime earnings.[38] As a result, people who earn above certain threshold have an incentive to save for retirement, and the saving rate may depend on lifetime earnings. For simplicity, we assume that there is no uncertainty in earnings, the retirement age, pension benefits or the length of life.

In each period, individual i can save. Savings can be invested in bank accounts (B_{it}) that yield a fixed gross interest R_f or in equity (S_{it}) that yield stochastic potential return $R_t \sim iid \log N(\mu, \sigma)$. The individual cannot borrow or short the risky asset so $B_{it} \geq 0$ and $S_{it} \geq 0$. Risky returns R_t are defined as potential returns in the sense that the actual returns individual i can earn come at a discount of τ so that effective returns are $R_t^e = R_t e^{-\tau} \sim \log N(\mu - \tau, \sigma)$. The idea here is that R_t denotes the yearly gain on an ideal portfolio of risky assets. In this paper we assume that the ideal portfolio is the stock market index fund, and therefore realizations of R_t are the realized returns on the stock market index. We assume that individuals earn less than the return on such an ideal portfolio because of proportional transaction costs (therefore the notation τ) and sub-optimal choice of underlying assets.

[38] The Social Security benefit formula is very concave indeed. It starts with defining average monthly earnings from the lifetime earnings history, in which months without earnings count as zero. Benefits are 90 per cent of that average up to a relatively low level of earnnings; earnings in the middle range are transformed into benefits by a 32 per cent factor; and a factor of 15 per cent is used for high levels of earnings.

Importantly, we assume that the survey questions in HRS (p_0 and p_{10}) ask about potential returns R_t, but individual investment decisions are based on effective returns R_t^e. R_t is a random variable but the factor τ is not. Realizations of potential returns R_t are common across all individuals. However, individuals may differ in their beliefs about the parameters of the distribution of R_t (but they all think it is i.i.d. lognormal). Individual beliefs about the parameters are denoted by tilde over the greek letters denoting true parameters, e.g., $\tilde{\mu}$.

In the beginning of period 1, individual i is endowed with a set of beliefs about the parameters of the distribution of potential returns R_t. Beliefs about parameter σ^2 are assumed to be the same for everybody (this setup is the same as the one used by Brennan, 1998). At the same time, there is uncertainty about μ with heterogeneous beliefs. Individuals have some belief $\tilde{\mu}$ but they know that they don't know the true μ. We refer to incomplete knowledge about μ as uncertainty and model it by a prior distribution of μ, which is normal, centered around $\tilde{\mu}_{i1}$, and its variance $\tilde{\sigma}^2_{\mu i1}$ is potentially heterogenous, too.

In this setup, the distribution of log gross potential returns is perceived as normal with mean $\tilde{\mu}_{i1} = \tilde{E}_{it}[\mu]$ (the individual-specific mean of the random variable μ) and variance $\tilde{\sigma}^2_{i1} = \tilde{\sigma}^2_{\mu i1} + \sigma^2$ (the reduced-form variance is the sum of variance due to individual-specific parameter uncertainty and fixed variance due to risk). When individual i makes the portfolio choice decision in period 1, she thinks that risky returns follow a lognormal distribution with parameters $(\tilde{\mu}_{i1}, \tilde{\sigma}^2_{i1})$. Heterogeneity in period 1 beliefs is predetermined by differences in what people may learn at home or in school, or differences in personality (degree of general optimism and general uncertainty). If they do not learn more about the returns, individuals enter period 2 with the same beliefs: $\tilde{\mu}_{i2} = \tilde{\mu}_{i1}$ and $\tilde{\sigma}^2_{i2} = \tilde{\sigma}^2_{i1}$. However, their beliefs can change as results of two kinds of learning.

The first kind is mechanical learning, or passive learning following the terminology of Veldkamp (2011). If an individual invests in S_{i1}, the realized returns will make her change her beliefs by Bayesian updating. Since the length of period 1 is unity, and the realized returns are R_1 for everyone, the results of passive learning are the Bayesian posteriors

$$\tilde{\mu}_{i2} = \frac{\sigma^2 \tilde{\mu}_{1i} + \tilde{\sigma}^2_{\mu i1} R_1}{\sigma^2 + \tilde{\sigma}^2_{\mu i1}} \tag{39}$$

$$\tilde{\sigma}^2_{\mu i2} = \left(\frac{1}{\tilde{\sigma}^2_{\mu i1}} + \frac{1}{\sigma^2} \right)^{-1} = \frac{\sigma^2 \tilde{\sigma}^2_{\mu i1}}{\sigma^2 + \tilde{\sigma}^2_{\mu i1}} \tag{40}$$

$$\tilde{\sigma}^2_{i2} = \tilde{\sigma}^2_{\mu i2} + \sigma^2 \tag{41}$$

As a result of passive learning, individuals update their $\tilde{\mu}$ in the direction of the realized stock market returns in period 1, and their uncertainty decreases.

The second kind of learning is active learning (again, following the terminology of Veldkamp, 2011). Individuals can invest in learning even if they do not invest in period 1. Also, those who are investors in period 1 may learn more than simply observing the returns they realize. Active learning is an investment in one's financial knowledge, which is a form of human capital. Many insights of the large literature on investment into human capital may apply to active learning (see, e.g., Becker, 1964).

Active learning affects attainable returns in two ways. The first is Bayesian updating of one's beliefs about μ and σ. The investor can update her beliefs by observing a history of past returns, where the length of the history is h_i. While h_i should be a decision variable in general, we abstract away from that in this simple model and fix it to h. We set $h > 1$ in order to reflect a longer horizon than available in mechanical learning. With history length h and observed average stock market returns $\bar{R}_h$ the result of active learning is the Bayesian posterior distribution

$$\tilde{\mu}_{i2} = \frac{\sigma^2 \tilde{\mu}_{1i} + h\tilde{\sigma}^2_{\mu i1}\bar{R}_h}{\sigma^2 + h\tilde{\sigma}^2_{\mu i1}} \tag{42}$$

$$\tilde{\sigma}^2_{\mu i2} = \left(\frac{1}{\tilde{\sigma}^2_{\mu i1}} + \frac{h}{\sigma^2}\right)^{-1} = \frac{\sigma^2\tilde{\sigma}^2_{\mu i1}}{\sigma^2 + h\tilde{\sigma}^2_{\mu i1}} \tag{43}$$

$$\tilde{\sigma}^2_{i2} = \tilde{\sigma}^2_{\mu i2} + \sigma^2 \tag{44}$$

Similarly to passive learning, individuals update their $\tilde{\mu}$ in the direction of the realized returns in the observed time horizon, and their uncertainty decreases. Those are ex post results of learning. Active learning is a choice based on results that are expected ex ante. Ex ante, individuals do not know in which direction their $\tilde{\mu}$ will be updated. In particular, they do not expect their mean to change after learning. But they know that learning will decrease their uncertainty.[39]

The second aspect of learning affects individual transaction costs τ that discount potential returns. Recall that although potential returns are R_t, individuals can attain returns of $R_t e^{-\tau}$ on their investment S_{it}. By active learning, we assume that individuals can decrease their transaction cost τ. For simplicity, we assume that active learning leads to $\tau = 0$ so that active learners can expect to realize (and do realize) R_t on their investment S_{it}.

Active learning is an investment. We assume that its two aspects are bundled so that those who choose to learn will see their beliefs updated as in (42) through (44) and their transaction costs τ reduced (to zero in this simple setup). Active learning entails individual-specific costs of D_i that are to be paid in period 1. Note a key aspect of this investment setup: while the benefits to active learning are related to the amount to invest into the risky assets, the costs are not. This aspect will drive many of our most important implications.

[39] In this setup, the decrease in uncertainty is a deterministic function of h because of the simplistic assumption of known σ^2. But uncertainty decreases in h in richer setups as well as long as the observed returns are from a stationary distribution.

Combining all the ingredients outlined above, the decision problem of the individual can be formulated the following way.

$$
\begin{aligned}
EU_i &= \sum_{t=1}^{3} \beta^{t-1} Eu\left(C_{it}\right) && (45)\\
u\left(C_{it}\right) &= \frac{1}{1-\gamma} C_{it}^{1-\gamma} && (46)\\
X_{it} &\geq C_{it} + B_{it} + S_{it} + f_{it} + D_{it} && (47)\\
f_{it} &= f \times 1\left(S_{it} > 0\right) && (48)\\
D_{i1} &= D \text{ if active learning in period 1} && (49)\\
D_{it} &= 0 \text{ otherwise} && (50)\\
X_{it} &= S_{i(t-1)}\left(R_t e^{-\tau_{it}} - R_f\right) + \left(B_{i(t-1)} + S_{i(t-1)}\right) R_f + Y_{it} && (51)\\
B_{it} &\geq 0, \quad S_{it} \geq 0 && (52)
\end{aligned}
$$

The utility function in (45) is standard expected utility, C_{it} is consumption, β is the discount factor. The instantaneous utility function is CRRA, and γ is the parameter for risk aversion and the inverse of the intertemporal elasticity of substitution at the same time. The budget line in (47) states that the sum of investments B_{it} and S_{it} (bonds and stocks, respectively), the fixed costs of investment (f_{it}), and the cost of active learning (D_{it}) cannot exceed cash on hand (X_{it}). Fixed costs need to be paid if one invests in the risky assets, and their role is to prevent very small investments. D needs to be paid if one invests in active learning. In our setup the only time people may invest into active learning is period 1 (no one wants to save in period 3, and thus it is never optimal to learn later than period 1). Equation (51) describes the equation of motion for cash on hand. In the beginning of every period t earnings (Y_{it}) are received, and the returns on previous period $(t-1)$ investments are collected. In case of stocks, these are net returns that include proportional transaction costs τ. Equation (52) states the nonnegativity constraints that make borrowing and short selling impossible.

This model is relatively simple, but it does not yield to analytical solutions. In order to get the implications for our empirical investigation, we simulated out the policy function. The model can be solved with backward induction. In the third period the optimal behavior is trivial. There is only one state variable, X_{i3} (cash-on-hand in period 3) and one control variable C_{i3} (consumption). The optimal policy is to consume everything, $C_{i3} = X_{i3}$. The second period is more complex. There are four state variables: X_{i2}, D_{i1} (whether the individual had active learning in period 1) and the belief parameters $\tilde{\mu}_{i2}$ and $\tilde{\sigma}_{i2}^2$. The two control variables are B_{i2} and S_{i2} which then imply C_{i2}. The optimal second period policy function and the implied value function can be computed by simulation. We computed the optimal decision for a large number of grid points on the state variables and then we used cubic splines to approximate the functions for their entire domains. In the first period there is no state variable, but there are three control variables: B_{i1}, S_{i2} and D_{i1}.

We solved the model for a large number of different parameter values. Some parameters values were borrowed from the literature such as:

$$\gamma = 3 \tag{53}$$

$$\beta = 0.97 \tag{54}$$

$$R_f = e^{0.02} \tag{55}$$

$$\mu = 0.07 \tag{56}$$

$$\sigma = 0.15 \tag{57}$$

The second set of variables are the wage variables. The heterogeneity of lifetime income and its link to learning and investment is our primary focus so we computed the optimal policy function for a large set of wage values. We generated a distribution of earnings that resembles the observed distribution. As a benchmark, we set the ratio of Y_{i2} to Y_{i1} to 2 (so that $Y_{i2} = 2Y_{i1}$). The distribution of earnings is set to lognormal in both the first and the second period (or generation). We have set the 5^{th} percentile of the Y_{i1} to be 0.4 and the 95^{th} percentile of the Y_{i1} to be 2.2. This way the population average of Y_{i1} is normalized to roughly 1.

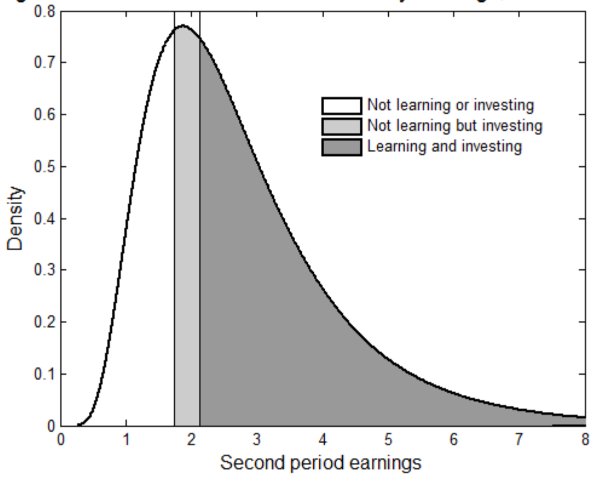
For the third period income we used a simplified social security formula. First we computed the average indexed monthly earnings (AIME) as the average of the prior wages, $PIA_i = 0.5\,(Y_{i2} + Y_{i1})$. Then we have chosen two bendpoints, Q_1 and Q_2. The third period social security income was defined as

$$Y_{i3} = \begin{cases} 0.9PIA_i & \text{if } \; PIA_i \leq Q_1 \\ 0.9Q_1 + 0.32\,(PIA_i - Q_1) & \text{if } \; Q_1 < PIA_i \leq Q_2 \\ 0.9Q_1 + 0.32\,(Q_2 - Q_1) + 0.15\,(PIA_i - Q_3) & \text{if } \; Q_2 < PIA_i \end{cases} \tag{58}$$

The bendpoints were chosen to be $Q_1 = 1.25$ (approximately the 40th percentile) and $Q_2 = 2.65$ (approximately the 90th percentile).

The rest of the parameters, due to lack of consensus about their values, had to be calibrated differently. We have chosen basic values that made the results interesting, and we have run sensitivity analyses to see how the results change as we move away from these

Figure A1: Active learners and investors by earnings, default values



values. The default values of these parameters were:

$$\tau = 0.025 \tag{59}$$

$$f = 0.06 \tag{60}$$

$$D = 0.02 \tag{61}$$

$$\tilde{\sigma}^2_{\mu 1} = 0.15 \tag{62}$$

$$h = 3 \tag{63}$$

Perhaps the most important but also a rather straightforward result of the model is that an increase in lifetime earnings leads to an increased propensity to learn and to invest. In the setup here, the only source of heterogeneity is in earnings. Figure A1 illustrates the results using our default parameter values. There is a first threshold value of second period earnings (~ 1.75) below which nobody learns and nobody participates on the stock market. Between this and a second threshold value (~ 2.13) people participate on the stock market but they do not acquire financial knowledge. People whose second period earnings are above the second threshold, and consequently who had the most incentive to save, both learn actively and participate on the stock market. The pattern that, other things equal, the lowest earners do not learn and do not invest, the middle income people do not learn but invest and the rich both invest and learn is universal in this model, but the two threshold values can coincide in which case all investors are knowledgeable.

This relationship between lifetime earnings and learning is due to saving motives in this model. Expected benefits of learning are increasing in the level of period-2 savings. Ceteris paribus, those who intend to save less will see lower benefits to learning than those who intend to save more. Since intended period-2 savings are increasing in lifetime earnings, expected benefits to learning are increasing in lifetime earnings as well. At the same time, the costs of investment, D, aren't directly related to the amount to invest. As a result, the likelihood of learning and investing is increasing in lifetime earnings.

In a richer and more realistic setting learning costs would also be heterogeneous. In reality, learning costs are likely to be negatively correlated with earnings. Heterogeneity in lifetime earnings reflects heterogeneity in general human capital (Becker, 1964). Heterogeneity in human capital is the result of differences in the costs as well as the benefits to investment into human capital (Willis, 1986, and Card, 1998). Those costs include general skills and family background, which likely play important roles in determining costs of learning about stock returns, too. Therefore, those who have higher lifetime earnings because of higher levels of human capital are also likely to face lower learning costs of stock returns. This amplifies the positive relationship between learning and earnings.

The 8 panels in Figures A2 show additional comparative static results. Each figure shows the fraction of individuals who choose to learn in period 1 and the fraction of individuals holding stocks in period 2**.** These fractions are calculated using the simulated distribution of earnings as described above.

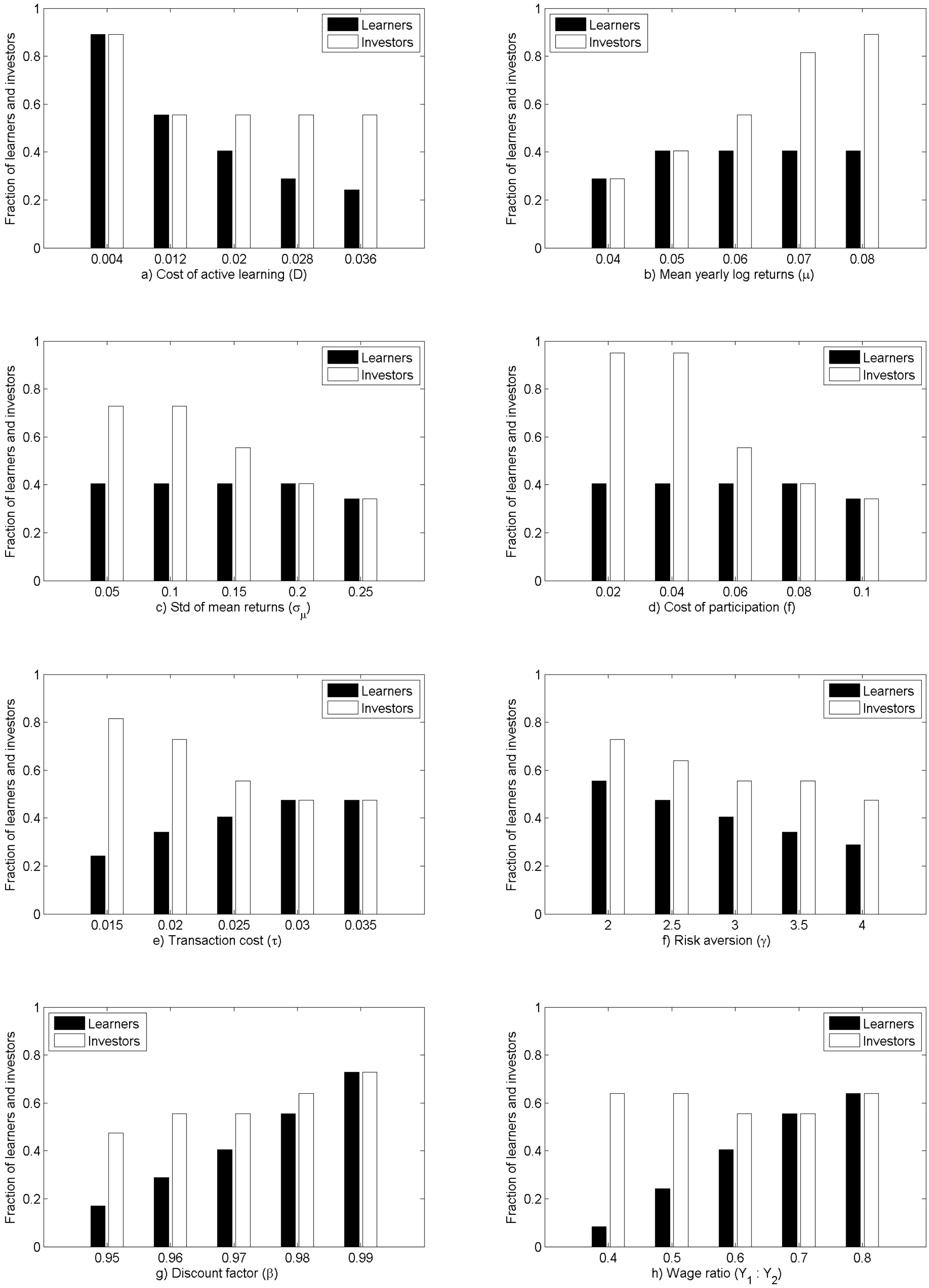
The results are very intuitive. Panel a) of Figure A2 shows that as the cost of learning increases, the fraction of people who choose to learn decreases. The increasing learnings costs make active learning less beneficial but that does not necessarily discourage stock market participation. As long as learning costs are sufficiently high to begin with, a further increase in it would only make people participate on the stock market without learning. This is the case on Figure A1, where an increase in learning cost leads to a monotonic decrease in active learning, but that does not fully translate into lower stockholding above some level of learning costs.

Panel b) shows a reverse picture. As the period-1 expectation of the mean of log potential returns increases, stockholding in period 2 increases dramatically, and the probability of learning increases as well up to a point. Above some expected potential returns, some people can acquire sufficiently high effective returns in the second period even without financial knowledge, but they choose not to invest in knowledge in the first period when they are relatively poor.

Panel c) shows that parameter uncertainty (i.e., uncertainty about μ) is negatively related to stock market participation and weakly negatively related to learning. The expected value of a lognormally distributed variable positively depends on the variance of its logarithm[40]. This panel nets out this effect. In order to show the pure effect of increasing uncertainty, we have imposed a mean-preserving spread such that expected returns are the same in all five cases. This result shows that, in this setup, the prospect of decreased uncertainty is not an

[40] $E\left(R_{it}\right) = \exp\left(\tilde{\mu}_{it} + 0.5\tilde{\sigma}_{it}^2\right)$

Figure A2: Fraction of active learners and second period stock market participants by different parameter values



important motive for learning. The expectation of gains is the important motive.[41] Panel d) shows that increasing the fixed costs of participation leads to decreasing stockholding, and albeit in a much less pronounced way, it also leads to less learning.

Panel e) shows the effect of increasing transaction costs τ, the discount from potential returns. Increasing this discount decreases effective attainable returns conditional on R_t, but it increases the expected gains from active learning. The results imply that the effects on both learning and on stockholding in period 2 are substantial, but the two effects go in the opposite direction. Higher discount makes participation without financial knowledge less beneficial. Some of these people would leave the market, but some would decide to learn and stay on the market.

Panel f) shows a strong and monotonic negative relationship between risk aversion on the one hand, and learning and subsequent stockholding on the other hand. Higher risk aversion leads to a smaller fraction of savings put into stocks, ceteris paribus, which decreases the value of learning about stock returns (especially since the primary effect of such learning is increased expected returns and not decreased risk). Panel g) shows that increased patience increases stockholding and learning as well. It is partly because more patient individuals plan to save more, and partly because they are more willing to pay the costs of learning in period 1 for its expected benefits in period 2.

Finally, panel h) shows that as the age-earnings profile gets flatter (period 1 earnings increase at the cost of period 2 earnings), the probability of learning increases and the effect on stock market participation is rather ambiguous. In order to net out wealth effects lifetime earnings are kept constant in all five specifications and only the ratio of first and second period wage is changing. A flatter wage profile makes any investment in the first period more likely as the marginal value of consumption loss in period 1 decreases. There are two opposing effects on stock market participation. First, a flatter wage profile decreases second period earnings, which makes people less likely to participate on the stock market. Second, if the earnings profile is sufficiently flat, the increasing number of financially knowledgeable people would push stock market participation up. In this particular setup, the first effect dominates at very steep profiles, and the second effect dominates at very flat profiles. In general, it is not evident which of the two effects is stronger.

[41] In case we do not make the adjustment of the mean log return the dependence between uncertainty and learning vanishes completely, and the dependence between uncertainty and participation becomes very weakly positive.

A.2 Data, descriptive statistics and detailed evidence on noise and information in the probability answers

A.2.1 Sample and stockholding

Table A2.1. Sample size

	Age 55-64[a]	Other respondents[b]
HRS 2002	4,056	12,074
HRS 2004	3,676	14,651
HRS 2006	3,182	14,027
HRS 2008 (before Sep)	2,512	11,161

[a]Individuals of age 55 to 64 and whose spouse is of age 55 to 64 as well (or have no spouse)

Table A2.2. Fraction of stockholders in the sample

	Stockholders		
	outside retirement acc.[a]	in retirement acc.[b]	All
HRS 2002	0.37	0.33	0.51
HRS 2004	0.34	0.32	0.49
HRS 2006	0.29	0.29	0.45
HRS 2008 (before Sep)	0.26	0.29	0.42

[a] Have investments in stocks or mutual funds outside retirement accounts

[b] Have stocks or mutual funds within retirement accounts

Table A2.3. Share of stocks in the portfolio among stockholders in the sample

	Stockholders		
	outside retirement acc.[a]	in retirement acc.[b]	All
HRS 2002	0.59	0.79	0.56
HRS 2004	0.58	0.75	0.56
HRS 2006	0.53	0.81	0.56
HRS 2008 (before Sep)	0.51	0.78	0.53

[a] Have investments in stocks or mutual funds outside retirement accounts

[b] Have stocks or mutual funds within retirement accounts

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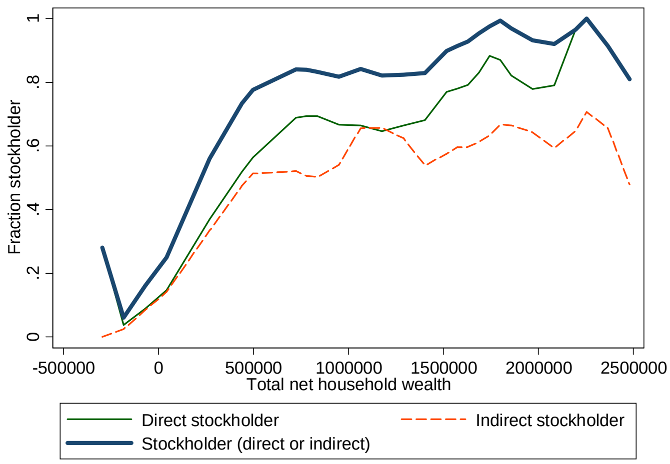

Figure A2.1. Fraction of stockholders and total net wealth. HRS 2002.

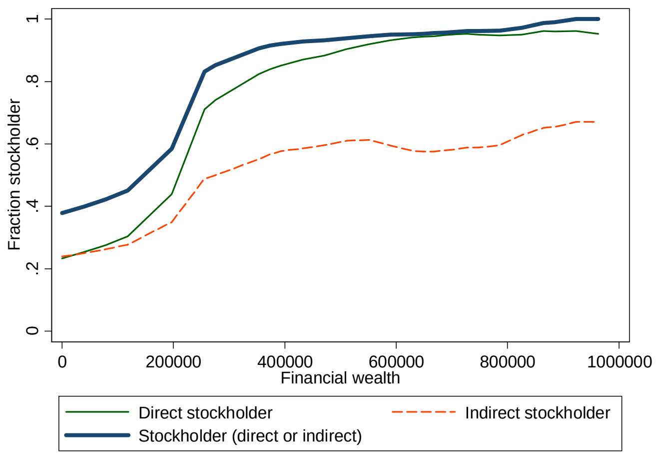

Figure A2.2. Fraction of stockholders and financial wealth. HRS 2002.

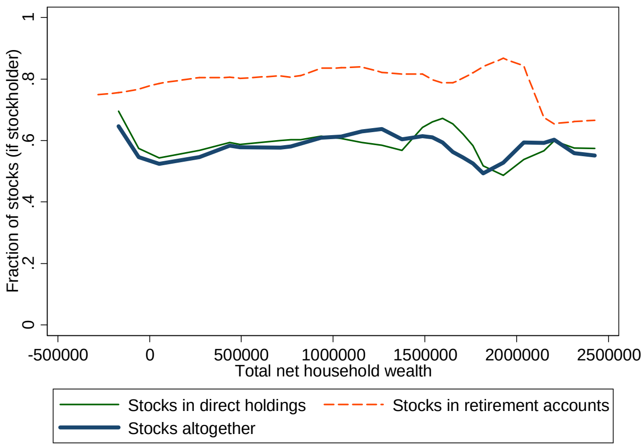

Figure A2.3. Share of stocks in the portfolio of stockholders, and total net wealth. HRS 2002.

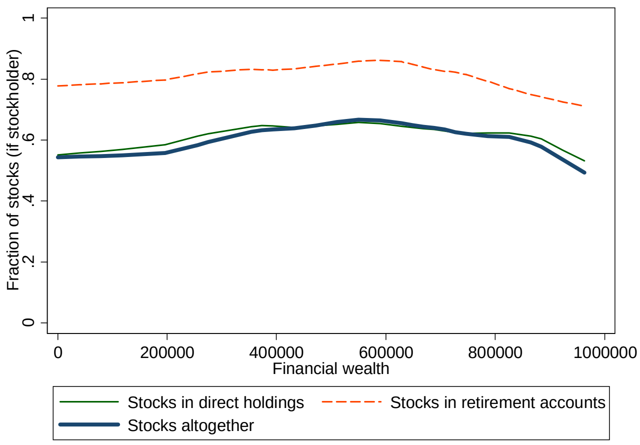

Figure A2.4. Share of stocks in the portfolio of stockholders, and financial wealth. HRS 2002.

A.2.2 The proxy variable for lifetime earnings

The source of the lifetime earnings data is the Detailed and the Summary Earnings Records (DER and SER) derived from the Master Earnings File (MEF) of the Social Security Administration that is linked to HRS. For details about the MEF and the linking procedure see Olsen and Hudson (2009) and the documents on the HRS website.[42]

The DER data is derived from the W-2 forms filed by employers to the Internal Revenue Service each year, and it is available from 1978 onward. The SER data is available since 1951, but it contains information only on jobs covered by social security and income up to the taxable maximum. In principle the DER data is superior to the SER as it covers more jobs and it provides more precise information on high income people whose earnings records are capped in SER but uncapped in DER. Therefore we gave priority to the DER data and we only used the SER in exceptional cases described below.

The main issue of the linking procedure is that HRS needed to acquire written consent from sample members in order to get the administrative information on them. HRS made a lot of effort to increase the participation rate, but it remained below 100 percent. Generally HRS has a relatively good coverage rate for earnings before 1992 (slightly above 80 percent) and moderately good coverage rate for earnings afterwards (around 60 percent). Below we provide precise numbers about the attrition rate for our target sample, which will be higher than these numbers. HRS asked for consent in each wave, but in some waves only people with prior consent were asked. Before 2006 the consent covered years up to the interview year, but since 2006 the consent covers future years as well. The consequence is that, as of now, the coverage rates are typically higher for earlier waves (people had more chance to provide consent), but in the future this difference will diminish. Another problem beyond coverage rate is selection. There is evidence that giving consent is not random. Men, the educated, the rich and minorities are underrepresented in the merged sample. See the text for details about how we handle this problem.

Our primary sample is a ten year cohort, people 55-64 years old in 2002 and whose spouse is in the same age range, too.[43] In some specifications we look at people in the same age range in 2004, 2006 and 2008 as well. As Table A24 shows our target sample size is 4056 in 2002, 3672 in 2004, 3174 in 2006 and 2506 in 2008. Hudomiet, Kézdi and Willis (2011) show that shortly after the fall of Lehman Brothers in September 2008 stock market beliefs

[42]There are two relatively detailed documents under the data section at hrsonline.isr.umich.edu that can be accessed after free registration. Note that the social security data is not public, and thus only these documents are available but not the data. The website also provides detailed information about how to get permission to use the restricted data.

[43]People who are at least 55 years old, but they haven't turned 65 yet.

of households changed substantially and in an unusual way. For this reason we decided to drop interviews that were made after September 2008 in this paper.

The earnings data we created is the average CPI-adjusted earnings in a 15 year period, between age 40 and 55.[44] The earliest year we used is 1978, which is for earnings at age 40 for people who were 64 in 2002 ($2002 - 64 + 40 = 1978$). The DER data in principle is available from 1978, but the version of the data stored at HRS only covers years 1980 onward. The HRS staff claims that there were some technical problems with the 1978 and the 1979 DER data, and therefore they decided against merging it to HRS. Therefore all the 1978 and 1979 earnings information is coming from the SER. Another issue happened in 1998-2000. HRS first acquired only the DER data until 1997, and then it acquired the SER data until 1999. Therefore for people who stopped giving consent after 2000, we only have SER information for their earnings from 1998 and 1999.

Table A2.4 shows the quality of the social security earnings information in HRS. As we can see we had no information about the earnings of 612 people in 2002 (15 percent). This number is similar in later waves as well, but due to the falling sample size the ratio of missing values is increasing. Among those who provided some information the majority did so for all the 15 years we needed for our lifetime earnings proxy. The nature of the data is such that missing years can only happen at the end of the period and only for those who stopped giving consent to HRS to collect the earnings data on them. In 2002 we have all the necessary years for 2733 people, we have 10-14 years of information for 590 people and less than 10 years for 121 people. The corresponding numbers for later waves are smaller in level but very similar as a percentage. Here the decision we made was to disregard the earnings data for everyone for whom we have less than 10 years of information, and use the available years for imputation for those who only have 5 or less missing years.

Table A2.4. Social Security earnings availability in our target samples

	2002	2004	2006	2008*
Target sample size	4056	3672	3174	2506
All 15 years available	2733	2521	1974	1337
10-14 years available	590	376	405	469
1-9 years available	121	91	64	44
no SSA information	612	684	731	656

* Interviews made prior to October 2008

For confidentiality reasons HRS top-coded all the earnings variables. For people whose earnings were above $250,000 in a given year, we only have interval information, where

[44] People who are at least 40 years old but haven't turned 55 yet.

the intervals are \$250,000-\$299,999; \$300,000-\$499,999 and \$500,000 and above. Topcoded responses were imputed with a procedure described below. HRS also rounded earnings below \$250,000 to the closest multiple of \$100, with the exception of \$0-\$49, where we can differentiate between a true \$0 and a \$1-\$49 value.

The DER data contains five earnings variables:

1. Total compensation: This variable amounts to the sum of the Box 1 values on each W-2 forms submitted on behalf of a person by all his employers. Total compensation includes wages, bonuses, non-cash payments and tips[45]. Total compensation typically does not include deferred payments such as contributions to a 401k plans, but certain plans are included. This variables is uncapped, meaning that high income vales are not censored, only topcoded.

2. Social security earnings: This variables is derived from the Box 3 values of the corresponding W-2 forms. There are two major differences between this variable and total compensation. The first difference is that social security earnings contain information on deferred compensation as well. The second difference is that this variable is capped at the taxable maximum. The taxable maximum was changing year by year. In 2002 it was \$80,400, for example, meaning that any earnings beyond this amount are missing.

3. Medicare earnings: This variable is based on the Box 5 values of the W-2 forms. Medicare earnings are almost identical to social security earnings. The main difference is between the taxable maximums used for the two measures. Before 1991 the medicare and the social security caps were identical. Since 1994 there is no limit on the taxable earnings for medicare, and between 1991 and 1993 the difference between the medicare and the social security taxable maximums were diverging.

4. FICA taxable self employment earnings: This variable is based on Form 1040 Schedule SE reported by the self employed to IRS. The variable is capped at the same amounts as the social security earnings.

5. Medicare taxable self employment earnings: This variable is almost identical to the previous, but here the less restrictive medicare caps are used.

The SER data contains only one variable which is the sum of all his wage, salary and self employment income. Similarly to social security DER earnings the variable has information only on jobs covered by social security and contains capped values at the social security taxable maximum.

[45] Only tips that the employee reported to the employer. Allocated tips are not part of Box 1.

The correlation between these variables are generally very high, but they are not identical. In principle the best quality data is the post 1994 values of the medicare earnings which is uncapped and it also contains information on deferred compensation. The decision we made was the following. First we took the maximum of the total compensation, the social security, and the medicare earnings. In case the maximum was capped or topcoded, we imputed a value with a procedure described below. Second, we took the maximum of the FICA and the medicare taxable self employment earnings. Again, if the maximum was capped or topcoded, we used imputation. Third, we added the employment and the self-employment values. Fourth, we compared this sum to the SER data and took the maximum. After this procedure we had an almost complete person-year-earnings dataset.[46] The final step was the imputation of the remaining missing values.

We needed to impute earnings in three cases. The first is topcoded and rounded responses; the second is for people who stopped giving consent to HRS and therefore their earnings are missing for their last years; and the third is for capped earnings values. Out of these three only the second one affected many respondents (590 in 2002), topcoding and capping were less severe issues.

Topcoded and rounded responses were imputed in a very simple way. For the \$250,000-\$299,999 interval we imputed \$270,000; for the \$300,000-\$499,999 interval we imputed \$370,000; for the \$500,000 and above interval we imputed \$710,000, and for the \$1-\$49 interval we imputed \$40. Other rounded responses were not imputed, we used the rounded values. The values we used were motivated by interval regressions for the logarithm of earnings. If one assumes log-normally distributed earnings, estimates an interval regression, and computes the conditional expected value of a given interval, then he gets numbers that are very close to the values we used. We estimated models with and without flexible time trends in earnings and with and without basic demographic variables such as gender, age and education, and the resulting conditional expectations were always very close to these numbers.

For people whose last earnings values were missing we used their earlier earnings for imputation. As described above, we only have people in our sample with at least ten years of information and thus maximum 5 years of missing earnings. We saw two possibilities for imputation. We could either impute the mean of earlier wages, or we could put more weight on recent years. We have found that many people in our sample had notable fluctuations in their earnings so we decided to use the second approach. First we identified the last four valid earning values for each missing value. Second, we adjusted all the four values with the cpi to get an initial guess for the missing earnings. Then we averaged these values with relative

[46] One technical issue was that missing values and zero earnings were hard to distinguish in the DER data, but it was precisely stored in the version of the SER data HRS provided.

weights $1/t$ if the initial guess was based on earnings t years before the missing response. For example let us say that for a given person we only had earnings up to 1991, and we wanted to impute values for 1992-1994. Let us take the 1994 value. We took the person's earnings from 1988,1989,1990 and 1991, adjusted these values with the cpi, and averaged them with relative weights $1/6, 1/5, 1/4$ and $1/3$.

Capped earning values were imputed in a very similar fashion to the previous. Recall that capping applied to people whose earnings were higher than the taxable maximum. As the taxable maximum increased over time we decided to use the next four earnings values instead of the last four. Moreover, when an initial guess turned out to be lower than the taxable maximum, we replaced the guess with the taxable maximum. When the final guess was equal to the taxable maximum (when all the four initial guesses were lower than that) we imputed 110 percent of the taxable maximum. Another problem was that sometime we had less than four initial guesses, in which case we used as many as we had. When there was no initial guess at all, we again imputed 110 percent of the taxable maximum.

Finally, we simply imputed the sample mean for all the missing observations. The last two rows of Table A2.4 shows that the number of imputed observations were 733 in 2002 and similar in magnitude in the later waves as well. In the regression analyses, we entered a dummy variable for missing (and therefore imputed) earnings data.

A.2.3 Noise in the probability answers

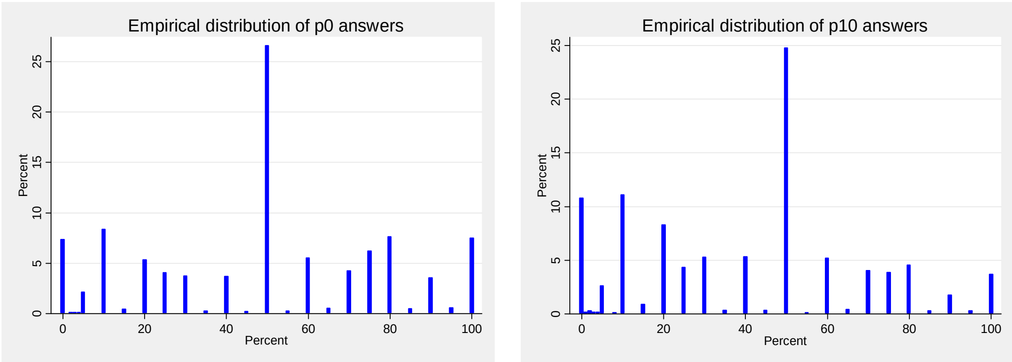

Figure A2.5. The distribution of reported subjective probabilities of a gain of the stock market (p_0) and the 10 per cent or larger gain (p_{10}). HRS 2002, estimation sample ($n = 2969$)

Table A2.5. Patterns of survey noise in the core questionnaire

Fraction of responses where	HRS 2002	HRS 2004	HRS 2006	HRS 2008
$p_0 = 0.5$	0.238	0.262	0.239	0.254
$p_0 = 0.0$ or $p\,0 = 1.0$	0.119	0.077	0.073	0.062
p_0 rounded other ten per cent	0.509	0.512	0.559	0.539
p_0 rounded 25% or 75%	0.088	0.096	0.082	0.096
p_0 not round number	0.047	0.054	0.047	0.048
Total	1.000	1.000	1.000	1.000
$p_0 > p_{10}$	0.425			
$p_0 = p_{10}$	0.439			
$p_0 < p_{10}$	0.136			
Total	1.000			

Table A2.6. Direct evidence on survey noise: Test-retest comparisons using core questionnaire and experimental module answers to the same probability questions from HRS 2002

	p_0	p_{10}
Mean answer in core questionnaire	0.486	0.396
Mean answer in module	0.479	0.334
Difference (core minus module)	0.007	0.063
Standard dev. in core	0.290	0.272
Standard dev. in module	0.272	0.303
Difference (core minus module)	0.018	−0.031
Fraction who gave the same answer in core and module	0.273	0.179
Absolute value of difference between core and module	0.231	0.240
Correlation core and module answers	0.467	0.356

Table A2.7. The propensity to give round answer to the to p_0 question (multiple of 10% or 25% or 75%)

OLS regression results for the noise patterns in HRS 2002-2008

LHS variable: p0 answer rounded (dummy). HRS 2002-2008						
Stockholder dummy	-0.004 [0.005]					
Log lifetime earnings		-0.001 [0.003]				
Education			-0.002 [0.001]**			
Cognition				0.002 [0.004]		
Single female					0.013 [0.007]	
Single male					0.011 [0.009]	
Female in couple					0.008 [0.007]	
Age					-0.001 [0.001]	
Black					-0.020 [0.009]*	
Hispanic					0.014 [0.009]	
Father manager/professional					-0.013 [0.008]	
Log risk tolerance					0.001 [0.011]	
Wealth non-positive						-0.004 [0.011]
Wealth in middle						-0.007 [0.007]
Wealth high						-0.011 [0.009]
Fin. wealth zero						0.000 [0.010]
Fin. wealth in middle						-0.004 [0.008]
Fin. wealth high						-0.005 [0.009]
Dummies for p0 categories	YES	YES	YES	YES	YES	YES
Observations	11,113	11,113	11,113	11,113	11,112	11,113
R-squared	0.09	0.09	0.09	0.10	0.10	0.06
F-test statistic for shown coeffs					1.27	1.01
p-value					0.257	0.418

Robust standard errors in brackets. * significant at 5%; ** significant at 1%

Mean fill for missing cognition, father's occ, risk tolearnce variables; dummies for missing values included

Table A2.8. The propensity to give the same answer to p_0 and p_{10}

OLS regression results for the noise patterns in HRS 2002

LHS variable: $p_0=p_{10}$ (dummy). HRS 2002						
Stockholder dummy	-0.016 [0.020]					
Log lifetime earnings		-0.020 [0.009]*				
Education			-0.008 [0.004]*			
Cognition				-0.007 [0.011]		
Single female					0.015 [0.026]	
Single male					0.047 [0.034]	
Female in couple					0.046 [0.023]*	
Age					0.011 [0.004]*	
Black					0.018 [0.028]	
Hispanic					0.007 [0.043]	
Father manager/professional					-0.032 [0.027]	
Log risk tolerance					0.001 [0.029]	
Wealth non-positive						0.088 [0.044]*
Wealth in middle						0.003 [0.029]
Wealth high						-0.034 [0.034]
Fin. wealth zero						-0.045 [0.037]
Fin. wealth in middle						0.021 [0.030]
Fin. wealth high						-0.017 [0.035]
Dummies for p0 categories	YES	YES	YES	YES	YES	YES
Observations	3,520	3,520	3,520	3,520	3,519	3,520
R-squared	0.06	0.06	0.06	0.07	0.06	0.00
F-test statistic for shown coeffs					2.31	0.73
p-value					0.019	0.623

Robust standard errors in brackets. * significant at 5%; ** significant at 1%

Mean fill for missing cognition, father'socc, risk tolearnce variables; dummies for missing values included t

Table A2.9. The propensity to give smaller answer to p_0 than p_{10}

OLS regression results for the noise patterns in HRS 2002

	μ		log(s)		log[Std(u)]	
	coef	interaction with stockholding	coef	interaction with stockholding	coef	interaction with stockholding
Constant	-0.091	0.087	-0.597	-0.134	-1.176	-0.229
	[0.007]**	[0.013]**	[0.026]**	[0.052]**	[0.082]**	[0.063]**
July 08 to Sep08	0.008	-0.013	0.135	-0.052	0.158	-0.066
	[0.018]	[0.022]	[0.067]*	[0.082]	[0.076]*	[0.098]
Oct 08 to Nov 08	0.091	-0.031	0.29	-0.066	0.465	0.135
	[0.041]*	[0.052]	[0.155]	[0.188]	[0.161]**	[0.203]
Dec 08 to Feb 09	-0.012	-0.006	-0.113	0.194	0.231	0.251
	[0.052]	[0.068]	[0.173]	[0.246]	[0.181]	[0.268]
Female	-0.103	0.05	0.234	0.076	0.157	0.024
	[0.016]**	[0.019]**	[0.059]**	[0.073]	[0.066]*	[0.087]
Single	0.012	-0.017	0.135	-0.169	0.244	-0.232
	[0.018]	[0.022]	[0.062]*	[0.081]*	[0.070]**	[0.096]*
Black	0.015	-0.126	0.539	0.101	0.483	0.276
	[0.028]	[0.062]*	[0.104]**	[0.228]	[0.107]**	[0.241]
Hispanic	0.013	-0.04	0.408	-0.107	0.357	-0.004
	[0.035]	[0.055]	[0.134]**	[0.223]	[0.139]*	[0.242]
Age	-0.002	-0.001	-0.011	0.011	-0.012	0.018
	[0.001]**	[0.001]	[0.003]**	[0.004]**	[0.003]**	[0.004]**
Years of education	0.004	-0.001	-0.04	-0.006	-0.033	-0.028
	[0.003]	[0.004]	[0.011]**	[0.015]	[0.012]**	[0.017]
P(economic recession)	-0.004	0.001				
2004-2006 average	[0.000]**	[0.000]				
Depressive symptoms	-0.025	0.007				
2004-2006 average	[0.008]**	[0.010]				
Ratio of fifty answers			1.673	-0.42		
2004-2006 average			[0.249]**	[0.261]		
Ll	-42278.6					
N	9348					
Mean(μ)	-0.083					
Mean(s)	0.622					
Mean(Std(u))	0.355					
Var[R*]/Var[R]	0.652					
Rho(v0,vx-)	-0.535					
Rho(v0,vx+)	0.225					

Standard errors in brackets. * Significant at 5%; ** significant at 1%.
Stockholders are those who own any stock-market based assets (stocks, mutual funds etc.) either directly or through retirement accounts. Members of the same households are assigned the same stockholding status.
For more details see main text and footnotes to Table 4.4

Table A2.10. Absolute value of the difference between p_0 in the core and p_0 in the module OLS regression results for the noise patterns in HRS 2002

LHS variable: $\mid p_0 - p_{0\_module} \mid$. HRS 2002						
Stockholder dummy	0.044 [0.037]					
Log lifetime earnings		-0.006 [0.017]				
Education			0.003 [0.007]			
Cognition				0.034 [0.020]		
Single female					0.006 [0.061]	
Single male					-0.056 [0.073]	
Female in couple					-0.041 [0.052]	
Age					-0.006 [0.007]	
Black					-0.087 [0.046]	
Hispanic					-0.025 [0.127]	
Father manager/professional					0.003 [0.056]	
Log risk tolerance					-0.028 [0.069]	
Wealth non-positive						-0.009 [0.055]
Wealth in middle						0.047 [0.060]
Wealth high						-0.007 [0.065]
Fin. wealth zero						0.027 [0.081]
Fin. wealth in middle						0.044 [0.057]
Fin. wealth high						0.024 [0.063]
Dummies for p0 categories	NO	NO	NO	NO	NO	NO
Observations	205	205	205	205	205	205
R-squared	0.01	0.00	0.00	0.01	0.02	0.02
F-test statistic for shown coeffs					1.26	1.19
p-value					0.272	0.314

Robust standard errors in brackets. * significant at 5%; ** significant at 1%

Mean fill for missing cognition, father'socc, risk tolearnce variables; dummies for missing values included l

Table A2.11. Absolute value of the difference between p_{10} in the core and p_{10} in the module OLS regression results for the noise patterns in HRS 2002

LHS variable: $\mid p_{10} - p_{10\_module} \mid$. HRS 2002						
Stockholder dummy	0.025					
	[0.036]					
Log lifetime earnings		0.003				
		[0.016]				
Education			-0.005			
			[0.008]			
Cognition				0.027		
				[0.020]		
Single female					-0.022	
					[0.059]	
Single male					-0.026	
					[0.062]	
Female in couple					-0.043	
					[0.048]	
Age					-0.009	
					[0.008]	
Black					0.012	
					[0.048]	
Hispanic					0.200	
					[0.124]	
Father manager/professional					0.093	
					[0.060]	
Log risk tolerance					0.075	
					[0.061]	
Wealth non-positive						0.051
						[0.063]
Wealth in middle						0.052
						[0.056]
Wealth high						0.039
						[0.064]
Fin. wealth zero						0.146
						[0.094]
Fin. wealth in middle						0.031
						[0.051]
Fin. wealth high						0.017
						[0.059]
Dummies for p0 categories	NO	NO	NO	NO	NO	NO
Observations	196	196	196	196	196	196
R-squared	0.00	0.00	0.00	0.01	0.06	0.03
F-test statistic for shown coeffs					1.26	1.19
p-value					0.272	0.314

Robust standard errors in brackets. * significant at 5%; ** significant at 1%

Mean fill for missing cognition, father'socc, risk tolearnce variables; dummies for missing values included

A.2.4 Relevant heterogeneity in the probability answers

Table A2.12. Descriptive statistics of the subjective probability answers to the stock market returns questions by survey wave. HRS 2002 through 2008.

	$\bar{p}_0$	$V(p_{0i})$	$\bar{p}_0 - \bar{p}_{10}$	Fraction missing p_0
All respondents				
2002	0.48	0.081	0.088	0.18
2004	0.52	0.068		0.12
2006	0.51	0.068		0.19
2008 (before September)	0.50	0.067	0.104	0.15
Stockholders				
2002	0.56	0.081	0.113	0.06
2004	0.58	0.056		0.03
2006	0.57	0.056		0.05
2008 (before September)	0.56	0.055	0.129	0.04
Not stockholders				
2002	0.40	0.081	0.064	0.27
2004	0.46	0.072		0.20
2006	0.46	0.072		0.28
2008 (before September)	0.46	0.071	0.086	0.21

Sample: Health and Retirement Study, waves 2002, 4, 6 and 8 ($\bar{p}_0 - \bar{p}_{10}$ is from HRS 2002 only).

Respondents of age 55 through 64 with a spouse of the same age range (and singles)

p_0 is the answer to the probability of positive returns on stock markets by following year

Table A2.13. OLS regression results for the stock market probability answers.

Panel 1: Without wealth on the right-hand side

	p0	resid square	p0 - p10	missing p0
Log lifetime earnings	0.010	0.000	0.003	0.032
	[0.004]*	[0.001]	[0.004]	[0.003]**
DB pension	-0.009	-0.002	0.009	-0.012
	[0.008]	[0.002]	[0.011]	[0.008]
DC pension	0.016	-0.004	0.010	-0.023
	[0.007]*	[0.002]	[0.010]	[0.008]**
Education	0.008	-0.001	0.005	-0.015
	[0.002]**	[0.000]**	[0.002]*	[0.002]**
Cognition	0.024	-0.007	0.007	-0.037
	[0.004]**	[0.001]**	[0.006]	[0.005]**
Financial respondent	0.029	0.006	0.000	0.164
	[0.009]**	[0.003]*	0.000	[0.011]**
Log risk tolerance	0.044	0.002	0.022	-0.033
	[0.012]**	[0.004]	[0.014]	[0.012]**
Single female	-0.082	-0.004	-0.045	0.092
	[0.009]**	[0.003]	[0.012]**	[0.010]**
Single male	-0.032	0.000	-0.018	0.038
	[0.012]**	[0.004]	[0.015]	[0.012]**
Female in couple	-0.065	-0.007	-0.023	0.071
	[0.008]**	[0.002]**	[0.011]*	[0.009]**
Age	-0.001	0.001	0.002	0.013
	[0.001]	[0.000]	[0.002]	[0.002]**
Black	-0.048	0.006	-0.019	0.022
	[0.009]**	[0.003]	[0.011]	[0.011]*
Hispanic	0.001	0.005	-0.029	0.112
	[0.013]	[0.004]	[0.017]	[0.016]**
Father manager/professional	0.019	0.001	0.018	0.004
	[0.009]*	[0.003]	[0.012]	[0.009]
Sunny day optimism	0.016	0.000	-0.006	0.004
	[0.007]*	[0.002]	[0.008]	[0.007]
Economic pessimism	-0.092	-0.002	0.000	-0.003
	[0.014]**	[0.004]	[0.017]	[0.014]
Depressive symptomes	-0.010	0.003	0.004	0.014
	[0.004]**	[0.001]*	[0.004]	[0.004]**
Fraction fifty answers	-0.041	-0.124	-0.193	-0.069
	[0.036]	[0.011]**	[0.045]**	[0.040]
Dummies for missing varaibles	YES	YES	YES	YES
Dummies for years	YES	YES	YES	YES
Observations	9131	9131	3323	10887
R-squared	0.1	0.05	0.03	0.23

Robust standard errors in brackets. * significant at 5%; ** significant at 1%

Table A2.13. OLS regression results for the stock market probability answers.
Panel 2: Wealth included on the right-hand side

	p0	resid square	p0 - p10	missing p0
Log lifetime earnings	0.007	0.000	0.001	0.035
	[0.004]	[0.001]	[0.004]	[0.003]**
DB pension	-0.010	-0.001	0.010	-0.008
	[0.008]	[0.002]	[0.011]	[0.008]
DC pension	0.017	-0.003	0.011	-0.022
	[0.007]*	[0.002]	[0.010]	[0.008]**
Education	0.005	-0.001	0.003	-0.012
	[0.002]**	[0.000]*	[0.002]	[0.002]**
Cognition	0.019	-0.007	0.006	-0.031
	[0.004]**	[0.001]**	[0.006]	[0.005]**
Financial respondent	0.033	0.006	0.000	0.156
	[0.009]**	[0.003]*	0.000	[0.011]**
Log risk tolerance	0.042	0.002	0.021	-0.031
	[0.012]**	[0.004]	[0.014]	[0.012]*
Single female	-0.068	-0.005	-0.038	0.078
	[0.009]**	[0.003]	[0.012]**	[0.010]**
Single male	-0.019	-0.001	-0.011	0.024
	[0.012]	[0.004]	[0.015]	[0.012]*
Female in couple	-0.068	-0.007	-0.026	0.074
	[0.008]**	[0.002]**	[0.011]*	[0.009]**
Age	-0.001	0.001	0.001	0.014
	[0.001]	[0.000]	[0.002]	[0.002]**
Black	-0.031	0.005	-0.010	0.001
	[0.009]**	[0.003]	[0.011]	[0.011]
Hispanic	0.013	0.005	-0.022	0.095
	[0.013]	[0.004]	[0.018]	[0.016]**
Father manager/professional	0.014	0.002	0.014	0.007
	[0.009]	[0.003]	[0.013]	[0.009]
Sunny day optimism	0.015	0.000	-0.006	0.004
	[0.007]*	[0.002]	[0.008]	[0.007]
Economic pessimism	-0.081	-0.003	0.008	-0.011
	[0.014]**	[0.004]	[0.017]	[0.014]
Depressive symptomes	-0.007	0.002	0.006	0.010
	[0.004]	[0.001]*	[0.004]	[0.004]*
Fraction fifty answers	-0.045	-0.116	-0.191	-0.044
	[0.036]	[0.011]**	[0.045]**	[0.040]
Wealth non-positive	0.017	0.002	-0.010	-0.014
	[0.015]	[0.005]	[0.016]	[0.016]
Wealth in middle	0.026	-0.002	0.002	-0.029
	[0.008]**	[0.003]	[0.011]	[0.010]**
Wealth high	0.049	0.000	0.025	-0.038
	[0.010]**	[0.003]	[0.014]	[0.012]**
Fin. wealth zero	-0.021	0.003	0.002	0.056
	[0.012]	[0.004]	[0.014]	[0.015]**
Fin. wealth in middle	0.025	-0.003	0.014	-0.041
	[0.008]**	[0.003]	[0.012]	[0.010]**
Fin. wealth high	0.046	-0.001	0.028	-0.043
	[0.010]**	[0.003]	[0.014]*	[0.011]**
Dummies for missing varaibles	YES	YES	YES	YES
Dummies for years	YES	YES	YES	YES
Observations	9131	9131	3323	10887
R-squared	0.11	0.05	0.04	0.24

Robust standard errors in brackets. * significant at 5%; ** significant at 1%

Table A2.14. OLS regression results for the stock market probability answers.

Panel 1: Without wealth or belief-specific variables on the right-hand side

	Stockholding		Share of stocks if stockholder	
	HRS 2002-8	HRS 2002	HRS 2002-8	HRS 2002
p0	0.222		0.077	
	[0.019]**		[0.020]**	
p0 missing	-0.140		0.005	
	[0.012]**		[0.021]	
p0 - p10		0.094	0.000	0.043
		[0.033]**	0.000	[0.030]
Log lifetime earnings	0.042	0.046	0.005	0.008
	[0.007]**	[0.008]**	[0.008]	[0.009]
DB pension	0.040	0.021	-0.023	-0.005
	[0.014]**	[0.021]	[0.012]	[0.018]
DC pension	0.019	0.040	0.000	0.003
	[0.013]	[0.020]*	[0.011]	[0.017]
Education	0.027	0.025	0.006	0.005
	[0.002]**	[0.003]**	[0.003]*	[0.004]
Cognition	0.055	0.063	0.001	0.001
	[0.006]**	[0.009]**	[0.008]	[0.011]
Financial respondent	-0.078	-0.159	0.001	0.001
	[0.010]**	[0.019]**	[0.011]	[0.037]
Log risk tolerance	0.038	0.037	0.024	0.031
	[0.021]	[0.024]	[0.021]	[0.023]
Single female	-0.096	-0.129	-0.001	-0.006
	[0.014]**	[0.019]**	[0.015]	[0.021]
Single male	-0.115	-0.130	0.022	-0.016
	[0.018]**	[0.025]**	[0.020]	[0.027]
Female in couple	0.029	0.011	0.007	0.018
	[0.008]**	[0.012]	[0.007]	[0.011]
Age	0.003	-0.001	0.002	0.009
	[0.002]	[0.003]	[0.002]	[0.003]**
Black	-0.196	-0.209	0.005	0.022
	[0.016]**	[0.022]**	[0.023]	[0.035]
Hispanic	-0.136	-0.172	0.012	0.018
	[0.019]**	[0.027]**	[0.031]	[0.047]
Father manager/professional	0.095	0.069	0.001	0.006
	[0.017]**	[0.022]**	[0.014]	[0.018]
Dummies for missing varaibles	YES	YES	YES	YES
Dummies for years	YES	YES	YES	YES
Observations	10901	4055	4850	1876
R-squared	0.27	0.26	0.01	0.01

Robust standard errors in brackets. * significant at 5%; ** significant at 1%

Table A2.14. OLS regression results for the stock market probability answers.

Panel 2: Wealth and belief-specific variables are included on the right-hand side

	Stockholding		Share of stocks if stockholder	
	HRS 2002-8	HRS 2002	HRS 2002-8	HRS 2002
p0	0.109		0.070	
	[0.017]**		[0.020]**	
p0 missing	-0.079		0.005	
	[0.011]**		[0.020]	
p0 - p10		0.026	0.000	0.044
		[0.030]	0.000	[0.030]
Log lifetime earnings	0.017	0.016	0.003	0.006
	[0.005]**	[0.006]*	[0.008]	[0.008]
DB pension	0.022	0.005	-0.019	-0.001
	[0.012]	[0.019]	[0.012]	[0.018]
DC pension	0.020	0.041	0.002	0.006
	[0.011]	[0.018]*	[0.011]	[0.017]
Education	0.007	0.002	0.005	0.004
	[0.002]**	[0.003]	[0.003]	[0.004]
Cognition	0.020	0.029	0.002	-0.001
	[0.005]**	[0.008]**	[0.008]	[0.011]
Financial respondent	-0.039	-0.068	-0.001	0.001
	[0.008]**	[0.016]**	[0.011]	[0.037]
Log risk tolerance	0.031	0.030	0.021	0.027
	[0.018]	[0.022]	[0.021]	[0.023]
Single female	0.005	-0.025	0.003	0.001
	[0.012]	[0.018]	[0.015]	[0.022]
Single male	-0.024	-0.035	0.027	-0.017
	[0.016]	[0.023]	[0.020]	[0.027]
Female in couple	0.009	-0.009	0.009	0.017
	[0.007]	[0.010]	[0.007]	[0.011]
Age	-0.001	-0.005	0.002	0.008
	[0.002]	[0.003]	[0.002]	[0.003]*
Black	-0.100	-0.095	0.005	0.010
	[0.014]**	[0.021]**	[0.022]	[0.036]
Hispanic	-0.074	-0.092	-0.006	-0.007
	[0.017]**	[0.025]**	[0.032]	[0.047]
Father manager/professional	0.053	0.027	-0.006	-0.004
	[0.015]**	[0.019]	[0.014]	[0.018]
Sunny day optimism	0.005	0.029	0.008	-0.016
	[0.010]	[0.013]*	[0.012]	[0.015]
Economic pessimism	-0.106	-0.082	-0.061	-0.089
	[0.018]**	[0.026]**	[0.023]**	[0.033]**
Depressive symptomes	-0.008	-0.012	0.011	0.009
	[0.005]	[0.007]	[0.007]	[0.010]
Fraction fifty answers	-0.092	-0.076	-0.149	-0.072
	[0.053]	[0.073]	[0.066]*	[0.089]
Wealth non-positive	-0.005	-0.011	0.083	0.180
	[0.012]	[0.020]	[0.050]	[0.077]*
Wealth in middle	0.138	0.134	0.023	0.011
	[0.016]**	[0.024]**	[0.021]	[0.031]
Wealth high	0.279	0.278	0.069	0.045
	[0.020]**	[0.031]**	[0.023]**	[0.035]
Fin. wealth zero	-0.006	-0.016	0.134	0.168
	[0.012]	[0.020]	[0.045]**	[0.067]*
Fin. wealth in middle	0.203	0.232	-0.113	-0.073
	[0.016]**	[0.025]**	[0.021]**	[0.032]*
Fin. wealth high	0.352	0.379	-0.099	-0.043
	[0.020]**	[0.031]**	[0.022]**	[0.035]
Dummies for missing varaibles	YES	YES	YES	YES
Dummies for years	YES	YES	YES	YES
Observations	10887	4054	4848	1876
R-squared	0.42	0.43	0.04	0.04

A.3 Details of the structural econometric model

A.3.1 The likelihood function

The hypothetical "before rounding" survey answers are the following:

$$p_{0i}^{br} = \Phi\left(\frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{0i}\right) \tag{64}$$

$$p_{10i}^{br} = \Phi\left(\frac{\tilde{\mu}_i - 0.1}{\tilde{\sigma}_i} + v_{10i}\right) \tag{65}$$

Observed probability answers are modeled as interval responses:

$$\begin{pmatrix} p_{0i} \\ p_{10i} \end{pmatrix} \in \mathbf{Q}_{kl} \Leftrightarrow \begin{pmatrix} \Phi\left(\frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{0i}\right) \\ \Phi\left(\frac{\tilde{\mu}_i - 0.1}{\tilde{\sigma}_i} + v_{10i}\right) \end{pmatrix} \in \mathbf{Q}_{kl} \tag{66}$$

$$\mathbf{Q}_{kl} = \begin{pmatrix} [q_k, q_{k+1}) \\ [q_l, q_{l+1}) \end{pmatrix} \tag{67}$$

Expressing the event in scalar terms makes it clear how we can invert the standard normal c.d.f. and get algebraic expressions in terms of the latent variables $\tilde{\mu}_i$, $\tilde{\sigma}_i$ and v_{0i} and v_{10i}.

$$p_{0i} \in [q_k, q_{k+1}) \Leftrightarrow q_k \leq \Phi\left(\frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{0i}\right) < q_{k+1} \tag{68}$$

$$\Leftrightarrow \frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{0i} \in \left[\Phi^{-1}(q_k), \Phi^{-1}(q_{k+1})\right]$$

$$p_{10i} \in [q_l, q_{l+1}) \tag{69}$$

$$\Leftrightarrow \frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{10i} \in \left[\Phi^{-1}(q_l) + \frac{0.1}{\tilde{\sigma}_i}, \Phi^{-1}(q_{l+1}) + \frac{0.1}{\tilde{\sigma}_i}\right] \tag{70}$$

We need distributional assumptions on the random variables to close the econometric model and make it suitable for Maximum Likelihood estimation. First of all, we assume that conditional on observables the individual mean, $\tilde{\mu}_i$ is distributed normally:

$$\tilde{\mu}_i = \beta'_\mu x_{\mu i} + \gamma'_\mu z_{\mu i} + u_{\mu i} \tag{71}$$

$$u_{\mu i} \sim N(0, V(u_\mu)) \tag{72}$$

Second, we assume that individual uncertainty, $\tilde{\sigma}_i$ can take two values $\tilde{\sigma}_i \in \{\tilde{\sigma}_{low}, \tilde{\sigma}_{high}\}$ where $\tilde{\sigma}_{low}$ is the low value corresponding to certain people and $\tilde{\sigma}_{high}$ is the high value for uncertain ones. These two cut points can be estimated, but sometimes we set $\tilde{\sigma}_{low} = 0.15$ which is the historical standard deviation of yearly log-returns. Whether someone has high or low uncertainty is a probit:

$$\tilde{\sigma}_i = \begin{cases} \tilde{\sigma}_{low} & \text{if} \quad \beta'_\sigma x_{\sigma i} + \gamma'_\sigma z_{\sigma i} + u_{\sigma i} \geq 0 \\ \tilde{\sigma}_{high} & \text{if} \quad \text{otherwise} \end{cases} \tag{73}$$

$$u_{\sigma i} \sim N(0, 1) \tag{74}$$

Third, we assume that the noise components, v_{0i} and v_{10i} follow a bivariate normal distribution:

$$\begin{bmatrix} v_{0i} \\ v_{10i} \end{bmatrix} \sim N\left(0, \sigma_v^2 \begin{bmatrix} 1 & \rho_v \\ \rho_v & 1 \end{bmatrix}\right) \tag{75}$$

Lastly, we assume that $u_{\mu i}, u_{\sigma i}$ and $\mathbf{v}_i \equiv (v_{0i},\ v_{10i})'$ are mutually independent.

Let $\tilde{\boldsymbol{\eta}}_i$ denote the vector of beliefs, $\mathbf{v}$ the vector of noise terms, $\mathbf{p}_i$ the vector of probability answers and let the parameter vector $\boldsymbol{\theta}$ denote all parameters of the conditional density and $\mathbf{X}_i$ denote the vector of all right hand-side variables. The quadrant $\mathbf{Q}_{kl}$ that contains the observed probability answers is defined above.

$$\begin{aligned}
\tilde{\boldsymbol{\eta}}_i &\equiv (\tilde{\mu}_i,\ \tilde{\sigma}_i)' && (76)\\
\mathbf{v}_i &\equiv (v_{0i},\ v_{10i})' && (77)\\
\mathbf{p}_i &\equiv (p_{0i},\ p_{10i})' && (78)\\
\boldsymbol{\theta} &= \left(\beta_\mu', \gamma_\mu', \tilde{\sigma}_{low}, \tilde{\sigma}_{high}, \beta_\sigma', \gamma_\sigma',\ V(u_\mu),\ \sigma_v^2,\ \rho_v\right)' && (79)\\
\mathbf{X}_i &\equiv \left(x_i',\ z_{\mu i}',\ z_{\sigma i}'\right) && (80)
\end{aligned}$$

Then the event described by (68) and (69) can be summarized as

$$\mathbf{p}_i \in \mathbf{Q}_{kl} | \tilde{\boldsymbol{\eta}}_i, \mathbf{v}_i \tag{81}$$

The individual (conditional) likelihood is the probability of observing that event conditional on observables.

$$\ell_i \equiv \ell(\mathbf{p}_i|\mathbf{X}_i;\ \boldsymbol{\theta}) = \Pr(\mathbf{p}_i \in \mathbf{Q}_{kl} \mid \mathbf{X}_i;\ \boldsymbol{\theta}) \tag{82}$$

It is worth expanding the likelihood by conditioning on $\tilde{\sigma}_i$

$$\begin{aligned}
\ell_i &= \ell(\mathbf{p}_i|\mathbf{X}_i;\ \boldsymbol{\theta}, \tilde{\sigma}_i = \tilde{\sigma}_{low}) \Pr(\tilde{\sigma}_i = \tilde{\sigma}_{low}|\mathbf{X}_i;\ \boldsymbol{\theta}) && (83)\\
&\quad + \ell(\mathbf{p}_i|\mathbf{X}_i;\ \boldsymbol{\theta}, \tilde{\sigma}_i = \tilde{\sigma}_{high}) \Pr(\tilde{\sigma}_i = \tilde{\sigma}_{high}|\mathbf{X}_i;\ \boldsymbol{\theta}) && (84)\\
&= \ell(\mathbf{p}_i|\mathbf{X}_i;\ \boldsymbol{\theta}, \tilde{\sigma}_i = \tilde{\sigma}_{low}) \Phi(\beta_\sigma' x_{\sigma i} + \gamma_\sigma' z_{\sigma i}) && (85)\\
&\quad + \ell(\mathbf{p}_i|\mathbf{X}_i;\ \boldsymbol{\theta}, \tilde{\sigma}_i = \tilde{\sigma}_{high}) (1 - \Phi(\beta_\sigma' x_{\sigma i} + \gamma_\sigma' z_{\sigma i})) && (86)
\end{aligned}$$

Thus it is enough to find an expression for $\ell(\mathbf{p}_i|\mathbf{X}_i;\ \boldsymbol{\theta}, \tilde{\sigma}_i)$. Let us denote $w_{0i} \equiv \frac{u_{\mu i}}{\tilde{\sigma}_i} + v_{0i}$ and $w_{10i} \equiv \frac{u_{\mu i}}{\tilde{\sigma}_i} + v_{10i}$. The conditional likelihood is

$$\begin{aligned}
\ell(\mathbf{p}_i|\mathbf{X}_i;\ \boldsymbol{\theta}, \tilde{\sigma}_i) &= \Pr\left(\begin{bmatrix} p_{0i} \\ p_{10i} \end{bmatrix} \in \begin{bmatrix} [q_k, q_{k+1}) \\ [q_l, q_{l+1}) \end{bmatrix} \middle| \mathbf{X}_i;\ \boldsymbol{\theta}, \tilde{\sigma}_i\right) && (87)\\
&= \Pr\left(\begin{bmatrix} w_{0i} \\ w_{10i} \end{bmatrix} \in \begin{bmatrix} [w_{0i}^k, w_{0i}^{k+1}) \\ [w_{10i}^l, w_{10i}^{l+1}) \end{bmatrix} \middle| \mathbf{X}_i;\ \boldsymbol{\theta}, \tilde{\sigma}_i\right) && (88)
\end{aligned}$$

with the notation $w_{0i}^{k+1} \equiv \Phi^{-1}(q_{k+1}) - \beta_{\mu}' x_{\mu i} + \gamma_{\mu}' z_{\mu i}$, $w_{0i}^{k} \equiv \Phi^{-1}(q_{k}) - \beta_{\mu}' x_{\mu i} + \gamma_{\mu}' z_{\mu i}$, $w_{10i}^{l+1} \equiv \Phi^{-1}(q_{l+1}) - \beta_{\mu}' x_{\mu i} + \gamma_{\mu}' z_{\mu i} + \frac{0.1}{\tilde{\sigma}_i}$ and $w_{10i}^{l} \equiv \Phi^{-1}(q_{l}) - \beta_{\mu}' x_{\mu i} + \gamma_{\mu}' z_{\mu i} + \frac{0.1}{\tilde{\sigma}_i}$

$\mathbf{w}_i \equiv (w_{0i},\ w_{10i})'$ has a centered bivariate normal distribution and thus the likelihood can be expressed from the bivariate normal c.d.f.

$$\ell(\mathbf{p}_i|\mathbf{X}_i;\ \boldsymbol{\theta},\tilde{\sigma}_i) = Binorm\left(w_{0i}^{k+1}, w_{10i}^{l+1}, \mathbf{C}_i\right) + Binorm\left(w_{0i}^{k}, w_{10i}^{l}, \mathbf{C}_i\right) \tag{89}$$

$$-Binorm\left(w_{0i}^{k}, w_{10i}^{l+1}, \mathbf{C}_i\right) - Binorm\left(w_{0i}^{k+1}, w_{10i}^{l}, \mathbf{C}_i\right) \tag{90}$$

with $\mathbf{C}_i$ representing the variance-covariance matrix of $\mathbf{w}_i|\mathbf{X}_i;\ \boldsymbol{\theta},\tilde{\sigma}_i$

$$\mathbf{C}_i = \begin{bmatrix} \frac{V(u_\mu)}{\tilde{\sigma}_i^2} + \sigma_v^2 & \frac{V(u_\mu)}{\tilde{\sigma}_i^2} + \rho_v \sigma_v^2 \\ \frac{V(u_\mu)}{\tilde{\sigma}_i^2} + \rho_v \sigma_v^2 & \frac{V(u_\mu)}{\tilde{\sigma}_i^2} + \sigma_v^2 \end{bmatrix} \tag{91}$$

As the bivariate normal distribution is available in standard econometric packages such as Stata 11 the likelihood can be evaluated using (90).

A.3.2 Expected $\tilde{\mu}$ and σ conditional on the probability answers

With the parameter estimates, we can "estimate" (predict) the latent variables for each individual. The goal is to get

$$\hat{\boldsymbol{\eta}}_i = \widehat{\mathbb{E}}\left[\tilde{\boldsymbol{\eta}}_i|\mathbf{p}_i \in \mathbf{Q}_{kl}\right] \tag{92}$$

Start from the individual likelihood (68) and (69). These describe the probability of the probability answers falling in a certain interval, conditional on $\tilde{\mu}_i$ and $\tilde{\sigma}_i$. In the parsimonious notation, (68) and (69) describe the event $\mathbf{p}_i \in \mathbf{Q}_{kl}|\tilde{\boldsymbol{\eta}}_i$. Our question is the reverse: it is the density (and then the expectation) of $\tilde{\mu}_i$ and $\tilde{\sigma}_i$ conditional on the probability answer:

$$\mathbb{E}\left[\tilde{\boldsymbol{\eta}}_i|\mathbf{p}_i \in \mathbf{Q}_{kl}\right] = \int \tilde{\boldsymbol{\eta}}_i \times f\left(\tilde{\boldsymbol{\eta}}_i|\mathbf{p}_i \in \mathbf{Q}_{kl}\right) d\left(\tilde{\boldsymbol{\eta}}_i\right) \tag{93}$$

By Bayes' theorem,

$$f\left(\tilde{\boldsymbol{\eta}}_i|\mathbf{p}_i \in \mathbf{Q}_{kl}\right) = \frac{\Pr\left(\mathbf{p}_i \in \mathbf{Q}_{kl}|\tilde{\boldsymbol{\eta}}_i\right) \times f\left(\tilde{\boldsymbol{\eta}}_i\right)}{\Pr\left(\mathbf{p}_i \in \mathbf{Q}_{kl}\right)} = \frac{\Pr\left(\mathbf{p}_i \in \mathbf{Q}_{kl}|\tilde{\boldsymbol{\eta}}_i\right)}{l_i} \times f\left(\tilde{\boldsymbol{\eta}}_i\right) \tag{94}$$

so that

$$\hat{\boldsymbol{\eta}}_i = \mathbb{E}\left[\tilde{\boldsymbol{\eta}}_i|\mathbf{p}_i \in \mathbf{Q}_{kl}\right] = \int \tilde{\boldsymbol{\eta}}_i \times \frac{\Pr\left(\mathbf{p}_i \in \mathbf{Q}_{kl}|\tilde{\boldsymbol{\eta}}_i\right)}{l_i} \times f\left(\tilde{\boldsymbol{\eta}}_i\right) d\left(\tilde{\boldsymbol{\eta}}_i\right) \tag{95}$$

The only unknown part is $\Pr(\mathbf{p}_i \in \mathbf{Q}_{kl}|\tilde{\boldsymbol{\eta}}_i)$. It can be computed similarly to the likelihood function.

$$\Pr(\mathbf{p}_i \in \mathbf{Q}_{kl}|\tilde{\boldsymbol{\eta}}_i) = \Pr\left(\left[\begin{array}{c} p_{0i} \\ p_{10i} \end{array}\right] \in \left[\begin{array}{c} [q_k, q_{k+1}) \\ [q_l, q_{l+1}) \end{array}\right] \middle| \mathbf{X}_i;\ \boldsymbol{\theta},\tilde{\sigma}_i, \tilde{\mu}_i\right) \tag{96}$$

$$= \Pr\left(\left[\begin{array}{c} v_{0i} \\ v_{10i} \end{array}\right] \in \left[\begin{array}{c} [v_{0i}^k, v_{0i}^{k+1}) \\ [v_{10i}^l, v_{10i}^{l+1}) \end{array}\right] \middle| \mathbf{X}_i;\ \boldsymbol{\theta},\tilde{\sigma}_i, \tilde{\mu}_i\right) \tag{97}$$

where $v_{0i}^{k+1} \equiv \Phi^{-1}(q_{k+1}) - \frac{\tilde{\mu}_i}{\tilde{\sigma}_i}$, $v_{0i}^k \equiv \Phi^{-1}(q_k) - \frac{\tilde{\mu}_i}{\tilde{\sigma}_i}$, $v_{10i}^{l+1} \equiv \Phi^{-1}(q_{l+1}) - \frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + \frac{0.1}{\tilde{\sigma}_i}$ and $v_{10i}^l \equiv \Phi^{-1}(q_l) - \frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + \frac{0.1}{\tilde{\sigma}_i}$. Note that this is different from the analogous formula in the likelihood function because, at this stage we "know" $(\tilde{\mu}_i, \tilde{\sigma}_i)$. (In practice, we simulate it out using the estimated parameters which completely specify its distribution.)

The probability in question is again the probability mass over a rectangle:

$$\Pr(\mathbf{p}_i \in \mathbf{Q}_i|\tilde{\boldsymbol{\eta}}_i) = Binorm\left(v_{0i}^{k+1}, v_{10i}^{l+1}, \mathbf{D}\right) + Binorm\left(v_{0i}^{k}, v_{10i}^{l}, \mathbf{D}\right) \tag{98}$$

$$-Binorm\left(v_{0i}^{k}, v_{10i}^{l+1}, \mathbf{D}\right) - Binorm\left(v_{0i}^{k+1}, v_{10i}^{l}, \mathbf{D}\right) \tag{99}$$

with covariance matrix $\mathbf{D}$ from (75) so that $\mathbf{D} = \sigma_v^2 \left[\begin{array}{cc} 1 & \rho_v \\ \rho_v & 1 \end{array}\right]$.

Having all elements in (95) the integration can be approximated by simulation. With drawing M simulation draws $\tilde{\boldsymbol{\eta}}_{i,s}$ from the distribution of $\tilde{\boldsymbol{\eta}}_i$ the approximation can be written as

$$\hat{\boldsymbol{\eta}}_i = \int \tilde{\boldsymbol{\eta}}_i \times \frac{\Pr(\mathbf{p}_i \in \mathbf{Q}_{kl}|\tilde{\boldsymbol{\eta}}_i)}{l_i} \times f(\tilde{\boldsymbol{\eta}}_i)\, d(\tilde{\boldsymbol{\eta}}_i) \tag{100}$$

$$\approx \frac{1}{K_i} \sum_{s=1}^{M} \tilde{\boldsymbol{\eta}}_{i,s} \times \frac{\Pr\left(\mathbf{p}_i \in \mathbf{Q}_{kl}|\tilde{\boldsymbol{\eta}}_{i,s}\right)}{l_i} \tag{101}$$

where K_i is a normalization factor:

$$K_i = \sum_{s=1}^{M} \frac{\Pr\left(\mathbf{p}_i \in \mathbf{Q}_i|\tilde{\boldsymbol{\eta}}_{i,s}\right)}{l_i} \tag{102}$$

A.3.3 Estimating the variance and correlation of survey noise

The goal of this exercise is to estimate moments of the noise distribution so that we can calibrate those in the estimation. We are interested in σ_v^2 and ρ_v. In this simple exercise, we make use of the probability answers in the core questionnaire (p_{0i}, p_{10i}) and the probability answers in the experimental module (p_{M0i}, p_{M10i}), and we ignore rounding.

The hypothetical "before rounding" survey answers are, conditional on the noise variables, the following

$$p_{0i}^{br} = \Phi\left(\frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{0i}\right) \tag{103}$$

$$p_{10i}^{br} = \Phi\left(\frac{\tilde{\mu}_i - 0.1}{\tilde{\sigma}_i} + v_{10i}\right) \tag{104}$$

$$p_{M0i}^{br} = \Phi\left(\frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{M0i}\right) \tag{105}$$

$$p_{M10i}^{br} = \Phi\left(\frac{\tilde{\mu}_i - 0.1}{\tilde{\sigma}_i} + v_{M10i}\right) \tag{106}$$

As a result, we have that $\Phi^{-1}\left(p_{0i}^{br}\right) = \frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{0i}$, $\Phi^{-1}\left(p_{10i}^{br}\right) = \frac{\tilde{\mu}_i - 0.1}{\tilde{\sigma}_i} + v_{10i}$, $\Phi^{-1}\left(p_{M0i}^{br}\right) = \frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{M0i}$, and $\Phi^{-1}\left(p_{M10i}^{br}\right) = \frac{\tilde{\mu}_i - 0.1}{\tilde{\sigma}_i} + v_{M10i}$.

By assumption, the noise components are jointly normally distributed, and they are uncorrelated across core questionnaire and the experimental module.

$$\begin{bmatrix} v_{0i} \\ v_{10i} \\ v_{M0i} \\ v_{M10} \end{bmatrix} \sim N\left(\mathbf{0}, \sigma_v^2 \begin{bmatrix} 1 & & & \\ \rho_v & 1 & & \\ 0 & 0 & 1 & \\ 0 & 0 & \rho_v & 1 \end{bmatrix}\right) \tag{107}$$

Moment conditions 1 and 2. Compare the inverse normal of the core and module answers to the same probability question (p_0 and p_{M0} or p_{10} and p_{M10}), and and take expectation of the squares:

$$\begin{aligned} E\left[\left\{\Phi^{-1}\left(p_{0i}^{br}\right) - \Phi^{-1}\left(p_{M0i}^{br}\right)\right\}^2\right] &= \left[\left\{\frac{\tilde{\mu}_i}{\tilde{\sigma}_i} + v_{0i} - \frac{\tilde{\mu}_i}{\tilde{\sigma}_i} - v_{M0i}\right\}^2\right] && (108) \\ &= E\left[\left(v_{0i} - v_{M0i}\right)^2\right] = 2\sigma_v^2 \\ E\left[\left\{\Phi^{-1}\left(p_{10i}^{br}\right) - \Phi^{-1}\left(p_{M10i}^{br}\right)\right\}^2\right] &= \left[\left\{\frac{\tilde{\mu}_i - 0.1}{\tilde{\sigma}_i} + v_{10i} - \frac{\tilde{\mu}_i - 0.1}{\tilde{\sigma}_i} - v_{M10i}\right\}^2\right] && (109) \\ &= E\left[\left(v_{10i} - v_{M10i}\right)^2\right] = 2\sigma_v^2 \end{aligned}$$

Moment conditions 3 and 4. Similar comparisons across questions (p_0 and p_{M10} or p_{10} and p_{M0}) yield moments that are similar to (108) and (109), but they also include the subjective beliefs about the standard deviation of stock market returns.

$$E\left[\left\{\Phi^{-1}\left(p^{br}_{0i}\right)-\Phi^{-1}\left(p^{br}_{M10i}\right)\right\}^2\right] = \left[\left\{\frac{\tilde{\mu}_i}{\tilde{\sigma}_i}+v_{10i}-\frac{\tilde{\mu}_i-0.1}{\tilde{\sigma}_i}-v_{M10i}\right\}^2\right] \tag{110}$$

$$= E\left[\left(v_{0i}-v_{M10i}+\frac{0.1}{\tilde{\sigma}_i}\right)^2\right]=2\sigma_v^2+0.01E\left[\frac{1}{\tilde{\sigma}_i^2}\right]$$

$$E\left[\left\{\Phi^{-1}\left(p^{br}_{10i}\right)-\Phi^{-1}\left(p^{br}_{M0i}\right)\right\}^2\right] = \left[\left\{\frac{\tilde{\mu}_i-0.1}{\tilde{\sigma}_i}+v_{10i}-\frac{\tilde{\mu}_i}{\tilde{\sigma}_i}-v_{M10i}\right\}^2\right] \tag{111}$$

$$= E\left[\left(v_{0i}-v_{M10i}-\frac{0.1}{\tilde{\sigma}_i}\right)^2\right]=2\sigma_v^2+0.01E\left[\frac{1}{\tilde{\sigma}_i^2}\right]$$

Moment condition 5. Compare the adjacent probability answers in the core questionnaire (p_{0i}, p_{10i}) and take expectation of the squares:

$$E\left[\left\{\Phi^{-1}\left(p^{br}_{0i}\right)-\Phi^{-1}\left(p^{br}_{10i}\right)\right\}^2\right] = E\left[\left\{\frac{\tilde{\mu}_i}{\tilde{\sigma}_i}+v_{0i}-\frac{\tilde{\mu}_i-0.1}{\tilde{\sigma}_i}-v_{10i}\right\}^2\right] \tag{112}$$

$$= E\left[\left(v_{0i}-v_{10i}+\frac{0.1}{\tilde{\sigma}_i}\right)^2\right]=2\left(1-\rho_v\right)\sigma_v^2+0.01E\left[\frac{1}{\tilde{\sigma}_i^2}\right]$$

In principle, one can do this for the answers from the experimental module, (p_{M0i}, p_{M10i}). Because of low number of observations in the experimental module, we do not make use of that comparison.

Estimation. In principle, this is a simple Minimum Distance problem with five moment conditions ((108) through (112)) in three parameters $\left(\sigma_v^2,\ \rho_v \text{ and } E\left[1/\tilde{\sigma}_i^2\right]\right)$. Of these three parameters, we are interested in two, $(\sigma_v^2$, and $\rho_v)$.

The first two moment conditions allow for a Minimum Distance estimation of σ_v^2, while the fifth moment, together with the third and the fourth moments, allows for a Minimum Distance estimation of ρ_v. To see the latter, consider the difference (110) – (112), and the difference (111) – (112) is, of course, analogous.

$$\begin{aligned} &E\left[\left\{\Phi^{-1}\left(p^{br}_{0i}\right)-\Phi^{-1}\left(p^{br}_{M10i}\right)\right\}^2\right] \\ &-E\left[\left\{\Phi^{-1}\left(p^{br}_{0i}\right)-\Phi^{-1}\left(p^{br}_{10i}\right)\right\}^2\right] = 2\sigma_v^2+0.01E\left[\frac{1}{\tilde{\sigma}_i^2}\right]-2\left(1-\rho_v\right)\sigma_v^2-0.01E\left[\frac{1}{\tilde{\sigma}_i^2}\right] \\ &= 2\rho_v\sigma_v^2 \end{aligned} \tag{113}$$

Unfortunately, we do not observe p^{br}_{0i} and p^{br}_{10i} only their survey response versions that are rounded versions for almost all respondents. We address rounding in the likelihood estimation is by interval regressions, which is consistent under any rounding model (as long as rounding is within the pre-defined intervals). In this simple exercise, we assume away rounding error and treat observed answers as if they were the hypothetical pre-rounding variables p_{0i}^{br} and p_{10i}^{br} .

However, an important practical consequence of rounding is the prevalence of answers at 0 and 1, and $\Phi^{-1}(p)$ is not defined for $p = 0$ or $p = 1$. In this simple exercise we opted for an ad-hoc solution replacing $p = 0$ to $p = 0 + \varepsilon$ and $p = 1$ to $p = 1 - \varepsilon$, respectively. Various values for ε were considered (0.05, 0.025, 0.01, and 0.005), and we present the results as a function of those values.

We have four equations in two unknowns, with the first two and second two equations being symmetric in p_0 and p_{10} (or their counterparts in the experimental module). This symmetry implies that the optimum Minimum Distance estimator has identity weights under the structure of our model.

The estimation results are the following.

Table A3.1. Estimated variance and correlation of survey noise.
Results of the Minimum Distance exercise by values of the auxiliary parameter ε

	$\varepsilon = 0.050$	$\varepsilon = 0.025$	$\varepsilon = 0.010$	$\varepsilon = 0.005$
σ_v	0.96	1.05	1.17	1.26
ρ_v	0.61	0.61	0.61	0.60

These results should be viewed as very crude approximations because they ignore rounding (we substituted in the actual answers p for the hypothetical, before-rounding answers p^{br}) and because they handle boundary values in a very ad-hoc way. The estimates of ρ_v seem robust to our handling the boundary problem, but the estimates of σ_v are not.

Estimation of ρ and σ_v^2 by Minimum Distance with covariates. The likelihood function and the estimator for $(\hat{\mu}_i, \hat{\sigma}_i)$ conditions on observed covariates $(\mathbf{X}_i)$ as well as the observed answers to the stock market probability questions $(\mathbf{p}_i)$. The variance and correlation coefficient of the noise variables (v_{0i}, v_{10i}) may be different if conditioned on those covariates.[47]

In this subsection we present estimates of the noise parameters that use moment conditions conditional on covariates. In practice, we repeated the Minimum Distance exercise described above, but instead of the inverse of the observed (and ε-adjusted) variables $\Phi^{-1}(p_{0i})$ etc. we used their residuals after having regressed on all covariates $(\mathbf{X}_i)$. The results are in Table A3.2.

[47] Tables B6 through B10 in the Online Appendix B show that the observed noise features are not strongly associated with covariates. That was the basis for our assumption of unbiased and homoskedastic noise. However, even those weak associations may result in a conditional noise variance that is somewhat smaller than the unconditional one, which may make a difference in the likelihood estimation procedure.

Table A3.2. Estimated variance and correlation of survey noise, conditional on covariates. Results of the Minimum Distance exercise by values of the auxiliary parameter ε

	$\varepsilon = 0.050$	$\varepsilon = 0.025$	$\varepsilon = 0.010$	$\varepsilon = 0.005$
σ_v	0.95	1.04	1.15	1.24
ρ_v	0.42	0.42	0.43	0.44

A.4 Detailed estimation results from the structural econometric model

A.4.1 Detailed estimates from the benchmark model

Table A4.1. Detailed structural estimates 2-point distribution for $\tilde{\sigma}$, low fixed at 0.15.

	E[mu]	P[sig=low] probit coeff	E[mu]	P[sig=low] probit coeff	E[mu]	P[sig=low] probit coeff
Log lifetime earnings			0.005	-0.055	0.003	-0.053
			[0.009]	[0.048]	[0.010]	[0.058]
Education (years)			-0.046	-0.309	-0.042	-0.294
			[0.021]*	[0.262]	[0.021]*	[0.291]
Cognitive score			0.003	-0.019	0.005	-0.009
			[0.018]	[0.179]	[0.018]	[0.197]
DB pension plan			0.01	-0.01	0.005	-0.009
			[0.004]*	[0.032]	[0.004]	[0.035]
DC pension plan			0.02	-0.121	0.017	-0.112
			[0.014]	[0.101]	[0.014]	[0.118]
Financial respondent			0.036	0.073	0.035	0.057
			[0.020]	[0.193]	[0.020]	[0.207]
Log risk tolerance			0.088	0.137	0.087	0.11
			[0.027]**	[0.239]	[0.026]**	[0.276]
Single female			-0.158	-0.763	-0.145	-0.833
			[0.033]**	[0.234]**	[0.034]**	[0.257]**
Single male			-0.076	-0.28	-0.061	-0.361
			[0.030]*	[0.238]	[0.029]*	[0.268]
Female in couple			-0.108	-0.715	-0.112	-0.798
			[0.028]**	[0.235]**	[0.031]**	[0.305]**
Age			0.003	0.079	0.002	0.072
			[0.004]	[0.039]*	[0.004]	[0.045]
Black			-0.072	-0.184	-0.041	-0.174
			[0.034]*	[0.269]	[0.032]	[0.267]
Hispanic			0	0.029	0.021	0.036
			[0.039]	[0.297]	[0.039]	[0.331]
Father professional			0.028	0.202	0.018	0.166
			[0.021]	[0.189]	[0.020]	[0.217]
Missing lifetime earnings			0.015	0.519	0.005	0.558
			[0.020]	[0.170]**	[0.021]	[0.189]**
Missing risk tolerance			0.023	0.32	0.018	0.314
			[0.022]	[0.212]	[0.021]	[0.235]
Missing father occupation			0.005	0.291	0.006	0.312
			[0.025]	[0.205]	[0.024]	[0.222]
Non-positive wealth					-0.008	-0.623
					[0.120]	[1.107]
Medium wealth					0.015	-0.396
					[0.025]	[0.209]
Hugh wealth					0.068	-0.061
					[0.030]*	[0.296]
Zero financial wealth					0.02	0.272
					[0.040]	[0.353]
Medium financial wealth					0.024	-0.215
					[0.027]	[0.237]
High finacial wealth					0.049	-0.138
					[0.031]	[0.278]
Sunshine optimism			0.04		0.038	
			[0.016]*		[0.015]*	
Pessimism in economic outlook			-0.206		-0.182	
			[0.041]**		[0.038]**	
Depressive symptomes			-0.018		-0.011	
			[0.010]		[0.009]	
Missing sunshine			-0.034		-0.033	
			[0.031]		[0.030]	
Missing economic pessimism			-0.038		-0.036	
			[0.038]		[0.038]	
Fraction fifty answers				-3.724		
				[1.025]**		
Constant	-0.009	-0.716	-0.148	-3.871	-0.052	-0.76

Table A4.2. Detailed probit estimates 2-point distribution for $\tilde{\sigma}$, low fixed at 0.15

	Pr(S=1)		E(s|s>0)	
	Probit coefficients		Truncated regression coefficients	
mu_hat		2.361***		0.302***
		[0.257]		[0.099]
sigma_hat		-0.160		0.144
		[0.285]		[0.111]
Log lifetime earnings	0.128***	0.109***	0.000	-0.003
	[0.027]	[0.026]	[0.008]	[0.008]
Education (years)	0.102***	0.075***	0.011**	0.008
	[0.013]	[0.013]	[0.005]	[0.005]
Cognitive score	0.214***	0.143***	0.002	-0.009
	[0.035]	[0.035]	[0.014]	[0.014]
DB pension plan	0.017	0.108	-0.020	-0.012
	[0.066]	[0.069]	[0.024]	[0.024]
DC pension plan	0.097	0.079	0.006	0.004
	[0.064]	[0.065]	[0.021]	[0.021]
Financial respondent	-0.071**	-0.152***	-0.029***	-0.040***
	[0.035]	[0.038]	[0.010]	[0.011]
Log risk tolerance	0.072	-0.127	0.048*	0.030
	[0.082]	[0.086]	[0.028]	[0.029]
Single female	-0.344***	0.067	-0.023	0.009
	[0.072]	[0.093]	[0.029]	[0.035]
Single male	-0.292***	-0.098	-0.022	-0.005
	[0.087]	[0.092]	[0.034]	[0.035]
Female in couple	-0.011	0.280***	0.010	0.029
	[0.039]	[0.060]	[0.013]	[0.020]
Age	-0.013	-0.016	0.006	0.006*
	[0.010]	[0.010]	[0.004]	[0.004]
Black	-0.728***	-0.582***	0.034	0.057
	[0.087]	[0.090]	[0.045]	[0.046]
Hispanic	-0.715***	-0.723***	-0.008	-0.004
	[0.125]	[0.124]	[0.065]	[0.063]
Father professional	0.162**	0.094	0.002	-0.003
	[0.074]	[0.075]	[0.023]	[0.023]
Constant	-1.452**	-1.066	0.110	0.060
	[0.690]	[0.724]	[0.276]	[0.282]
Observations	3323	3323	974	974
Log likelihood	-1078	-1049	-129	-128

Standard errors are clustered at the household level

* significant at 5%; ** significant at 1%

A.4.2 Results for financial respondents

Table A4.3. Relevant heterogeneity in stock market beliefs. Estimates from the structural model Financial respondents, 2-point distribution for $\tilde{\sigma}$, low fixed at 0.15.

	Model w/o covariates		Model with covariates	
	Point estimate	SE*	Point estimate	SE*
Population average of $\tilde{\mu}$	−0.048	0.011	−0.038	0.056
Population standard deviation of $\tilde{\mu}$	0.147	0.010	0.202	0.094
Population average of $\tilde{\sigma}$	0.170	0.002	0.449	0.086

*Bootstrap standard errors

Sample: Health and Retirement Study, wave 2002. Financial respondents, age 55-64 (age of spouse also 55-64)

Table is analogous to Table 2.2 in the main text

Table A4.4. Estimated mean of the structural parameters of stock market beliefs in various subpopulations. HRS 2002
Financial respondents, 2-point distribution for $\tilde{\sigma}$, low fixed at 0.15.

	Average $\hat{\mu}_i$	Average $\hat{\sigma}_i$
Top 25 per cent of lifetime earnings	0.134	0.434
Bottom 25 per cent of lifetime earnings	−0.078	0.481
Education college or more	0.061	0.461
Education high school or less	−0.074	0.469
Has DC pension (top 25% lifetime earnings)	0.140	0.437
Has DB pension (top 25% lifetime earnings)	0.092	0.471
Top 25 per cent of cognitive capacity	0.042	0.474
Bottom 25 per cent of cognitive capacity	−0.128	0.455
Father was manager or professional	0.047	0.442
Father had other occupation	−0.021	0.477
Top 25 per cent of risk tolerance	0.033	0.437
Bottom 25 per cent of risk tolerance	−0.141	0.504
Entire sample of financial respondents	−0.020	0.468
Total number of observations	2,313	2,313

Sample: Health and Retirement Study, wave 2002. Financial respondents, age 55-64 (age of spouse also 55-64)

$\hat{\mu}_i$ and $\hat{\sigma}_i$ are the subjective mean and subjective standard deviation of the one-year ahead stock return, predicted value. Table is analogous to Table 2.3 in the main text

Table A4.5. Subjective stock market beliefs and stockholding at the extensive margin. Financial respondents, 2-point distribution for $\tilde{\sigma}$, low fixed at 0.15.

	$\Pr(s_i > 0)$, partial effects		$E(s_i \mid s_i > 0)$	
	(1)	(2)	(3)	(4)
$\hat{\mu}_i$		0.862		0.355
		(0.097)**		(0.129)**
$\hat{\sigma}_i$		−0.158		0.170
		(0.115)		(0.146)
Log lifetime earnings	0.031	0.019	−0.003	−0.008
	(0.008)**	(0.007)**	(0.008)	(0.008)
Education	0.037	0.029	0.010	0.008
	(0.004)**	(0.004)**	(0.06)**	(0.007)
Cognitive capacity	0.060	0.039	−0.002	−0.013
	(0.013)**	(0.013)**	(0.018)	(0.018)
Log risk tolerance	0.015	−0.064	0.049	0.028
	(0.030)	(0.030)*	(0.035)	(0.036)
Single female	−0.132	0.025	−0.032	0.001
	(0.025)**	(0.034)	(0.031)	(0.042)
Single male	−0.108	−0.034	−0.028	−0.014
	(0.029)**	(0.031)	(0.032)	(0.038)
Female in couple	−0.026	0.101	−0.003	0.018
	(0.026)	(0.033)**	(0.032)	(0.039)
African American	−0.218	−0.155	0.036	0.058
	(0.027)**	(0.028)**	(0.044)	(0.045)
Hispanic	−0.210	−0.162	−0.004	0.002
	(0.043)**	(0.042)**	(0.067)	(0.064)
Other variables	YES	YES	YES	YES

Table analogous to Table 2.4 in main text. Probit models (1) and (2); truncated regression models (3) and (4).
Sample: Health and Retirement Study, wave 2002. Financial respondents, age 55-64 (age of spouse also 55-64)
Clustered standard errors in parentheses; bootstrapped for models (2).and (4); ** significant at 1%; * at 5%

A.4.3 Results with freely estimated 2-point distributions for $\tilde{\sigma}$

Table A4.6. Relevant heterogeneity in stock market beliefs. Estimates from the structural model 2-point distribution for $\tilde{\sigma}$, low point estimated as well.

	Model w/o covariates		Model with covariates	
	Point estimate	SE*	Point estimate	SE*
Population average of $\tilde{\mu}$	−0.054	0.015	−0.036	0.010
Population standard deviation of $\tilde{\mu}$	0.210	0.027	0.178	0.013
Population average of $\tilde{\sigma}$	0.516	0.055	0.165	0.011

*Bootstrap standard errors

Sample: Health and Retirement Study, wave 2002. All respondents, age 55-64 (age of spouse also 55-64)

Table is analogous to Table 2.2 in the main text

Table A4.7. Estimated mean of the structural parameters of stock market beliefs in various subpopulations. HRS 2002
2-point distribution for $\tilde{\sigma}$, low point estimated as well.

	Average $\hat{\mu}_i$	Average $\hat{\sigma}_i$
Top 25 per cent of lifetime earnings	0.703	0.164
Bottom 25 per cent of lifetime earnings	−0.004	0.164
Education college or more	0.054	0.164
Education high school or less	−0.041	0.164
Has DC pension (top 25% lifetime earnings)	0.071	0.164
Has DB pension (top 25% lifetime earnings)	0.068	0.165
Top 25 per cent of cognitive capacity	0.041	0.164
Bottom 25 per cent of cognitive capacity	−0.071	0.163
Father was manager or professional	0.042	0.164
Father had other occupation	−0.008	0.164
Top 25 per cent of risk tolerance	0.021	0.164
Bottom 25 per cent of risk tolerance	−0.055	0.164
Entire sample respondents	−0.036	0.164
Total number of observations	3,314	3,314

Sample: Health and Retirement Study, wave 2002. All respondents, age 55-64 (age of spouse also 55-64)

$\hat{\mu}_i$ and $\hat{\sigma}_i$ are the subjective mean and subjective standard deviation of the one-year ahead stock return, predicted value. Table is analogous to Table 2.3 in the main text

Table A4.8. Subjective stock market beliefs and stockholding at the extensive margin. 2-point distribution for $\tilde{\sigma}$, low point estimated as well.

	$\Pr(s_i > 0)$, partial effects		$E(s_i \mid s_i > 0)$	
	(1)	(2)	(3)	(4)
$\hat{\mu}_i$		0.712		2.283
		(0.080)**		(0.265)**
$\hat{\sigma}_i$		−11.62		−37.2
		(13.95)		(44.7)
Log lifetime earnings	0.041	0.037	0.000	0.121
	(0.008)**	(0.008)**	(0.008)	(0.027)**
Education	0.033	0.026	0.011	0.080
	(0.004)**	(0.004)**	(0.05)**	(0.013)**
Cognitive capacity	0.069	0.054	0.002	0.176
	(0.011)**	(0.013)**	(0.014)	(0.037)
Log risk tolerance	0.023	−0.008	0.049	−0.027
	(0.026)	(0.027)*	(0.029)	(0.084)
Single female	−0.111	−0.033	−0.023	−0.107
	(0.027)**	(0.027)	(0.029)	(0.083)
Single male	−0.093	−0.051	−0.022	−0.164
	(0.029)**	(0.028)	(0.034)	(0.090)
Female in couple	−0.003	0.050	0.006	0.161
	(0.012)	(0.018)**	(0.013)	(0.056)**
African American	−0.233	−0.204	0.034	−0.655
	(0.027)**	(0.028)**	(0.045)	(0.089)**
Hispanic	−0.229	−0.227	−0.008	−0.729**
	(0.024)**	(0.038)**	(0.065)	(0.125)
Other variables	YES	YES	YES	YES

Table analogous to table 4 in main text. Probit models (1) and (2); truncated regression models (3) and (4).

Sample: Health and Retirement Study, wave 2002. All respondents, age 55-64 (age of spouse also 55-64)

Clustered standard errors in parentheses; bootstrapped for models (2).and (4); ** significant at 1%; * significant at

A.4.4 Results with including all the belief-specific right hand-side variables (z) in all models with the other covariates (x)

Table A4.9. Relevant heterogeneity in stock market beliefs. Estimates from the structural model 2-point distribution for $\tilde{\sigma}$, low point fixed to 0.15. z_μ and z_σ are always included with x

	Model w/o covariates		Model with covariates	
	Point estimate	SE*	Point estimate	SE*
Population average of $\tilde{\mu}$	-0.066	0.018	-0.046	0.021
Population standard deviation of $\tilde{\mu}$	0.197	0.019	0.213	0.036
Population average of $\tilde{\sigma}$	0.576	0.077	0.524	0.089

*Bootstrap standard errors

Sample: HRS 2002, 55 to 64 years old financial respondents (partner is also 55 to 64)

Table is analogous to Table 2.2 in the main text

Table A4.10. Estimated mean of the structural parameters of stock market beliefs in various subpopulations. HRS 2002
2-point distribution for $\tilde{\sigma}$, low point fixed to 0.15. z_μ and z_σ are always included with x

	Average $\hat{\mu}_i$	Average $\hat{\sigma}_i$
Top 25 per cent of lifetime earnings	0.065	0.546
Bottom 25 per cent of lifetime earnings	−0.092	0.535
Education college or more	0.042	0.539
Education high school or less	−0.090	0.532
Has DC pension (top 25% lifetime earnings)	0.073	0.540
Has DB pension (top 25% lifetime earnings)	0.049	0.576
Top 25 per cent of cognitive capacity	0.024	0.551
Bottom 25 per cent of cognitive capacity	−0.132	0.509
Father was manager or professional	0.030	0.520
Father had other occupation	−0.046	0.548
Top 25 per cent of risk tolerance	0.011	0.512
Bottom 25 per cent of risk tolerance	−0.142	0.562
Financial respondent in couple	0.032	0.511
Non-financial respondent in couple	−0.049	0.556
Entire sample	−0.038	0.537
Total number of observations	3,314	3,314

Sample: Health and Retirement Study, wave 2002. Respondents of age 55 through 64 (partner also 55-64)
$\hat{\mu}_i$ and $\hat{\sigma}_i$: subjective mean and subjective standard deviation of the one-year ahead stock return, predicted value. Table is analogous to Table 2.3 in the main text

Table A4.11. Subjective stock market beliefs and stockholding at the extensive margin. 2-point distribution for $\tilde{\sigma}$, low point fixed to 0.15. z_μ and z_σ are always included with x

	$\Pr(s_i > 0)$, partial effects		$E(s_i \mid s_i > 0)$	
	(1)	(2)	(3)	(4)
$\hat{\mu}_i$		0.608		0.240
		(0.095)**		(0.117)*
$\hat{\sigma}_i$		−0.071		0.240
		(0.096)		(0.131)
Log lifetime earnings	0.037	0.033	−0.000	−0.003
	(0.008)**	(0.008)**	(0.009)	(0.012)
Education	0.030	0.023	0.009	0.007
	(0.004)**	(0.004)**	(0.05)	(0.005)
Cognitive capacity	0.054	0.042	−0.002	−0.011
	(0.011)**	(0.012)**	(0.015)	(0.015)
Log risk tolerance	0.026	−0.025	0.045	0.034
	(0.026)	(0.029)	(0.028)	(0.030)
Single female	−0.088	0.015	−0.017	0.010
	(0.023)**	(0.032)	(0.029)	(0.037)
Single male	−0.079	−0.030	−0.024	−0.021
	(0.029)**	(0.031)	(0.035)	(0.038)
Female in couple	0.005	0.078	0.017	0.018
	(0.013)	(0.020)**	(0.013)	(0.024)
African American	−0.233	−0.190	0.028	0.047
	(0.026)**	(0.030)**	(0.045)	(0.046)
Hispanic	−0.220	−0.216	−0.005	−0.006
	(0.039)**	(0.040)**	(0.063)	(0.061)
Other variables	YES	YES	YES	YES

Table analogous to table 4 in main text. Probit models (1) and (2); truncated regression models (3) and (4).

Sample: Health and Retirement Study, wave 2002. All respondents, age 55-64 (age of spouse also 55-64)

Clustered standard errors in parentheses; bootstrapped for models (2).and (4); ** significant at 1%; * at 5%

B Appendix to Chapter 3

B.1 Descriptive statistics

Table B1.1. Summary statistics of the control variables in the entire HRS sample as well as the subsamples for the analysis

	Entire sample		50-70 years old subsample					
			All		Financial knowedge subsample		Personality subsample	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
Number series score	-0.05	1.01	0.16	0.96	0.08	1.00	0.24	0.95
Education (years)	12.55	3.18	12.99	3.02	13.11	2.92	13.13	2.88
Female	0.60	0.49	0.60	0.49	0.61	0.49	0.60	0.49
Age in 2010	70.52	10.42	62.57	5.08	62.37	5.11	62.70	5.05
African American	0.14	0.35	0.15	0.36	0.17	0.38	0.14	0.34
Hispanic	0.10	0.30	0.11	0.32	0.11	0.32	0.10	0.30
Wealth non-positive	0.09	0.29	0.10	0.30	0.12	0.32	0.10	0.30
Log wealth	10.87	3.90	10.76	4.05	10.53	4.27	10.86	4.03
Observations	15372		7446		685		3547	

Standard errors in brackets are clustered at the household level. * significant at 5%; ** 1%.

HRS 2002, 50 to 70 years old respondents and HRS 2010, 50 to 70 years old respondents.

Table B1.2. The propensity to give various answer combinations to p_0 and p_{10} in 2002 and p_0, p_{20} and p_{ltn20} in 2010 that imply zero probability mass or violate the laws of probability. Linear probability estimates (OLS regressions) age is 50 to 70 at the time of the survey

	Implied zero probability mass			Apparent violation of the laws of probability		
	2002	2010		2002	2010	
	$p_0=p_{10}$	$p_0=p_{20}$	$p_0+p_{ltn0}=1$	$p_0<p_{10}$	$p_0<p_{20}$	$p_0+p_{ltn0}>1$
Number series score	-0.010	-0.013	0.013	-0.012	-0.013	-0.018
	[0.007]	[0.006]*	[0.005]**	[0.005]*	[0.005]**	[0.005]**
Education (years)	-0.001	-0.002	0.005	0.000	-0.002	-0.004
	[0.002]	[0.002]	[0.002]**	[0.002]	[0.002]	[0.002]*
Female	0.040	0.043	-0.003	-0.004	0.015	0.026
	[0.011]**	[0.009]**	[0.008]	[0.008]	[0.007]*	[0.008]**
Age	0.001	0.002	-0.001	0.000	-0.001	-0.003
	[0.001]	[0.001]	[0.001]	[0.001]	[0.001]	[0.001]**
African American	-0.013	0.030	-0.015	0.008	0.070	0.004
	[0.017]	[0.015]	[0.011]	[0.013]	[0.012]**	[0.012]
Hispanic	-0.080	0.017	0.015	0.043	0.074	-0.010
	[0.024]**	[0.019]	[0.015]	[0.020]*	[0.015]**	[0.014]
Wealth nonpositive	-0.071	-0.021	-0.006	-0.038	0.012	-0.032
	[0.049]	[0.038]	[0.030]	[0.035]	[0.029]	[0.030]
Log wealth	-0.006	-0.003	-0.001	-0.003	-0.002	-0.004
	[0.004]	[0.003]	[0.002]	[0.003]	[0.002]	[0.002]
p_0 category dummies	YES	YES	YES	YES	YES	YES
Observations	7560	6416	6416	7560	6416	6416
R-squared	0.04	0.24	0.03	0.07	0.11	0.25

Standard errors in brackets are clustered at the household level. * significant at 5%; ** 1%.

HRS 2002, 50 to 70 years old respondents and HRS 2010, 50 to 70 years old respondents.

Table B1.3. The Big Five personality measures and their predictors.
Linear regression estimates (OLS regressions), 50 to 70 years old respondents in HRS 2010.

	Agreableness	Conscientiousness	Extroversion	Neuroticism	Openness
Agreableness		0.342	0.404	0.074	0.262
		[0.024]**	[0.020]**	[0.028]**	[0.021]**
Conscientiousness	0.244		-0.085	-0.151	0.128
	[0.017]**		[0.017]**	[0.024]**	[0.017]**
Extroversion	0.384	-0.114		-0.059	0.439
	[0.019]**	[0.023]**		[0.028]*	[0.019]**
Neuroticism	0.041	-0.117	-0.034		-0.011
	[0.015]**	[0.019]**	[0.016]*		[0.016]
Openness	0.254	0.174	0.447	-0.018	
	[0.021]**	[0.023]**	[0.019]**	[0.028]	
Number series score	-0.022	0.011	0.009	-0.039	-0.002
	[0.015]	[0.017]	[0.015]	[0.020]*	[0.015]
Education (years)	-0.02	-0.002	0.02	-0.013	0.041
	[0.005]**	[0.007]	[0.006]**	[0.007]	[0.006]**
Female	0.185	0.087	0.087	0.051	-0.153
	[0.023]**	[0.028]**	[0.024]**	[0.031]	[0.023]**
Age	0.001	-0.005	0.007	-0.017	-0.005
	[0.002]	[0.003]	[0.002]**	[0.003]**	[0.002]*
African American	0.051	-0.137	0.046	-0.31	-0.004
	[0.037]	[0.045]**	[0.037]	[0.052]**	[0.040]
Hispanic	-0.073	0.168	0.07	0.075	-0.005
	[0.044]	[0.051]**	[0.043]	[0.069]	[0.049]
Wealth nonpositive	-0.111	0.238	0.081	-0.058	0.077
	[0.095]	[0.106]*	[0.097]	[0.125]	[0.103]
Log wealth	-0.011	0.022	0.017	-0.017	0.003
	[0.007]	[0.008]**	[0.008]*	[0.009]	[0.008]
Constant	0.212	0.133	-0.946	1.563	-0.087
	[0.183]	[0.218]	[0.180]**	[0.247]**	[0.188]
Observations	3247	3247	3247	3247	3247
R-squared	0.51	0.21	0.53	0.06	0.51

Standard errors in brackets are clustered at the household level. * significant at 5%; ** 1%.

HRS 2002, 50 to 70 years old respondents and HRS 2010, 50 to 70 years old respondents.

Table B1.4. The measure of general optimism and its predictors.
Linear regression estimates (OLS), 50 to 70 years old respondents in HRS 2010.

	(1)	(2)
Sunshine optimism	0.103	
	[0.047]*	
Number series score	0.038	0.024
	[0.030]	[0.020]
Education (years)	0.025	0.020
	[0.010]*	[0.007]**
Female	0.065	0.067
	[0.048]	[0.031]*
Age	0.008	0.008
	[0.007]	[0.003]*
African American	0.402	0.421
	[0.077]**	[0.052]**
Hispanic	0.412	0.530
	[0.094]**	[0.062]**
Wealth nonpositive	0.477	0.499
	[0.195]*	[0.133]**
Log wealth	0.048	0.057
	[0.015]**	[0.010]**
Constant	-1.571	-1.566
	[0.552]**	[0.257]**
Observations	1540	3513
R-squared	0.04	0.05

Standard errors in brackets are clustered at the household level. * significant at 5%; ** 1%. HRS 2010, 50 to 70 years old respondents. Subset with sunshine optimism is smaller because that measure is derived from earlier observations in the HRS (1994 and 2000).

Table B1.5.

Pairwise correlation of financial knowledge, Big Five personality and optimism

	Hist. returns	Financial knowledge	Agreeable-ness	Conscien-tiousness	Extro-version	Neuro-ticism	Open-ness
Financial knowlege	0.19*						
	(685)						
Agreeableness	0.04	0.07					
	(349)	(310)					
Conscientiousness	−0.02	0.04	0.42*				
	(349)	(310)	(3543)				
Extroversion	0.09	0.13*	0.62*	0.25*			
	(349)	(310)	(3543)	(3542)			
Neuroticism	0.00	0.07	−0.03	−0.14*	−0.08*		
	(350)	(311)	(3545)	(3542)	(3543)		
Openness	0.04	0.07	0.57*	0.33*	0.65*	−0.08*	
	(348)	(309)	(3535)	(3534)	(3535)	(3535)	
Optimism	0.10	0.15*	0.26*	0.02	0.33*	−0.20*	0.28*
	(349)	(311)	(3529)	(3526)	(3527)	(3527)	(3520)

Number of observations in parentheses. * significant at 5%.

HRS 2010, Financial literacy experimental module and "Participant Lifestyle Questionnaire’" 50-70 years of age.

B.2 Detailed estimates of the structural models with 2 probability answers

Table B2.1

Stock market expectations and financial knowledge. Parameter estimates of the structural econometric model with other right hand-side variables

($n = 619$, log likelihood $= -2617.0$)			
	Perceived mean (β_μ)	Log unobserved heterogeneity in (β_u)	Probit coefficient for high perceived risk (β_σ)
Historical returns	0.06	-0.55	-0.28
(S.E.)	(0.03)	(0.36)	(0.56)
Other financial knowledge	0.05	-0.37	-0.54
(S.E.)	(0.02)*	(0.13)**	(0.30)
Female	-0.08	0.32	3.4
(S.E.)	(0.04)*	(0.28)	(35.9)
Age	-0.00	-0.01	-0.00
(S.E.)	(0.00)	(0.02)	(0.04)
Education	0.01	-0.08	0.01
(S.E.)	(0.00)*	(0.04)*	(0.08)
Number series score	-0.03	-0.33	-0.11
(S.E.)	(0.02)	(0.12)**	(0.29)
Constant	-0.06	-1.65	0.91
(S.E.)	(0.04)	(0.34)**	(0.60)

Historical returns: dummy for response stocks have had higher returns than bonds and savings accounts.

Other financial knowledge: standardized score. See the methods section for details of the model.

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. Financial literacy module, 50 to 70 years old respondents. No other RHS variables

Table B2.2

Stock market expectations and agreeableness. Parameter estimates of the structural econometric model

	Perceived mean returns (β_μ)		Probit coefficient for high perceived risk (β_σ)	
Agreeableness above average	0.00	−0.00	0.16	0.16
(S.E.)	(0.01)	(0.01)	(0.13)	(0.11)
Female		−0.08		0.55
(S.E.)		(0.01)**		(0.13)**
Age		−0.00		−0.01
(S.E.)		(0.01)		(0.01)
Education		0.02		−0.05
(S.E.)		(0.00)**		(0.04)
Number series score		0.05		−0.33
(S.E.)		(0.01)**		(0.11)**
Constant	−0.07	−0.04	0.92	0.62
(S.E.)	(0.01)**	(0.02)*	(0.10)**	(0.20)**
Log likelihood	−13099	−13016		
Observations	3062	3062		

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant lifestyle" questionnaire subsample, 50 to 70 years old respondents.

Table B2.3

Stock market expectations and conscientiousness. Parameter estimates of the structural econometric model

	Perceived mean returns (β_μ)		Probit coefficient for high perceived risk (β_σ)	
Conscientiousness above average	0.00	0.01	0.00	0.02
(S.E.)	(0.01)	(0.01)	(0.11)	(0.11)
Female		-0.08		0.59
(S.E.)		$(0.01)^{**}$		$(0.10)^{**}$
Age		-0.00		-0.01
(S.E.)		(0.01)		(0.01)
Education		0.02		-0.05
(S.E.)		$(0.00)^{**}$		(0.04)
Number series score		0.04		-0.35
(S.E.)		$(0.00)^{**}$		$(0.17)^{*}$
Constant	-0.06	-0.04	1.02	0.79
(S.E.)	$(0.01)^{**}$	$(0.02)^{*}$	$(0.10)^{**}$	$(0.25)^{**}$
Log likelihood	-13093	-13010		
Observations	3060	3060		

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant lifestyle" questionnaire subsample, 50 to 70 years old respondents.

Table B2.4

Stock market expectations and extroversion. Parameter estimates of the structural econometric model

	Perceived mean returns (β_μ)		Probit coefficient for high perceived risk (β_σ)	
Extroversion above average	0.02	0.00	−0.04	−0.02
(S.E.)	(0.01)	(0.01)	(0.13)	(0.11)
Female		−0.08		0.60
(S.E.)		(0.01)**		(0.15)**
Age		−0.00		−0.01
(S.E.)		(0.01)		(0.01)
Education		0.02		−0.04
(S.E.)		(0.00)**		(0.04)
Number series score		0.04		−0.34
(S.E.)		(0.00)**		(0.14)*
Constant	−0.08	−0.05	1.03	0.71
(S.E.)	(0.01)**	(0.02)*	(0.11)**	(0.22)**
Log likelihood	−13103	−13021		
Observations	3062	3062		

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant lifestyle" questionnaire subsample, 50 to 70 years old respondents.

Table B2.5

Stock market expectations and Openness. Parameter estimates of the structural econometric model using

	Perceived mean returns (β_μ)		Probit coefficient for high perceived risk (β_σ)	
Openness above average	0.05	0.02	−0.03	0.02
(S.E.)	(0.01)	(0.01)	(0.13)	(0.10)
Female		−0.08		0.58
(S.E.)		(0.01)**		(0.10)**
Age		−0.00		−0.01
(S.E.)		(0.01)		(0.01)
Education		0.02		−0.04
(S.E.)		(0.00)**		(0.04)
Number series score		0.04		−0.33
(S.E.)		(0.00)**		(0.17)*
Constant	−0.10		1.02	
(S.E.)	(0.01)**		(0.11)**	
Log likelihood	−13074			
Observations	3057	3057		

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant lifestyle" questionnaire subsample, 50 to 70 years old respondents.

B.3 Detailed estimates of the structural models using all three probability answers

Table B2.6

Stock market expectations and financial knowledge. Parameter estimates of the structural econometric model using all three probability answers (p_0, p_{20}, p_{ltn20})

($n = 433$, log likelihood $= -2791.3$)

	Perceived mean (β_μ)	Log unobserved heterogeneity in (β_u)	Probit coefficient for high perceived risk (β_σ)
Historical returns	0.02	−0.08	−0.05
(S.E.)	(0.02)	(0.21)	(0.29)
Other financial knowledge	0.02	−0.15	0.10
(S.E.)	(0.01)*	(0.10)	(0.13)
Constant	−0.07	−1.95	1.21
(S.E.)	(0.02)**	(0.16)**	(0.23)**

Historical returns: dummy for response stocks have had higher returns than bonds and savings accounts.

Other financial knowledge: standardized score. See the methods section for details of the model.

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. Financial literacy module, 50 to 70 years old respondents. No other RHS variables

Table B2.7

Stock market expectations and financial knowledge. Parameter estimates of the structural econometric model using all three probability answers (p_0, p_{20}, p_{ltn20})

($n = 433$, log likelihood $= -2781.1$)

	Perceived mean (β_μ)	Log unobserved heterogeneity in (β_u)	Probit coefficient for high perceived risk (β_σ)
Historical returns	0.01	−0.06	0.15
(S.E.)	(0.02)	(0.23)	(0.38)
Other financial knowledge	0.02	−0.17	0.09
(S.E.)	(0.01)	(0.11)	(0.17)
Female	0.01	0.28	0.52
(S.E.)	(0.02)	(0.25)	(0.33)
Age	0.00	0.03	−0.02
(S.E.)	(0.00)	(0.02)	(0.04)
Education	0.01	−0.04	0.12
(S.E.)	(0.00)	(0.04)	(0.09)
Number series score	0.00	0.19	−0.29
(S.E.)	(0.01)	(0.17)	(0.27)
Constant	−0.07	−2.71	1.08
(S.E.)	(0.03)*	(0.38)**	(0.56)

Historical returns: dummy for response stocks have had higher returns than bonds and savings accounts.

Other financial knowledge: standardized score. See the methods section for details of the model.

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. Financial literacy module, 50 to 70 years old respondents. No other RHS variables

Table B2.8

Stock market expectations and Agreeableness. Parameter estimates of the structural econometric model using all three probability answers (p_0, p_{20}, p_{ltn20})

	Perceived mean returns (β_μ)		Probit coefficient for high perceived risk (β_σ)	
Agreeableness above average	−0.01	−0.01	0.04	0.01
(S.E.)	(0.01)	(0.01)	(0.13)	(0.11)
Female		−0.02		0.21
(S.E.)		(0.01)*		(0.12)
Age		0.00		−0.02
(S.E.)		(0.01)		(0.01)
Education		0.01		0.01
(S.E.)		(0.00)**		(0.02)
Number series score		0.02		0.01
(S.E.)		(0.00)**		(0.07)
Constant	−0.05	−0.06	1.06	1.15
(S.E.)	(0.01)**	(0.01)**	(0.10)**	(0.19)**
Log likelihood	−14749	−14707		
Observations	2291	2291		

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant lifestyle" questionnaire subsample, 50 to 70 years old respondents.

Table B2.9

Stock market expectations and Consicientiousness. Parameter estimates of the structural econometric model using all three probability answers (p_0, p_{20}, p_{ltn20})

	Perceived mean returns (β_μ)		Probit coefficient for high perceived risk (β_σ)	
Conscientiousness above average	-0.01	-0.01	-0.06	-0.08
(S.E.)	(0.01)	(0.01)	(0.12)	(0.12)
Female		-0.02		0.22
(S.E.)		(0.01)*		(0.12)
Age		0.00		-0.02
(S.E.)		(0.01)		(0.01)
Education		0.01		0.00
(S.E.)		(0.00)**		(0.02)
Number series score		0.02		0.01
(S.E.)		(0.00)**		(0.07)
Constant	-0.05	-0.06	1.12	1.20
(S.E.)	(0.01)**	(0.01)**	(0.10)**	(0.18)**
Log likelihood	-14736	-14694		
Observations	2289	2289		

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant lifestyle" questionnaire subsample, 50 to 70 years old respondents.

Table B2.10

Stock market expectations and Extroversion. Parameter estimates of the structural econometric model using all three probability answers (p_0, p_{20}, p_{ltn20})

	Perceived mean returns (β_μ)		Probit coefficient for high perceived risk (β_σ)	
Extroversion above average	−0.00	−0.01	−0.07	−0.08
(S.E.)	(0.01)	(0.01)	(0.13)	(0.13)
Female		−0.03		0.21
(S.E.)		(0.01)*		(0.12)
Age		0.00		−0.02
(S.E.)		(0.01)		(0.01)
Education		0.01		0.00
(S.E.)		(0.00)**		(0.02)
Number series score		0.02		0.00
(S.E.)		(0.00)**		(0.07)
Constant	−0.06	−0.06	1.13	1.21
(S.E.)	(0.01)**	(0.01)**	(0.11)**	(0.20)**
Log likelihood	−14745	−14703		
Observations	2291	2291		

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant lifestyle" questionnaire subsample, 50 to 70 years old respondents.

Table B2.11

Stock market expectations and Neuroticism. Parameter estimates of the structural econometric model using all three probability answers (p_0, p_{20}, p_{ltn20})

	Perceived mean returns (β_μ)		Probit coefficient for high perceived risk (β_σ)	
Neuroticism above average	-0.02	0.00	-0.04	-0.05
(S.E.)	(0.01)	(0.01)	(0.12)	(0.12)
Female		-0.03		0.21
(S.E.)		$(0.01)^{**}$		(0.12)
Age		0.00		-0.02
(S.E.)		(0.01)		(0.01)
Education		0.01		-0.00
(S.E.)		$(0.00)^{**}$		(0.02)
Number series score		0.02		0.01
(S.E.)		$(0.00)^{**}$		(0.07)
Constant	-0.05	-0.06	1.11	1.19
(S.E.)	$(0.01)^{**}$	$(0.01)^{**}$	$(0.10)^{**}$	$(0.20)^{**}$
Log likelihood	-14748	-14707		
Observations	2291	2291		

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant lifestyle" questionnaire subsample, 50 to 70 years old respondents.

Table B2.12

Stock market expectations and Openness. Parameter estimates of the structural econometric model using all three probability answers (p_0, p_{20}, p_{ltn20})

	Perceived mean returns (β_μ)		Probit coefficient for high perceived risk (β_σ)	
Openness above average	0.01	0.00	0.06	0.07
(S.E.)	(0.01)	(0.01)	(0.12)	(0.11)
Female		−0.03		0.21
(S.E.)		(0.01)*		(0.12)
Age		0.00		−0.01
(S.E.)		(0.01)		(0.01)
Education		0.01		0.01
(S.E.)		(0.00)**		(0.02)
Number series score		0.02		0.01
(S.E.)		(0.00)**		(0.07)
Constant	−0.06	−0.07	1.05	1.11
(S.E.)	(0.01)**	(0.01)**	(0.10)**	(0.19)**
Log likelihood	−14718	−14677		
Observations	2286	2286		

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant lifestyle" questionnaire subsample, 50 to 70 years old respondents.

Table B2.13

Stock market expectations and Optimism. Parameter estimates of the structural econometric model using all three probability answers (p_0, p_{20}, p_{ltn20})

	Perceived mean returns (β_μ)		Probit coefficient for high perceived risk (β_σ)	
Optimism above average	0.04	0.03	0.07	0.10
(S.E.)	$(0.01)^{*}$	$(0.01)^{**}$	(0.12)	(0.12)
Female		−0.03		0.22
(S.E.)		$(0.01)^{**}$		(0.12)
Age		0.00		−0.02
(S.E.)		(0.01)		(0.01)
Education		0.01		0.00
(S.E.)		$(0.00)^{**}$		(0.02)
Number series score		0.02		0.02
(S.E.)		$(0.00)^{**}$		(0.07)
Constant	−0.07	−0.08	1.06	1.11
(S.E.)	$(0.01)^{**}$	$(0.01)^{**}$	$(0.09)^{**}$	$(0.18)^{**}$
Log likelihood	−14716	−14676		
Observations	2286	2286		

Standard errors in parentheses are clustered at the household level. * significant at 5%; ** 1%.

HRS 2010. "Participant lifestyle" questionnaire subsample, 50 to 70 years old respondents.

C Appendix to Chapter 4

C.1 Detailed estimates from the models with interactions

Table C1.1.

Stockholders versus non-stockholders. Date of interview in 2008 and average subjective expected value of yearly stock returns (μ), average subjective standard deviation (σ) and unobserved cross-sectional heterogeneity in expectations ($Std\,(u)$). Results from structural regressions including full interactions. HRS 2008.

	μ		log(s)		log[Std(u)]	
	coef	interaction with stockholding	coef	interaction with stockholding	coef	interaction with stockholding
Constant	-0.091	0.087	-0.597	-0.134	-1.176	-0.229
	[0.007]**	[0.013]**	[0.026]**	[0.052]**	[0.082]**	[0.063]**
July 08 to Sep08	0.008	-0.013	0.135	-0.052	0.158	-0.066
	[0.018]	[0.022]	[0.067]*	[0.082]	[0.076]*	[0.098]
Oct 08 to Nov 08	0.091	-0.031	0.29	-0.066	0.465	0.135
	[0.041]*	[0.052]	[0.155]	[0.188]	[0.161]**	[0.203]
Dec 08 to Feb 09	-0.012	-0.006	-0.113	0.194	0.231	0.251
	[0.052]	[0.068]	[0.173]	[0.246]	[0.181]	[0.268]
Female	-0.103	0.05	0.234	0.076	0.157	0.024
	[0.016]**	[0.019]**	[0.059]**	[0.073]	[0.066]*	[0.087]
Single	0.012	-0.017	0.135	-0.169	0.244	-0.232
	[0.018]	[0.022]	[0.062]*	[0.081]*	[0.070]**	[0.096]*
Black	0.015	-0.126	0.539	0.101	0.483	0.276
	[0.028]	[0.062]*	[0.104]**	[0.228]	[0.107]**	[0.241]
Hispanic	0.013	-0.04	0.408	-0.107	0.357	-0.004
	[0.035]	[0.055]	[0.134]**	[0.223]	[0.139]*	[0.242]
Age	-0.002	-0.001	-0.011	0.011	-0.012	0.018
	[0.001]**	[0.001]	[0.003]**	[0.004]**	[0.003]**	[0.004]**
Years of education	0.004	-0.001	-0.04	-0.006	-0.033	-0.028
	[0.003]	[0.004]	[0.011]**	[0.015]	[0.012]**	[0.017]
P(economic recession) 2004-2006 average	-0.004	0.001				
	[0.000]**	[0.000]				
Depressive symptoms 2004-2006 average	-0.025	0.007				
	[0.008]**	[0.010]				
Ratio of fifty answers 2004-2006 average			1.673	-0.42		
			[0.249]**	[0.261]		
Ll	-42278.6					
N	9348					
Mean(μ)	-0.083					
Mean(s)	0.622					
Mean(Std(u))	0.355					
Var[R*]/Var[R]	0.652					
Rho(v0,vx-)	-0.535					
Rho(v0,vx+)	0.225					

Standard errors in brackets. * Significant at 5%; ** significant at 1%.

Stockholders are those who own any stock-market based assets (stocks, mutual funds etc.) either directly or through retirement accounts. Members of the same households are assigned the same stockholding status.

For more details see main text and footnotes to Table 4.4

Table C1.2.

Informed versus uninformed respondents. Date of interview in 2008 and average subjective expected value of yearly stock returns (μ), average subjective standard deviation (σ) and unobserved cross-sectional heterogeneity in expectations ($Std\,(u)$). Results from structural regressions including full interactions. HRS 2008.

	μ		log(s)		log[Std(u)]	
	coef	interaction with informed	coef	interaction with informed	coef	interaction with informed
Constant	-0.085	0.06	-0.588	-0.123	-1.084	-0.087
	[0.006]**	[0.012]**	[0.025]**	[0.050]*	[0.084]**	[0.054]
July 08 to Sep08	0.001	-0.004	0.106	0.005	0.165	-0.079
	[0.020]	[0.023]	[0.072]	[0.085]	[0.080]*	[0.097]
Oct 08 to Nov 08	0.136	-0.108	0.311	-0.08	0.528	-0.047
	[0.049]**	[0.056]	[0.186]	[0.210]	[0.192]**	[0.218]
Dec 08 to Feb 09	0.014	-0.064	-0.114	0.138	-0.101	0.582
	[0.054]	[0.068]	[0.211]	[0.256]	[0.225]	[0.276]*
Female	-0.096	0.058	0.166	0.128	0.107	0.081
	[0.016]**	[0.019]**	[0.063]**	[0.076]	[0.070]	[0.086]
Single	0.01	-0.029	0.111	-0.096	0.183	-0.058
	[0.018]	[0.022]	[0.069]	[0.084]	[0.075]*	[0.093]
Black	0.01	-0.077	0.458	0.221	0.461	0.244
	[0.032]	[0.048]	[0.122]**	[0.180]	[0.124]**	[0.184]
Hispanic	0.066	-0.158	0.179	0.347	0.173	0.311
	[0.032]*	[0.054]**	[0.135]	[0.211]	[0.140]	[0.219]
Age	-0.003	0.001	-0.011	0.01	-0.01	0.011
	[0.001]**	[0.001]	[0.003]**	[0.004]**	[0.003]**	[0.004]**
Years of education	0.006	0	-0.034	-0.018	-0.026	-0.046
	[0.003]	[0.004]	[0.011]**	[0.014]	[0.012]*	[0.016]**
P(economic recession)	-0.003	0				
2004-2006 average	[0.000]**	[0.000]				
Depressive symptoms	-0.022	0				
2004-2006 average	[0.008]**	[0.010]				
Ratio of fifty answers			1.634	-0.537		
2004-2006 average			[0.252]**	[0.230]*		
Ll	-42300.4					
N	9348					
Mean(μ)	-0.08					
Mean(s)	0.608					
Mean(Std(u)	0.376					
$Var[R^*]/Var[R]$	0.676					
Rho(v0,vx-)	-0.699					
Rho(v0,vx+)	0.138					

Standard errors in brackets. * Significant at 5%; ** significant at 1%.
Informed are those who claim to follow the stock market at least occasionally.
For more details see main text and footnotes to Table 4.4

Table C1.3.

Stockholders versus non-stockholders. Date of interview in 2008 and average subjective expected value of yearly stock returns (μ), average subjective standard deviation (σ) and unobserved cross-sectional heterogeneity in expectations ($Std\,(u)$). Results from structural regressions including full interactions. HRS 2008.

	μ		log(s)		log[Std(u)]	
	coef	interaction with cognition	coef	interaction with cognition	coef	interaction with cognition
Constant	-0.085	0.029	-0.6	-0.116	-1.175	-0.194
	[0.007]**	[0.013]*	[0.025]**	[0.051]*	[0.075]**	[0.060]**
July 08 to Sep08	-0.03	0.043	0.152	-0.056	0.169	-0.081
	[0.019]	[0.022]	[0.070]*	[0.084]	[0.079]*	[0.099]
Oct 08 to Nov 08	0.078	-0.023	0.286	0.003	0.531	0.037
	[0.049]	[0.056]	[0.185]	[0.211]	[0.186]**	[0.218]
Dec 08 to Feb 09	-0.027	0.005	-0.249	0.405	0.31	0.017
	[0.058]	[0.070]	[0.188]	[0.245]	[0.191]	[0.262]
Female	-0.097	0.04	0.2	0.129	0.1	0.155
	[0.017]**	[0.019]*	[0.063]**	[0.075]	[0.071]	[0.088]
Single	-0.016	0.013	0.128	-0.105	0.259	-0.189
	[0.019]	[0.022]	[0.069]	[0.084]	[0.077]**	[0.097]
Black	0.011	-0.113	0.485	0.285	0.457	0.336
	[0.030]	[0.053]*	[0.113]**	[0.199]	[0.115]**	[0.208]
Hispanic	0.024	-0.074	0.325	0.134	0.284	0.159
	[0.036]	[0.054]	[0.146]*	[0.214]	[0.151]	[0.228]
Age	-0.003	0.001	-0.008	0.005	-0.011	0.01
	[0.001]**	[0.001]	[0.003]**	[0.004]	[0.003]**	[0.004]*
Years of education	0.009	-0.003	-0.046	0.011	-0.037	-0.014
	[0.003]**	[0.004]	[0.011]**	[0.014]	[0.012]**	[0.017]
P(economic recession)	-0.003	0				
2004-2006 average	[0.000]**	[0.000]				
Depressive symptoms	-0.025	0.005				
2004-2006 average	[0.008]**	[0.010]				
Ratio of fifty answers			1.408	0.239		
2004-2006 average			[0.215]**	[0.268]		
Ll	-42319.7					
N	9348					
Mean(μ)	-0.083					
Mean(s)	0.617					
Mean(Std(u)	0.351					
Var[R*]/Var[R]	0.649					
Rho(v0,vx-)	-0.518					
Rho(v0,vx+)	0.235					

Standard errors in brackets. * Significant at 5%; ** significant at 1%.
Cognitive capacity is measured by a score from memory and numeracy tasks.
For more details see main text and footnotes to Table 4.4

C.2 Estimates based on alternative functional form assumptions

In this appendix we re-estimate our structural models with two alternative specifications for the return distribution, Student-t with various degrees of freedom and shifted log-normal.

Within the Student-t framework, the relation between the probability answers and the estimated parameters is the following:

$$P_{0i} = T\left(\frac{\mu_i}{\sigma_i}, df\right) \tag{114}$$

$$P_{x+i} = T\left(\frac{\mu_i - x/100}{\sigma_i}, df\right) \tag{115}$$

$$P_{x-i} = T\left(\frac{x/100 - \mu_i}{\sigma_i}, df\right) \tag{116}$$

where df is the degrees of freedom of the distribution. Note that since the t distribution is symmetric, μ can still be interpreted as the mean of the subjective distribution. However, σ will not be the subjective standard deviation anymore. According to the properties of the Student-t distribution, the true standard deviation will be

$$Std\left(R_i\right) = \sigma_i \sqrt{\frac{df}{df-2}} \tag{117}$$

Within the shifted log-normal framework, returns follow $R_i \sim \left[\ln N\left(\mu_i, \sigma_i^2\right) - 1\right]$. In this case the probability answers are:

$$P_{0i} = \Phi\left(\frac{\mu_i}{\sigma_i}\right) \tag{118}$$

$$P_{x+i} = \Phi\left(\frac{\mu_i - x/100}{\sigma_i}\right) \tag{119}$$

$$P_{x-i} = \Phi\left(\frac{x/100 - \mu_i}{\sigma_i}\right) \tag{120}$$

For small return realizations the normal and the shifted log-normal distributions are very similar. We, however, have quite high subjective variance, and thus a very large fraction of the return distribution is in a range where the true distributions are different. An important difference between the normal and the shifted lognormal model is that they have different moments. It can be shown that the first two moments of the shifted lognormal distribution are:

$$E\left[R_i\right] = \exp\left(\mu_i + \frac{\sigma_i^2}{2}\right) - 1 \tag{121}$$

$$V\left[R_i\right] = \left(\exp\left(\sigma_i^2\right) - 1\right)\left(\exp\left(\mu_i + \sigma_i^2\right)\right) \tag{122}$$

Because it is skewed, the mean of the distribution will be larger than the estimated μ. The potential asymmetry of the subjective distribution, thus, can be a reason for having small, usually negative values for μ.

As we can see, the parameter estimates are very much the same in all specifications, and the qualitative results do not change. All specifications agree that the crash brought a moderate increase in uncertainty and a huge increase in disagreement. The normal and the Student-t models agree that the mean of the subjective distribution was weakly positively affected by the crash. The log-normal model is also equivalent to the other models if we talk about log-returns instead of actual returns. As we saw earlier, actual returns are non-linear functions of μ and σ, and the moderate increase in σ must have a positive effect on μ_R. According to these estimates, the average μ_R increased after the crash from 16.5 percent to 72.2 percent, and the overall mean is 21.2 percent. This is clearly implausible.

We can also see that the likelihood is the highest in the original normal case and thus this model fits the data the best. While asymmetric models could explain how the mean of the subjective distribution can be positive even if the average P_0 value is less than 50, we can see that the log-normal model does not fit the data well. Further investigation is needed to test for the potential non-normality of the subjective return distribution

Table C2.1.

Stock returns assumed to be distributed Student-t with 3 degrees of freedom

Date of interview in 2008 and average subjective expected value of yearly stock returns (μ), average subjective standard deviation (σ) and unobserved cross-sectional heterogeneity in expectations $(Std\,(u))$. Results from structural regressions. HRS 2008.

	μ	log(s)	log(Std(u))
Constant	-0.090	-0.871	-1.165
	[0.006]**	[0.024]**	[0.074]**
July 08 to September 08	0.000	0.114	0.122
	[0.010]	[0.038]**	[0.046]**
October 08 to November 08	0.059	0.277	0.552
	[0.025]*	[0.090]**	[0.099]**
December 08 to February 09	-0.027	0.003	0.364
	[0.034]	[0.122]	[0.135]**
Female	-0.063	0.248	0.157
	[0.009]**	[0.035]**	[0.043]**
Single	0.004	0.040	0.118
	[0.010]	[0.040]	[0.048]*
Black	-0.017	0.592	0.562
	[0.026]	[0.095]**	[0.098]**
Hispanic	0.000	0.383	0.327
	[0.027]	[0.109]**	[0.114]**
Age	-0.002	-0.006	-0.004
	[0.000]**	[0.002]**	[0.002]
Years of education	0.001	-0.033	-0.032
	[0.002]	[0.007]**	[0.009]**
Above average cognition	0.031	-0.094	-0.195
	[0.010]**	[0.038]*	[0.047]**
Follow the stock market	0.048	-0.125	-0.075
	[0.010]**	[0.039]**	[0.046]
Stockholder	0.071	-0.047	-0.177
	[0.010]**	[0.039]	[0.049]**
P(economic recession)	-0.003		
2004-2006 average	[0.000]**		
Depressive symptoms	-0.018		
2004-2006 average	[0.005]**		
Ratio of fifty-fifty answers		1.545	
2004-2006 average		[0.196]**	
Log-likelihood	-43091.9		
N	9348		
Mean(μ)	-0.087		
Mean(s)	0.818		
Mean(Std(u)	0.364		
$s^2/(s^2+V(v))$	0.763		
Rho(v0,vx-)	-0.574		
Rho(v0,vx+)	0.193		

Robust standard errors in parentheses. * significant at 5%; ** significant at 1%

Reference categories: Interview date March to June, male, non-Black and non-Hispanic, married.

Table C2.2.

Stock returns assumed to be distributed Student-t with 10 degrees of freedom

Date of interview in 2008 and average subjective expected value of yearly stock returns (μ), average subjective standard deviation (σ) and unobserved cross-sectional heterogeneity in expectations $(Std\,(u))$. Results from structural regressions. HRS 2008.

	μ	log(s)	log(Std(u))
Constant	-0.088	-0.678	-1.213
	[0.006]**	[0.023]**	[0.077]**
July 08 to September 08	0.001	0.113	0.128
	[0.010]	[0.038]**	[0.047]**
October 08 to November 08	0.061	0.288	0.565
	[0.025]*	[0.088]**	[0.099]**
December 08 to February 09	-0.028	0.015	0.376
	[0.033]	[0.120]	[0.135]**
Female	-0.062	0.239	0.149
	[0.009]**	[0.034]**	[0.043]**
Single	0.004	0.040	0.120
	[0.010]	[0.039]	[0.048]*
Black	-0.017	0.590	0.560
	[0.025]	[0.093]**	[0.097]**
Hispanic	0.001	0.386	0.330
	[0.027]	[0.107]**	[0.114]**
Age	-0.002	-0.005	-0.004
	[0.000]**	[0.002]**	[0.002]
Years of education	0.002	-0.034	-0.031
	[0.002]	[0.007]**	[0.009]**
Above average cognition	0.031	-0.099	-0.197
	[0.010]**	[0.038]**	[0.048]**
Follow the stock market	0.049	-0.128	-0.073
	[0.010]**	[0.038]**	[0.046]
Stockholder	0.071	-0.055	-0.180
	[0.010]**	[0.038]	[0.049]**
P(economic recession)	-0.003		
2004-2006 average	[0.000]**		
Depressive symptoms	-0.017		
2004-2006 average	[0.005]**		
Ratio of fifty-fifty answers		1.522	
2004-2006 average		[0.192]**	
Log-likelihood	-42425.5		
N	9348		
Mean(μ)	-0.085		
Mean(s)	0.641		
Mean(Std(u)	0.348		
$s^2/(s^2+V(v))$	0.663		
Rho(v0,vx-)	-0.512		
Rho(v0,vx+)	0.238		

Robust standard errors in parentheses. * significant at 5%; ** significant at 1%

Reference categories: Interview date March to June, male, non-Black and non-Hispanic, married.

Table C2.3.
Stock returns assumed to be distributed log-normal with parameters μ and σ (which, in this case, are different from the mean and the standard deviation, respectively) Date of interview in 2008 and average subjective expected value of yearly stock returns (μ), average subjective standard deviation (σ) and unobserved cross-sectional heterogeneity in expectations $(Std\,(u))$. Results from structural regressions. HRS 2008.

	μ	log(s)	log(Std(u))
Constant	-0.108	-0.565	-1.185
	[0.006]**	[0.023]**	[0.078]**
July 08 to September 08	0.000	0.098	0.116
	[0.010]	[0.038]*	[0.047]*
October 08 to November 08	0.065	0.296	0.572
	[0.026]*	[0.093]**	[0.103]**
December 08 to February 09	-0.027	0.018	0.379
	[0.035]	[0.124]	[0.139]**
Female	-0.067	0.234	0.145
	[0.009]**	[0.035]**	[0.044]**
Single	0.005	0.044	0.127
	[0.011]	[0.040]	[0.049]**
Black	-0.013	0.554	0.527
	[0.025]	[0.092]**	[0.094]**
Hispanic	0.005	0.382	0.328
	[0.028]	[0.109]**	[0.115]**
Age	-0.002	-0.006	-0.005
	[0.000]**	[0.002]**	[0.002]*
Years of education	0.002	-0.031	-0.029
	[0.002]	[0.007]**	[0.009]**
Above average cognition	0.032	-0.081	-0.179
	[0.010]**	[0.038]*	[0.048]**
Follow the stock market	0.050	-0.101	-0.046
	[0.010]**	[0.038]**	[0.047]
Stockholder	0.074	-0.054	-0.177
	[0.010]**	[0.039]	[0.050]**
P(economic recession)	-0.003		
2004-2006 average	[0.000]**		
Depressive symptoms	-0.018		
2004-2006 average	[0.005]**		
Ratio of fifty-fifty answers		1.521	
2004-2006 average		[0.196]**	
Log-likelihood	-42360.2		
N	9348		
Mean(μ)	0.210		
Mean(s)	9.017		
Mean(Std(u)	0.355		
$s^2/(s^2+V(v))$	0.644		
Rho(v0,vx-)	-0.484		
Rho(v0,vx+)	0.231		

Robust standard errors in parentheses. * significant at 5%; ** significant at 1%
Reference categories: Interview date March to June, male, non-Black and non-Hispanic, married.

References

[1] Agarwal, Sumit, John C. Driscoll, Xavier Gabaix and David I. Laibson (2009). "The Age of Reason: Financial Decisions over the Life-Cycle with Implications for Regulation," Brookings Papers on Economic Activity, Fall 2009, 51-101.

[2] Ameriks, John, and Stephen P. Zeldes (2004), "How Do Household Portfolio Shares Vary With Age? " *Working Paper, Columbia Business School.*

[3] Amromin, Gene and Steven A. Sharpe (2006), "From the horse's mouth: Gauging conditional expected stock returns from investor surveys." *Working paper, Federal Reserve Bank of Chicago.*

[4] Anderson, Michael L. (2008): "Multiple Inference and Gender Differences in the Effects of Early Intervention: A Reevaluation of the Abecedarian, Perry Preschool, and Early Training Projects." *Journal of the American Statistical Association*, 103:484, 1481-1495

[5] Anderson, Jon, Stephen Burks, Colin DeYoung and Aldo Rustichini (2011), "Toward the Integration of Personality Theory and Decision Theory in the Explanation of Economic Behavior." *Working Paper, University of Minnesota.*

[6] Anderson, Lisa and Jennifer Mellor (2009) "Are risk preferences stable? Comparing an experimental measure with a validated survey-based measure," *Journal of Risk and Uncertainty*, 39(2), 137-160.

[7] Barsky, Robert B, F. Thomas Juster, Miles S. Kimball and Matthew D. Shapiro. (1997), "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study." *The Quarterly Journal of Economics,* 112, 567-579.

[8] Bassett, William F. and Robin L. Lumsdaine (1999), "Outlook, Outcomes and Optimism." *Unpublished manuscript.*

[9] Becker, Anke, Thomas Deckers, Thomas Dohmen, Armin Falk and Fabian Kosse (2012), "The Relationship Between Economic Preferences and Psychological Personality Measures." *IZA Discussion Paper* 6470.

[10] Becker, Gary S. (1964, 1993, 3rd ed.), *Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education.* Chicago, University of Chicago Press.

[11] Brennan, Michael (1998), "The role of learning in dynamic portfolio decisions." *European Finance Review*, 1:295–306.

[12] Bruine de Bruin, Wandi and C. G. Carman. (2011), "Measuring risk perceptions: What does the excessive use of 50% mean?" *Medical Decision Making*, 32(2), 232-236.

[13] Calvet, Laurent E., John Y. Campbell and Paolo Sodini (2007) "Down or Out: Assessing the Welfare Costs of Household Investment Mistakes." *Journal of Political Economy*, 115, 707-747.

[14] Calvet, Laurent E., John Y. Campbell and Paolo Sodini (2009a) "Fight or Flight? Portfolio Rebalancing by Individual Investors." *Quarterly Journal of Economics,* February 2009, 301-348.

[15] Calvet, Laurent E., John Y. Campbell and Paolo Sodini (2009b) "Household Heterogeneity in Financial Markets – Measuring the Financial Sophistication of Households." *American Economic Review*, 99:2, 393-398.

[16] Campbell, John Y. (2006), "Household Finance." *The Journal of Finance* 61(4), 1553-1604.

[17] Card, David (1999), "The causal effect of education on earnings," in: O. Ashenfelter & D. Card (ed.), *Handbook of Labor Economics*, 3, chapter 30, 1801-1863. Elsevier.

[18] Christelis, Dimitrios, Tullio Jappelli and Mario Padula (2010), "Cognitive Abilities and Portfolio Choice." *European Economic Review*, 54(1), 18-38.

[19] Darmodaran, Aswhat (2012) "Equity Risk Premiums (ERP): Determinants, Estimation and Implications – The 2012 Edition." *Working Paper, New York University Stern School of Business.*

[20] Dave, Chetan, Catherine Eckel, Cathleen Johnson and Christian Rojas (2010), "Eliciting risk preferences: When is simple better?" *Journal of Risk and Uncertainty*, 41(3), 219-243.

[21] Delavande A., S. Rohwedder and R. J. Willis (2008), "Preparation for Retirement, Financial Literacy and Cognitive Resources," *Michigan Retirement Research Center Working Paper* No. 2008-190.

[22] Dohmen, Thomas, Armin Falk, David Huffman and Uwe Sunde (2010). "Are Risk Aversion and Impatience Related to Cognitive Ability?," *American Economic Review*, 100(3), 1238-60.

[23] Dominitz, J. and C. Manski (2005), "Measuring and Interpreting Expectations of Equity Returns" *NBER Working Paper*, 11313.

[24] Dominitz, J. and C. Manski (2007), "Expected Equity Returns and Portfolio Choice: Evidence from the Health and Retirement Study" *Journal of the European Economic Association*, 5(2-3), 369-379.

[25] Dominitz, J. and C. F. Manski (2011) "Measuring and Interpreting Expectations of Equity Returns." *Journal of Applied Econometrics*, 26(3), 352-370.

[26] Duckworth, Angela Lee , and David R. Weir (2010), "Personality, Lifetime Earnings, and Retirement Wealth." *Michigan Retirement Research Center Working Paper* 235.

[27] Ellsberg, Daniel (1961). "Risk, Ambiguity, and the Savage Axioms". *Quarterly Journal of Economics* 75(4): 643–669.

[28] Fischhoff, B . and W. Bruine de Bruin (1999), "Fifty Fifty = 50%?" *Journal of Behavioral Decision Making*, 12, 149 163

[29] Gigerenzer, G. (2008). *Rationality for mortals: How people cope with uncertainty.* New York: Oxford University Press.

[30] Gilboa, Itzhak (2010), "Questions in Decision Theory." *Annual Review of Economics* 2010:2, 1-19.

[31] Gouret, F. and Hollard, G. (2011) "When Kahneman Meets Manski: Using Dual Systems of Reasoning to Interpret Subjective Expectations of Equity Returns" *Journal of Applied Econometrics*, 26(3), 371-392.

[32] Grinblatt, Mark Matti Keloharju and Juhani Linnainmaa (2011)," IQ and Stock Market Participation." *The Journal of Finance*, 66(6), 2121-2164.

[33] Guiso, Luigi, Michalis Haliassos, Tullio Jappelli (2002), *Household Portfolios.* Cambridge: MIT Press.

[34] Guiso, Luigi and Tullio Jappelli (2009), "Financial Literacy and Portfolio Diversification." *CSEF Working Paper.*

[35] Guiso, Luigi, Paola Sapienza, and Luigi Zingales (2004), "The Role of Social Capital in Financial Development," *American Economic Review* 94, 526-556.

[36] Haliassos, Michalis, and Carol Bertaut (1995), "Why Do So Few Hold Stocks?" *The Economic Journal.* 195, 1110-29.

[37] Haliassos, Michael, and Alexander Michaelides (2002), "Calibration and Computation of Household Portfolio Models." In: Luigi Guiso, Michael Haliassos and Tulio Japelli, eds., *Household Portfolios.* Cambridge: M.I.T. Press

[38] Harris, M., and A. Raviv (1993). "Differences of Opinion Make a Horse Race." *Review of Financial Studies*, 6, 473-506

[39] Heckman, James J. and Burton S. Singer (1984), "A Method for Minimizing the Impact of Distributional Assumptions in Econometric Models for Duration Data." *Econometrica*, 1984, 52(2), pp. 271-320.

[40] Hill, Daniel, M. Perry and R. J. Willis (2005), "Estimating Knightian Uncertainty from Probability Questions on the HRS." *Presented at World Congress of the Econometric Society*, London, August 19-24, 2005.

[41] Hong, Harrison, Jeffrey D. Kubik, and Jeremy C. Stein (2004), "Social Interaction and Stock Market Participation," *Journal of Finance,* 59, 137-63.

[42] Hong, Harrison and Jeremy C. Stein (2007), "Disagreement and the Stock Market." *Journal of Economic Perspectives*, 21(2), 109-128.

[43] Horn, J. L., and McArdle, J. J. (2007), "Understanding Human Intelligence since Spearman." In: Robert Cudeck and Robert C. MacCallum, eds., *Factor Analysis at 100. Historical Developments and Future Directions.* LEA Publishers.

[44] Hudomiet, P, G. Kezdi and R. J. Willis (2011), "Stock Market Crash and Expectations of American Households", *Journal of Applied Econometrics*, 26(3), 393-415.

[45] Hurd, M. (2009), "Subjective Probabilities in Household Surveys." *Annual Review of Economics* 2009(1), 543-62.

[46] Hurd, M. and K. McGarry (1995), "Evaluation of the Subjective Probabilities of Survival in the Health and Retirement Study," *Journal of Human Resources*, 30, S268-S292.

[47] Hurd, M., S. Rohwedder and J. Winter (2005), "Subjective Probabilities of Survival: An International Comparison," *RAND manuscript.*

[48] Hurd, M, M. van Rooij, and J. Winter (2011) "Stock market expectations of Dutch households." *Journal of Applied Econometrics*, Vol. 26, No. 3, 2011, pp. 416-436.

[49] JUSTER, T. (1966), *Consumer Buying Intentions and Purchase Probability: An Experiment in Survey Design.* NBER Occasional Paper 99.

[50] JUSTER, T. AND R. SUZMAN (1995), "An Overview of the Health and Retirement Study," *Journal of Human Resources*, 30, S7-S56.

[51] KANDEL, E., AND N. D. PEARSON (1995). "Differential Interpretation of Public Signals and Trade in Speculative Markets." *Journal of Political Economy*, 103, 831-872

[52] KÉZDI, G., AND R. J. WILLIS (2003) "Who Becomes a Stockholder? Expectations, Subjective Uncertainty, and Asset Allocation." *Michigan Retirement Research Center Working Paper* 039.

[53] KÉZDI, G., AND R. J. WILLIS (2008) "Stock Market Expectations and Portfolio Choice of American Households." *University of Michigan Workign Paper.*

[54] KÉZDI, G., AND R. J. WILLIS (2010) "Household Stock Market Beliefs and Learning." *NBER Working Paper* 17614

[55] KIMBALL, MILES S. AND TYLER SHUMWAY (2010). "Investor Sophistication and the Home Bias, Diversification, and Employer Stock Puzzles." *University of Michigan Workign Paper.*

[56] KIMBALL, M. S., C. R. SAHM AND M. D. SHAPIRO (2008), "Imputing Risk Tolerance from Survey Responses." *Journal of the American Statistical Association* 103 (Sept 2008) 1028-1038

[57] KOCHERLAKOTA, N. R. (1996), "The Equity Premium: It's Still a Puzzle," *Journal of Economic Literature*, 34(1), 42-47.

[58] KONDOR, PETER (2012), "The more we know about the fundamental, the less we agree on the price." *Review of Economic Studies*, 79(3), 1175-1207.

[59] LILLARD, L. A. AND ROBERT J. WILLIS (2001), "Cognition and Wealth: The Importance of Probabilistic Thinking," *Michigan Retirement Research Center Working Paper* UM00-04.

[60] LUCAS, ROBERT E. JR. (1976), " Econometric Policy Evaluation: A Critique," *Carnegie-Rochester Conference Series on Public Policy* 1 (1): 19-46.

[61] LUSARDI, ANNAMARIA AND OLIVIA S. MITCHELL (2008)."Planning and Financial Literacy: How Do Women Fare?," *American Economic Review*, vol. 98(2), 413-17.

[62] Machina Mark J. (2008), "Non-expected utility theory." In: Steven N. Durlauf and Lawrence E. Blume, eds., *The New Palgrave Dictionary of Economics*, Second Edition, 2008 http://www.dictionaryofeconomics.com/article?id=pde2008_N000081

[63] Malloy, Christopher J., Tobias J. Moskowitz, and Annette Vissing-Jorgensen (2008), " Long-Run Stockholder Consumption Risk and Asset Returns," *Journal of Finance,* 64(6), 2427-2479.

[64] Mankiw, N. Gregory and Zeldes, Stephen P. (1991), "The consumption of stockholders and nonstockholders," *Journal of Financial Economics*, 29(1), pages 97-112.

[65] Manski, Charles F. (2004), "Measuring Expectations," *Econometrica*, 72, 1329-1376.

[66] Manski, Charles F. and Franceska Molinari (2009). "Rounding Probabilistic Expectations in Surveys," *Journal of Business and Economic Statistics*, 28(2), 219-231.

[67] McArdle, John J., Fisher, Gwenith G., Kadlec, Kelly M. (2007) "Latent Variable Analyses of Age Trends of Cognition in the Health and Retirement Study, 1992-2004" *Psychology and Aging.* 22:3 p.525-545

[68] Mehra, Rajnish and Edward C. Prescott (1985), "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15, 145-161.

[69] Merton, Robert C. (1969), "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case," *The Review of Economics and Statistics*, 51-3, 247-257.

[70] National Institute of Aging. (2007), *Growing Old in America. The Health and Retirement Study.* NIH Publications 07- 5757, Washington D.C.

[71] Olsen, Anya and Russell Hudson (2009), "Social Security Administration's Master Earnings File: Background Information." *Social Security Bulletin.* 69(3).

[72] Poterba, J., J. Rauh, S. Venti and D. Wise (2005), "Utility Evaluation of Risk in Retirement Saving Accounts." In David Wise, ed., *Analyses in the Economics of Aging.* University of Chicago Press.

[73] Pounder, Laurie (2007) *Life Cycle Consumption Examined.* Ph.D. Thesis, Department of Economics, University of Michigan.

[74] Ricther, Marcel K. (2008), "Revealed Preferences Theory." In: Steven N. Durlauf and Lawrence E. Blume, eds., *The New Palgrave Dictionary of Economics*, Second Edition, 2008. http://www.dictionaryofeconomics.com/article?id=pde2008_R000135

[75] Rosen, Harvey S. and Stephen Wu (2004), "Portfolio Choice and Health Status," *Journal of Financial Economics,* 72, 457-84.

[76] Sahm, Claudia R. (2007), "Stability of Risk Preference." Federal Reserve Board Working Paper 2007-66.

[77] Scheier, Michael F. and Charles S. Carver (1985), "Optimism, coping, and health: Assessment and implications of generalized outcome expectancies." *Health Psychology*, Vol 4(3), 1985, 219-247.

[78] Scheier, Michael F., Charels S. Carver, and M. V. Bridges (1994). "Distinguishing optimism from neuroticism (and trait anxiety, self-mastery, and self-esteem): A reevaluation of the Life Orientation Test." *Journal of Personality and Social Psychology*, 67(6), 1063-1078.

[79] Seligman, Martin E. P. and Mihaly Csikszentmihalyi (2010), "Positive Psychology – An Introduction." *American Psychologist*, 55(1), 5-14.

[80] Sharpe, J. Patrick , Nicholas R. Martin and Kelly A. Roth (2011), "Optimism and the Big Five factors of personality: Beyond Neuroticism and Extraversion." *Personality and Individual Differences*, 51(8), 946–951.

[81] Tversky, Amos and Daniel Kahneman (1974). "Judgment under uncertainty: Heuristics and biases". *Science,* 185 (4157): 1124–1131.

[82] van Rooij Maarten, Annamaria Lusardi and Rob Alessie (2011), "Financial literacy and stock market participation." *Journal of Financial Economics*, 101, 449-472.

[83] Veldkamp, Laura (2011), *Information Choice in Macroeconomics and Finance.* Princeton University Press.

[84] Viceira, L.M. (2001), "Optimal portfolio choice for long-horizon investors with nontradeable labor income," *Journal of Finance*, 56(2), 433-470.

[85] Vissing-Jorgensen, Annette (2004), "Perspectives on Behavioral Finance: Does Irrationality Disappear with Wealth? Evidence from Expectations and Actions," in: M. Gertler and K. Rogoff eds., *The NBER Macroeconomics Annual 2003*, Cambridge: MIT Press.

[86] Weitzman, Martin L. (2007), "Subjective Expectations and Asset-Returns Puzzles." *American Economic Review.* 97(4): 1102–1130.

[87] WILLIS, ROBERT J. (1986) "Wage Determinants: A Survey and Reinterpretation of Human Capital Earnings Functions" In O. Ashenfelter and R. Layard, eds. *Handbook of Labor Economics*, (Amsterdam: North Holland, 1987): 525 602.