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**Review of Dr. Robert Fullér's dissertation "Multicriteria
Decision Models with Imprecise Information" submitted
with respect to his application to be granted the Doctor of
Science degree at the Hungarian Academy of Sciences**

This review has been written in response to the request of Professor László Amand Palkovics, Secretary of the Doctoral Council in the Hungarian Academy of Sciences to provide my opinion on Dr. Robert Fullér's dissertation "Multicriteria Decision Models with Imprecise Information" published in Budapest in 2014, and its summary (thesis).

First, I wish to reconfirm the following. In addition to the attached work by the Candidate, I have been familiar with both him and his research work for a long time as I have been reading his articles and listened to his conference talks for probably some 20-25 years. I have also had a chance to learn about a very positive opinion about him and his works by the international fuzzy systems community during my Presidency of IFSA (International Fuzzy Systems Association) and as a member of the Administrative Committee of the Computational Intelligence Society, Institute of Electronics and Electrical Engineers (IEEE). As a frequent reviewer and member of various commissions, I am fully familiar with formal and customary requirements for the D.Sc. and/or "habilitation" degree in all countries in which they exist, notably in the Central and Eastern Europe, France, Germany, etc. Moreover, as a member of the Polish National Testimonial Commission (CK) which supervises the granting of Ph.D.'s, D.Sc./habilitations and professorships, I know in detail requirements, procedures, standards, etc. which are practically the same as in Hungary.

Finally, I wish to reconfirm that I know well and fully understand the Hungarian system of universities and institutes of the Hungarian Academy of Sciences because, first, I have had contacts with those institutes of the Academy since my first visits in ca. the mid-1970s. Over the last decades I have been collaborating closely with many Hungarian scientists, and those contacts have culminated in 2014 – what I am particularly honored and proud of – the Doctorate Honoris Causa from the Széchenyi István University in Győr.

Before commenting upon the particular aspects of Dr. Robert Fullér’s scientific results, I wish to add some more general comments. They are, in my opinion, important in view of the fact that he is applying for the D.Sc. degree in mathematics, clearly applied mathematics, specifically the broadly perceived operations research. This aspect is related to the education and formation that has always an impact on the future career of a scientist. First, one should emphasize that Dr. Robert Fullér is a mathematician who graduated from the Eötvös Loránd University in Budapest, one of the grand European universities, one of Europe’s best known centers of mathematics that cultivates the legacy of the famous Hungarian school of mathematics. Such an environment can very well form a young adept, and it has done this in the case of the Candidate. Then, he spent some time at another famous center of mathematics, at the Lomonosov University in Moscow. So, I am totally convinced that he has gained a first class knowledge in mathematics, though maybe initially in its more theoretical direction.

Then, as far as I remember, in the early 1990s, he was invited to join a world famous group of Professor Christer Carlsson at the Abo Akademi (Swedish) University in Turku (Abo), Finland. I had known Professor Carlsson since the late 1970s or early 1980s because he was very active in EURO, the Federation of the European Operational Research Societies. Professor Carlsson was a real “applied mathematician” (though I am not sure if he was trained as a mathematician) who was basically interested at that time in multicriteria decision making, and its industrial and business applications, more generally, operations research. What was important for me, and probably for all of us from the Eastern and Central Europe, was that – first – Professor Carlsson was an “applied mathematician”, because there was no “applied mathematics”, as a separate area of mathematics, in our part of Europe though, for instance, Professor Hugo Steinhaus, who “discovered” our greatest mathematician, Stefan Banach, was trying hard to convince the Ministry of Higher Education and the Polish Academy of Sciences that applied mathematics was crucial and a “real mathematics”, and also more and more professors of mathematics who came back from the USA started to mention the power of SIAM, Society of Industrial and Applied Mathematics, and applied mathematics as an area.

Professor Carlsson has been very influential for Dr. Robert Fullér. But also more generally for the entire fuzzy community. Namely, he has managed to maintain a difficult balance in the sense that he has always tried to solve real problems but using high level mathematical tools, complemented with information technology (IT). Moreover, he has always had extremely good contacts with Finnish companies who have been interested in the use of sophisticated mathematical and computer tools, exemplified by Nokia, extremely rich and powerful at that time, many Finnish paper and pulp companies, machinery manufacturers, banks and investment companies, etc. This was a real scientific work for “real money”, with and for companies interested in new technologies and solutions, etc. something we, in our countries, rarely have. So, Dr. Robert Fullér was a perfect candidate for

Professor Carlsson's team because he was, on the one hand, a very good mathematician, maybe at that time with some theoretical inclination but ready to get into the applied field, and some "chemistry about the two people had happened. By the way, in my opinion Professor Carlsson was somehow similar to Professor John Ockendon from Oxford University, a famous applied mathematician who had practically founded, at least in Europe, the so called "industrial mathematics", and initiated both the more academic OCCIAM (Oxford Centre for Industrial and Applied Mathematics) and OCCAM (Oxford Centre for Collaborative Applied Mathematics) and the more commercial Smith Institute for Industrial Mathematics and System Engineering.

Those remarks are, in my opinion, important to properly evaluate the academic achievements of Dr. Robert Fullér. I will therefore try to show that his works can be seen as an example of that new wave in science. That is, to stay in the context of mathematics, his works can be surely considered as part of mathematics, but applied, and many of his works, notably those written while working part time in Finland in Professor Carlsson's serious Finnish and European projects, can be considered to belong to operations research, one of the most important parts of applied mathematics with roots in the foundation in the late 1930s of research groups of eminent mathematicians, statisticians, physicists, etc. in the UK and USA who have greatly helped the Allies during World War II, and then the field had flourished and became now an extremely important field of applied mathematics, decision sciences, management science, etc.

Now, I will comment upon the particular elements listed in the instructions for the review provided by the Secretariat.

First, I will comment upon Dr. Robert Fullér's D.Sc. dissertation on "Multicriteria decision models with imprecise information" which subsumes his best known works. The contents and composition of the dissertation is interesting, slightly unorthodox and original, but – as I will try to show – is well justified.

Chapter 1, "Introduction", is very important because it sets the stage, that is, indicates the class of problems to be discussed, and why the author has selected the presentation of the particular tools and techniques.

Chapter 2 "Preliminaries" is maybe standard as it presents the essence of fuzzy sets theory, fuzzy implications and approximate reasoning. This is well done and pretty standard, so that I will not comment upon this chapter leaving comments to the evaluation of the next chapters.

Chapter 3 "OWA operators in multiple criteria decisions" is a very important chapter the results of which are used in many later works of the candidate.

To start comments about this chapter, one should mention the first sacramental question whether the problem considered in the dissertation is clear, important and not trivial. Basically, the author is concerned with the broadly perceived multicriteria decision making or multiobjective optimization problem which boils down to the determination of an optimal solution (values of decision variables, usually from the real line) which "best" satisfy a number of criteria which are usually contradictory. Obviously, from the purely mathematical point of view, this problem has no solution and some "trickery" has to be applied. Usually, a set of Pareto optimal solutions is determined, which may be very

large, and then from among those solutions some best one(s) are determined by, for instance, using some aggregation, interaction with the human decision maker, etc.

Another aspect that makes the problem considered attractive for research is the assumption the information related to the problem is imprecise, not in a probabilistic sense of being uncertain, but mainly resulting from the use of inherently imprecise natural language descriptions of the values of variables and criteria, the very essence of what is a best (optimal) solution, etc. Zadeh's fuzzy sets theory is presumably the best, simple and efficient, apparatus that make it possible to formally and computationally handle such situations in a relatively simple and natural way, and the author has properly chosen it.

Dr. Fullér has been mostly involved in the use of the aggregation approach, that is, the partial scores resulting from the fulfilment of the particular criteria have been first assigned weights and then aggregated. Finally, a best solution has to be chosen as that which has the highest value of that aggregated score.

Usually, for mathematical simplicity, some weighted average was employed which could well be used in optimization. Another popular choice was the use of the minimum operation which reflected a pessimistic or safety-first attitude, or the maximum operation which reflected the opposite, optimistic attitude, but they implied obviously some problems in optimization due to non-differentiability. A popular choice is also the algebraic product. Notice that the use of the weighted average and algebraic product are related to the use of an additive and multiplicative multiattribute utility function which are popular in decision analyses. The other types of aggregation are less obvious from the decision theoretic point of view and maybe I would be glad to find some comment about this decision theoretic relation in the dissertation.

The candidate has chosen the so-called OWA (ordered weighted average) operators introduced by Professor Yager. They are simple and ingenious aggregation operators in which, first, the set of values to be aggregated is rank ordered from the largest to the smallest, and then the weights are applied to those rank ordered values, not to the values themselves. By a proper choice of the weights one can obtain the aggregation from the minimum to the maximum, through the algebraic product and many other operators. These operators, introduced in 1988 by Yager, have since then experienced a huge popularity, and have been presented in full in our volume: Yager and Kacprzyk (1997) and Yager, Kacprzyk and Beliakov (2012).

To properly view the importance of Dr. Fullér's results in this context, one should take into account that Professor Yager is not a mathematician, but an engineer, and his ideas have been more intuitive than formal although in no case one can accuse him of formal errors. Dr. Fullér has therefore extended the basic idea of the OWA operator using, first, some formal analyses, and as a result has proposed some new types of the OWA operators, for instance the so-called window OWA operators. However, I would see as his great achievement the development of some new approaches to analytically solve, by using the Lagrange multipliers and then constrained optimization, the fundamental problem of how to find proper weights of the OWA operators. A series of works on this topic, coauthored with Majlender and Carlsson, has been influential and highly cited, and also employed in real applications. His works with Carlsson on more general issue of weighted aggregation using possibility theory to handle benchmarks and ratings are also highly appreciated worldwide.

The last issue considered in this chapter, that is optimization with linguistic variables, is very interesting and useful. Basically, it is concerned with a highly original and novel formulation of fuzzy mathematical programming problem proposed in 2000 by Fullér and Carlsson in which the concept of fairness has been employed in the sense of using a rule base to model the objective functions. A solution that somehow treats the objective functions in a fair way, satisfying some constraints, is then obtained by solving a nonlinear programming problem. I like very much this paper for the following reason. Fuzzy optimization, or maybe better to say fuzzy mathematical programming, was practically first proposed by Tanaka and Okuda in the very early 1980s, or maybe even by Orlovski, though in a slightly different setting, in the very late 1970s, and the first really constructive formulation was given by Zimmermann in 1976, followed by many problem formulations with fuzzy coefficients in the objective function, fuzzy constraints, fuzzy right-hand-sides, etc. etc. In our first book on fuzzy optimization, Kacprzyk and Orlovski from 1987, we presented a full view of the area. However, all those methods followed the traditional “egoistic” attitude of the maximization of an objective (utility) function, under some constraints. In recent years, however, notably in the context of multicriteria optimization, meant in terms of resource allocation, there is much research on a fair distribution of resources, which results in new problem formulations in terms of the so-called equitable optimization (cf. Hanan Luss’ book “Equitable Resource Allocation: Models, Algorithms, and Applications”, Wiley, 2012). Fullér and Calsson’s works are close in spirit to those new approaches and they are real pioneers in the field of fuzzy optimization in that equitable direction. Maybe I would gladly see some reference to that fairness and equitable optimization in their works. Moreover, maybe a better place for this part of Chapter 3 would be in Chapter 4 but I would not insist.

Chapter 4 “Stability in fuzzy systems” is extremely important. First, let me start with some small (but really small) criticism. The title may be misleading. First, for a person of Dr. Fullér’s background, that is applied mathematics, more specifically operations research, but even more specifically optimization, the title can be justified with respect to “stability” though maybe less with respect to “fuzzy systems”. Basically, he deals with the stability (sensitivity) of optimal solutions of some fuzzy mathematical programming problems to changes of parameters exemplified by changes to coefficients. However, even to applied mathematicians but those who work in broadly perceived systems theory/analysis or control theory, stability of fuzzy systems could mean something different, namely they would consider fuzzy systems as not necessarily sets of (fuzzy) equations or optimization problems, but “real” systems with inputs, outputs, disturbances, controls, etc. This remark is not very serious, just to indicate a possibility of misunderstanding.

In general, the particular sections of this chapter, notably those closely related to fuzzy optimization and dealing with the stability of optimal solutions are very interesting and important from both the theoretical and practical point of view. After the first section, that is a point of departure for the next sections, in which the stability of a system of fuzzy equalities is considered and Kovács’ method is described, the next parts are co-authored by Dr. Robert Fullér, starting with the stability of solutions of ordinary fuzzy linear programming to fuzzy nonlinear programming problems. As to the latter, the authors have limited their analysis to fuzzy quadratic programming which is clearly justified from the point of view of a need for constructive results because, in my opinion, it would be very difficult, if not impossible at all, to obtain similar results for general nonlinear

programming problems. The last part, a relatively simple extension to multiobjective possibilistic linear programming, is very good too as it includes a non-trivial analyses and yet applicable results.

To summarize, I like this chapter very much, maybe because of my personal interests too, but also because the candidate is “at home” for sure in the optimization field. The results obtained by him are very significant and have a high applicability.

Chapter 5 “A normative view of possibility distribution” is very important too. Namely, we can view the former chapters as dealing with imprecise information, so that fuzzy sets theory and possibility theory could be a very good, if not the best choice. One can try to extend to the fuzzy/possibilistic setting some well-known concepts from the traditional probability theory, exemplified by lower and upper possibilistic mean values, marginal possibility distributions, variance, correlation, etc. Also in this field, Dr. Robert Fullér has published very important publications. Maybe I would welcome in this place some comments about approaches to deal with a joint occurrence of fuzziness and randomness though this is maybe somehow outside of the scope of his interests.

Chapter 6 “Operations on interactive fuzzy numbers” is very interesting. It concerns the case when we have fuzzy numbers and their joint possibility distribution is defined by a triangular norm like the minimum, algebraic product, Łukasiewicz, or some parametric one exemplified by those of Yager, Hamacher, Weber, etc. The results of Fullér, and Fullér and Keresztfalvi are here very interesting and have an application potential.

Chapter 7 “Selected industrial applications” is, on the one hand, very interesting, even if the author could not include all details, for obvious reasons as the works have been done for real business partners, and – on the other hand – very relevant from a more general point of view. Namely, as I have already mentioned, Dr. Robert Fullér has had an opportunity to work over some decades in Finland within a novel system in the academia, which has never been present in the Central and Eastern Europe, but which seems to become soon a norm in the entire Europe, i.e. that virtually all research and scholarly institutions will soon have to secure additional financial means, from outside sources (both from various EU programs and business) to cover all needs exceeding bare necessities.

Those conditions have implied that he, as a member of Professor Carlsson’s group, has participated at serious works for business and industrial customers in which non-trivial analyses and a strong application orientation has been combined.

I would cite in this context works on various computerized decision support systems, the so-called knowledge mobilization (the project started many years ago when mobile devices like smartphones have not yet been so developed like now), use of ontologies in information retrieval, financial engineering, notably real option pricing, grid computing, scheduling research and development projects, etc. These works are very valuable, maybe even extremely good examples of how applied mathematics could be used to solve real problems, for real money, to meet real needs.

To summarize my assessment of the dissertation, I would say that it is logically structured, presents the problems considered in Dr. Robert Fullér's works in a clear way, points out correctly main difficulties and their original solutions by the author, and can be very useful both for applied mathematicians interested in more theoretical issues and practitioners. My only suggestions have been that maybe it would be good to clarify some issues and indicate relations to other well established results in other fields of science exemplified by decision theory/analysis.

Now, I will proceed to more detailed comments following the instruction sent by the Doctoral Council of the Hungarian Academy of Sciences.

I will start with my responses to the more general questions:

- Dr. Robert Fullér has an internationally recognized PhD,
- his scientific activities are internationally known and highly appreciated as:
 - his number of citation is very high: due to WoS: citations – 1389, h-index – 19, Google Scholar: 5085, h-index - 34,
 - he has been awarded with the prestigious title of Finland's Distinguished Professor for 2007-2011,
- his research record is very rich and valuable, notably in the areas of: aggregation operators (in particular the OWA operators, generalized means, etc.), various novel extensions of fuzzy optimization and mathematical programming problems, notably multiobjective ones; these are real, worldwide recognized contributions to the field, moreover, optimization models are presumably key models for all kinds of applications,
- he has actively participated, notably during his stay in Finland, in high level applied research combining sophisticated mathematical analyses with real applications, and his publications on those topics, even if probably with somehow limited details, are extremely valuable, and concern novel and visionary problems like knowledge mobilization or information retrieval based on ontologies,
- as far as I know, he had actively participated in the Hungarian scientific life because, first, he had always emphasized that he was from Hungary even during his long time stay in Finland, and second, I had a chance to regularly meet him in Hungary during my frequent visits to Hungary, notably at conferences and congresses, even if he was staying at that time in Finland.

What concerns the requirements set by the Section of Mathematics of the Hungarian Academy of Sciences for the field of operations research, I have practically included above my positive responses to all of them. To summarize them, Dr. Robert Fullér's works:

- include new models in a relevant area, namely linear and nonlinear programming, and multiobjective optimization under imprecise information, which constitute an important part of operations research, without doubt one of the most important areas of applied mathematics,
- those new models have been implemented and used to solve real world, non-trivial practical problem in business and industry,
- his research record is very rich and he has a highly respected international stature.

To conclude, I would definitely recommend that the D.Sc. of the Hungarian Academy of Sciences be granted to Dr. Robert Fullér.